On $S$ fuzzy soft sub hemi rings of a hemi ring

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Abstract

In this expose, on endeavors equipped on the way to achieve comprehension of the arithmetical character of $S$-fuzzy soft sub hemi rings of a hemi ring.

Keywords: Fuzzy soft set, $S$ fuzzy soft sub hemi ring, anti-$S$-fuzzy soft sub hemi ring, and pseudo Fuzzy soft co-set.

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1 Introduction

A small amount of researchers done their works in near rings and a few kinds of semi rings contain conventional part. Semi rings emerge in a natural approach in a few applications in the theory of automata and formal languages. Soft set premise as a novel mathematical device means and deals with uncertainty which seems to be gratis from the intrinsic difficulties disturbing the obtainable works. The introduction of fuzzy sets as a result of Zadeh. L.A $[16]$, a few scholars developed fuzzy concepts lying on the impression of the concept of fuzzy sets. Dubois.D and Prade. H $[8]$, were urbanized the concept of fuzzy Sets and Systems: Theory and Applications. Aktas. H, Casman.N $[3]$ were developed by Soft sets and soft groups. In this article, $S$-Fuzzy soft sub hemi ring of a hemi ring is initiated in addition to the theorems in the company of various example.

2 Preliminaries

Definition 2.1. Let $\mathbb{R}$ be a hemi ring. A Fuzzy soft sub set $(H, C)$ of $\mathbb{R}$ is supposed to be a $S$-Fuzzy soft sub hemi ring (SFSHR) of $\mathbb{R}$ if it satisfies the subsequent circumstances:

(i) $\mu_{(H, C)}(a, b) \geq S\{\mu_{(H, C)}(a), \mu_{(H, C)}(b)\}$,

(ii) $\mu_{(H, C)}(ab) \geq S\{\mu_{(H, C)}(a), \mu_{(H, C)}(b)\}$, in favor of each and every one $a$ and $b$ in $\mathbb{R}$.

Definition 2.2. Let $(\mathbb{R}, +, \cdot)$ be a hemi ring. A $S$-Fuzzy soft sub hemi ring $(H, C)$ of $\mathbb{R}$ is said to be an Fuzzy soft normal sub hemi ring (SFSNSHR) of $\mathbb{R}$ if it satisfies the subsequent conditions:

(i) $\mu_{(H, C)}(ab) = \mu_{(H, C)}(ba)$ on behalf of all $a$ and $b$ in $\mathbb{R}$.

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Theorem 3.1. If \((H, C)\) is an S-Fuzzy soft sub hemi ring of a hemi ring \((\mathbb{R}, +, \cdot)\), then \((H, C)\) is an S-Fuzzy soft sub hemi ring of \(\mathbb{R}\).

Proof. Let \((H, C)\) be an \(S\)-fuzzy soft sub hemi ring of a hemi ring \(\mathbb{R}\). With the purpose of \((H, C) = \{ \langle a, \mu(H, C)(a) \rangle \},\) in favor of every one \(a \in \mathbb{R}\). Let \((H, C) = (H, D) = \{ \langle a, \mu(H, D)(a) \rangle \},\) designed for the entire \(a\) along with \(b \in \mathbb{R}\). In view of the fact that \((H, B)\) is an \(S\)-fuzzy soft sub hemi ring of \(\mathbb{R}\), which implies to facilitate \(1 - \mu(H, D)(ab) \leq S\{1 - \mu(H, D)(a), 1 - \mu(H, D)(b)\}\), which implies so as to \(\mu(H, D)(ab) \geq 1 - S\{1 - \mu(H, D)(a), 1 - \mu(H, D)(b)\} = S\{\mu(H, D)(a), \mu(H, D)(b)\}\). As a result, \(\mu(H, D)(ab) \geq S\{\mu(H, D)(a), \mu(H, D)(b)\}\), intended for every one of \(a\) furthermore \(b \in \mathbb{R}\). Consequently \((H, D) = (H, C)\) is an \(S\)-fuzzy soft sub hemi ring of a hemi ring \(\mathbb{R}\).

Theorem 3.2. If \((H, C)\) is an \(S\)-fuzzy soft sub hemi ring of a hemi ring \((\mathbb{R}, +, \cdot)\), then \((H, C)\) is an \(S\)-fuzzy soft sub hemi ring of \(\mathbb{R}\).

Proof. Let \((H, C)\) be an \(S\)-fuzzy soft sub hemi ring of a hemi ring \(\mathbb{R}\). With the purpose of \((H, C) = \{ \langle a, \mu(H, C)(a) \rangle \},\) in favor of every one \(a \in \mathbb{R}\). Let \((H, C) = (H, D) = \{ \langle a, \mu(H, D)(a) \rangle \},\) designed for the entire \(a\) along with \(b \in \mathbb{R}\). In view of the fact that \((H, B)\) is an \(S\)-fuzzy soft sub hemi ring of \(\mathbb{R}\), which implies to facilitate \(1 - \mu(H, D)(ab) \leq S\{1 - \mu(H, D)(a), 1 - \mu(H, D)(b)\}\), which implies so as to \(\mu(H, D)(ab) \geq 1 - S\{1 - \mu(H, D)(a), 1 - \mu(H, D)(b)\} = S\{\mu(H, D)(a), \mu(H, D)(b)\}\). As a result, \(\mu(H, D)(ab) \geq S\{\mu(H, D)(a), \mu(H, D)(b)\}\), intended for every one of \(a\) furthermore \(b \in \mathbb{R}\). Consequently \((H, D) = (H, C)\) is an \(S\)-fuzzy soft sub hemi ring of a hemi ring \(\mathbb{R}\).

