Intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring

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Abstract

In this paper, a generalized intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring is proposed. Further, some important notions and basic algebraic properties of intuitionistic fuzzy sets are discussed.

Keywords: $Q$-fuzzy subhemiring, $Q$-fuzzy ternary subhemiring, intuitionistic fuzzy ternary subhemiring, homomorphism, anti-homomorphism.

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1 Introduction

An algebra $(R; +; \cdot)$ is said to be a semiring if $(R; +)$ and $(R; \cdot)$ are semigroups satisfying $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all $a, b$ and $c$ in $R$. A Semiring $R$ is said to be additively commutative if $a + b = b + a$ for all $a$ and $b$ in $R$. Ternary rings are introduced by Lister [9]. And he investigated some of their properties and radical theory of such rings. A Semiring $R$ may have an identity 1, defined by $1 \cdot a = a = a \cdot 1$ and a zero 0, defined by $0 + a = a = a + 0$ and $a \cdot 0 = 0 = 0 \cdot a$ for all $a$ in $R$. Ternary semirings arise naturally as follows-consider the ring of integers $\mathbb{Z}$ which plays a vital role in the theory of ring. The concept of intuitionistic fuzzy subsets (IFS) was presented by K.T.Atanassov [5], as a generalization of the notion of fuzzy set. Solairaju.A and R.Nagarajan, have given a new structure in construction of $Q$-fuzzy groups [14]. Also Giri.R.D and Chide.B.R [8], given the structure of Prime Radical in Ternary Hemiring. In this paper, we introduce some properties and theorems in intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring.

2 Preliminaries

**Definition 2.1.** Let $X$ be a non-empty set and $Q$ be a non-empty set. A $Q$-fuzzy subset $A$ of $X$ is function $A : X \times Q \to [0, 1]$.

**Definition 2.2.** Let $R$ be a hemiring. A fuzzy subset $A$ of $R$ is said to be a $Q$-fuzzy ternary subhemiring (FTSHR) of $R$ if it satisfies the following conditions:

(i) $A(x + y, q) \geq \min\{A(x, q), A(y, q)\}$,

(ii) $A(xyz, q) \geq \min\{A(x, q), A(y, q), A(z, q)\}$, for all $x, y$ and $z$ in $R$ and $q$ in $Q$.

**Definition 2.3.** Let $R$ be a hemiring. A $Q$-fuzzy subset $A$ of $R$ is said to be an anti $Q$-fuzzy subhemiring (AFTSHR) of $R$ if it satisfies the following conditions:

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Theorem 3.3. If \( A \) is an intuitionistic \( Q \)-fuzzy ternary subhemiring of a hemiring \( R \), then for all \( x, y, z \in R \) and \( q \in Q \),

(i) \( A(x + y, q) \leq \max\{A(x, q), A(y, q)\} \),
(ii) \( A(xyz, q) \leq \max\{A(x, q), A(y, q), A(z, q)\} \).

Definition 2.4. An intuitionistic fuzzy subset (IFS) \( A \) in \( X \) is defined as an object of the form \( A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \), where \( \mu_A : X \to [0, 1] \) and \( \nu_A : X \to [0, 1] \) define the degree of membership and the degree of non-membership of the element \( x \in X \) respectively and for every \( x \in X \) satisfying \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \).

Definition 2.5. Let \( R \) be a hemiring. An intuitionistic \( Q \)-fuzzy subterary subhemiring (IFTSHR) of \( R \) if it satisfies the following conditions:

(i) \( \mu_A(x + y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\} \),
(ii) \( \nu_A(x + y, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\} \),
(iii) \( \mu_A(xyz, q) \geq \min\{\mu_A(x, q), \mu_A(y, q), \mu_A(z, q)\} \),
(iv) \( \nu_A(xyz, q) \leq \max\{\nu_A(x, q), \nu_A(y, q), \nu_A(z, q)\} \), for all \( x, y, z \in R \) and \( q \in Q \).

Definition 2.7. Let \( (R, +, \cdot) \) and \( (R', +, \cdot) \) be any two hemirings. Then the function \( f : R \to R' \) is called a homomorphism if \( f(x + y, q) = f(x, q) + f(y, q) \) and \( f(xyz, q) = f(x, q)f(y, q)f(z, q) \), for all \( x, y, z \in R \) and \( q \in Q \).