Theorem 3.3. Accede to \((\mathbb{R}, +, \cdot)\) survive a hemi ring and \((H, C)\) be present a non unfilled subset of \(\mathbb{R}\). Then \((H, C)\) is a sub hemi ring of \(\mathbb{R}\) merely if \((H, D) = \langle \chi(H, C), \chi(H, C) \rangle\) is a \(S\)-fuzzy soft sub hemi ring of \(\mathbb{R}\), where \(\chi(H, C)\) is the characteristic function.

Proof. Let \((\mathbb{R}, +, \cdot)\) be a hemi ring in addition to \((H, C)\) be a unbalance subset of \(\mathbb{R}\). Primary agree to \((H, C)\) be a sub hemi ring of \(\mathbb{R}\). Obtain \(a\) with \(b \in \mathbb{R}\).

**Case (i):** Condition \(a\) furthermore \(b \in (H, C)\) afterward \(a + b, ab\) inside \((H, C)\), given that \((H, C)\) is a sub hemi ring of \(\mathbb{R}\), \(\chi(H, C)(a) = \chi(H, C)(b) = \chi(H, C)(a + b) = \chi(H, C)(ab) = 1\) with \(\chi(H, C)(a) = \chi(H, C)(b) = \chi(H, C)(a + b) = \chi(H, C)(ab) = 0\). As a result, \(\chi(H, C)(a + b) \geq S\{\chi(H, C)(a), \chi(H, C)(b)\}\), meant for every one of \(a\) also \(b \in \mathbb{R}\), \(\chi(H, C)(a + b) \geq S\{\chi(H, C)(a), \chi(H, C)(b)\}\), behalf of all \(a\) along with \(b \in \mathbb{R}\). Subsequently, \(\chi(H, C)(a + b) \leq S\{\chi(H, C)(a), \chi(H, C)(b)\}\), in
favor of every part of $a$ in addition to $b$ into $\mathbb{R}$, $\chi_{(H,C)}(ab) \leq \{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, intended for each and every one $a$ as well as $b$ within $\mathbb{R}$.

Case (ii) Either $a$ or $b$ in $(H,C)$, then $a + b, ab$ may or may not be in $(H,C)$, $\chi_{(H,C)}(a) = 1, \chi_{(H,C)}(b) = 0$ (or) $\chi_{(H,C)}(a) = 0, \chi_{(H,C)}(b) = 1$, $\chi_{(H,C)}(a + b) = 1, \chi_{(H,C)}(ab) = 1$ (or 0) and $\chi_{(H,C)}(a) = 0, \chi_{(H,C)}(b) = 1$ (or $\chi_{(H,C)}(a) = 1, \chi_{(H,C)}(b) = 0$). $\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 1$ (or 1). Obvously $\chi_{(H,C)}(a + b) \geq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all that $a$ along with $b$ in $\mathbb{R}, \chi_{(H,C)}(ab) \geq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, intended for every $a$ and $b$ in $\mathbb{R}$, and $\chi_{(H,C)}(a + b) \leq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all $a$ and $b$ in $\mathbb{R}$.

Case (iii) If $a$ and $b$ somewhere else $(H,C)$, at that time $a + b, ab$ may well otherwise may not in $(H,C), \chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 0$, $\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 1$ or $0$ and $\chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 1, \chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 0$ or $1$. Evidently $\chi_{(H,C)}(a + b) \geq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all $a$ and $b$ in $\mathbb{R}, \chi_{(H,C)}(ab) \geq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all $a$ and $b$ in $\mathbb{R}$ and $\chi_{(H,C)}(a + b) \leq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all $a$ and $b$ in $\mathbb{R}$. Subsequently in above conditions we comprise $B$ is a fuzzy soft sub hemi ring of $(H,C)$ hemi ring $R$. On the contrary, Accede to $a$ and $b$ in $(H,C)$, In view of the fact that $(H,C)$ is non blank subset of $\mathbb{R}$, thus $\chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 1, \chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 0$.

While $B = \{x(H,C), \nabla(H,C)\}$ is a fuzzy soft sub hemi ring of $\mathbb{R}$, we have $\chi_{(H,C)}(a + b) \geq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = S\{1, 1\} = 1, \chi_{(H,C)}(ab) \geq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = S\{1, 1\} = 1$. For that reason $\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 1$. and $\chi_{(H,C)}(a + b) \leq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = \max\{0, 0\} = 0, \chi_{(H,C)}(ab) \leq S\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = \max\{0, 0\} = 0$. Therefore $\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 1$. Thus $a + b$ as well as $ab$ in $(H,C)$, as a result $(H,C)$ is a sub hemi ring of $\mathbb{R}$.

Theorem 3.4. Let $(H,C)$ be an $S$-fuzzy soft sub hemi ring of a hemi ring $H$ and $f$ is an isomorphism from a hemi ring $\mathbb{R}$ onto $H$. Then $(H,C) \circ f$ is an $S$- fuzzy soft sub hemi ring of $\mathbb{R}$.

Proof. Consent to $a$ and $b$ in $\mathbb{R}$ as well as $(H,C)$ be a fuzzy soft sub hemi ring of a hemi ring $H$. Subsequently, it is encompassed, $(\mu_{(H,C)} \circ f)(a + b) = \mu_{(H,C)}f(a + b) = \mu_{(H,C)}\{f(a) + f(b)\}$, as $f$ is an isomorphism $\geq S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\} = S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$, which implies so to $(\mu_{(H,C)} \circ f)(a + b) \geq S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$, which also implies to $\mu_{(H,C)}f(a + b) = \mu_{(H,C)}\{f(a) + f(b)\} = \mu_{(H,C)}\{f(a)\} + \mu_{(H,C)}\{f(b)\}$, as $f$ is an isomorphism $\geq S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\} = S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$, which implies that $(\mu_{(H,C)} \circ f)(ab) \geq S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$. For that reason $(H,C) \circ f$ is a $S$- Fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$.

Theorem 3.5. Let $(H,C)$ be an $S$-fuzzy soft sub hemi ring of a hemi ring $h$ and $f$ is an anti-isomorphism from a hemi ring $\mathbb{R}$ onto $H$. Then $(H,C) \circ f$ is a $S$-fuzzy soft sub hemi ring of $\mathbb{R}$.

Proof. Accede to $a$ and $b$ in $\mathbb{R}$ in addition to $(H,C)$ be an $S$-fuzzy soft sub hemi ring of a hemi ring $H$. Afterward we have $(\mu_{(H,C)} \circ f)(a + b) = \mu_{(H,C)}(f(a + b)) = \mu_{(H,C)}(f(b) + (f(a))$, as $f$ is an anti-isomorphism $\geq \min\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\} = S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$, which implies to facilitate $(\mu_{(H,C)} \circ f)(a + b) \geq S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$. In addition to, $(\mu_{(H,C)} \circ f)(ab) = \mu_{(H,C)} \circ f(ab) = \mu_{(H,C)} \circ (f(b)f(a))$, as $f$ is an anti-isomorphism $\geq S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\} = S\{\mu_{(H,C)}f(a), \mu_{(H,C)}f(b)\}$, which implies to $(\mu_{(H,C)} \circ f)(ab) \geq \{((\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b))\}$. Thus $(G, B) \circ f$ is an S-fuzzy soft sub hemi ring of the hemi ring $\mathbb{R}$.

Theorem 3.6. Let $(H,C)$ be an $S$-fuzzy soft sub hemi ring of a hemi ring $(R, +, \cdot)$, then the pseudo fuzzy soft co-set $(x(H,C))^p$ is an $S$-fuzzy soft hemi ring of a hemi ring $\mathbb{R}$, for every $x$ in $\mathbb{R}$.
Proof. Consent to \((H, C)\) be an \(S\)-fuzzy soft sub hemi ring of a hemi ring \(R\). In favor of each \(a\) and \(b\) in \(R\), we have
\[
((x\mu_{(H,C)})^p(a+b) = p(x)\mu_{(H,C)}(a+b) \geq p(x)S\{(\mu_{(H,C)}(a), (\mu_{(H,C)}(b))
= S\{(p(x)\mu_{(H,C)}(a), (\mu_{(H,C)}(b))
= S\{(x\mu_{(H,C)})^p(a), ((x\mu_{(H,C)})^p(b)).
\]
As a result, \(((x\mu_{(H,C)})^p(a+b) \geq S\{(x\mu_{(H,C)})^p(a), (x\mu_{(H,C)})^p(b)).\) At this istant
\[
((x\mu_{(H,C)})^p(ab) = p(x)\mu_{(H,C)}(ab) \geq p(x)S\{(\mu_{(H,C)}(a), (\mu_{(H,C)}(b))
= S\{(p(x)\mu_{(H,C)}(a), p(x)\mu_{(H,C)}(b))
= S\{((x\mu_{(H,C)})^p(a), ((x\mu_{(H,C)})^p(b)).
\]
Consequently, \(((x\mu_{(H,C)})^p(ab) \geq S\{((x\mu_{(H,C)})^p(a), (x\mu_{(H,C)})^p(b)).\) From now \((x(H,C))^p\) is an \(S\)-fuzzy soft sub hemi ring of a hemi ring \(R\).

4 Conclusion

In the current work, a novel concept of \(S\)-fuzzy soft sub hemi ring of a hemi ring which are defined with some properties and related theorems are studied.

References


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