Definition 2.8. Let \( (R, +, \cdot) \) and \( (R', +, \cdot) \) be any two hemirings. Then the function \( f : R \to R' \) is called an isomorphism if \( f \) is bijection.

Definition 2.9. Let \( (R, +, \cdot) \) and \( (R', +, \cdot) \) be any two hemirings. Then the function \( f : R \to R' \) is called an anti-isomorphism if \( f \) is bijection.

3 Some properties of intuitionistic \( Q \)-fuzzy ternary subhemiring of a hemiring

Theorem 3.1. If \( A \) is an intuitionistic \( Q \)-fuzzy ternary subhemiring of a hemiring \( (R, +, \cdot) \), then \( H = \{ (x, q) \mid x \in R : \mu_A(x, q) = 1, \nu_A(x, q) = 0 \} \) is either empty or is a ternary subhemiring of \( R \).

Proof. If none of the elements satisfies this condition, then \( H \) is empty. If \( (x, q) \) and \( (y, q) \) in \( H \), then \( \mu_A(x + y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\} = \min\{1, 1\} = 1 \). Therefore \( \mu_A(x + y, q) = 1 \), for all \( (x, q) \) and \( (y, q) \) in \( H \). And \( \mu_A(xyz, q) \geq \min\{\mu_A(x, q), \mu_A(y, q), \mu_A(z, q)\} = \min\{1, 1, 1\} = 1 \). Therefore \( \mu_A(xyz, q) = 1 \), for all \( (x, q), (y, q) \) and \( (z, q) \) in \( H \). And \( \nu_A(x + y, q) \leq \max\{\nu_A(x, q), \nu_A(y, q)\} = \max\{0, 0\} = 0 \). Therefore \( \nu_A(x + y, q) = 0 \), for all \( (x, q) \) and \( (y, q) \) in \( H \). And \( \nu_A(xyz, q) \leq \max\{\nu_A(x, q), \nu_A(y, q), \nu_A(z, q)\} = \max\{0, 0, 0\} = 0 \). Therefore \( \nu_A(xyz, q) = 0 \), for all \( (x, q), (y, q) \) and \( (z, q) \) in \( H \). Therefore \( H \) is a ternary subhemiring of \( R \). Hence \( H \) is either empty or is a ternary subhemiring of \( R \).

Theorem 3.2. If \( A \) is an intuitionistic \( Q \)-fuzzy ternary subhemiring of a hemiring \( (R, +, \cdot) \), then \( H = \{ (x, q) \mid \mu_A(x, q) \leq 1 \text{ and } \nu_A(x, q) = 0 \} \) is either empty or is a \( Q \)-fuzzy ternary subhemiring of \( R \).

Proof. By using Theorem 3.1.

Theorem 3.3. If \( A \) is an intuitionistic \( Q \)-fuzzy ternary subhemiring of a hemiring \( (R, +, \cdot) \), then \( H = \{ (x, q) \mid \mu_A(x, q) \leq 1 \} \) is either empty or is a \( Q \)-fuzzy ternary subhemiring of \( R \).

Proof. By using Theorem 3.2.
Theorem 3.4. If $A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $(R,+)$, then $A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $R$.

Proof. Let $A$ be an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $R$. Consider $A = \{\{(x,q),\mu_A(x,q)\};\}$ for all $x \in R$. $A = B = \{(x,q)\}$, where $\mu_B(x,q) = \mu_A(x,q) = 1 - \mu_A(x,q)$. Clearly, $\mu_B(x+y,q) \geq \min\{\mu_B(x,q),\mu_B(y,q)\}$ and $\mu_B(xyz,q) \geq \min\{\mu_B(x,q),\mu_B(y,q),\mu_B(z,q)\}$. Since $A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $R$, we have, $\mu_A(x+y,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$ for all $x$ and $y$ in $R$, which implies that $1 - \nu_B(x+y,q) \geq \min\{(1 - \nu_B(x,q)),(1 - \nu_B(y,q))\}$ which implies that $\nu_B(x+y,q) \leq 1 - \min\{1 - \nu_B(x,q),(1 - \nu_B(y,q))\} = \max\{\nu_B(x,q),\nu_B(y,q)\}$. Therefore, $\nu_B(x+y,q) \leq \max\{\nu_B(x,q),\nu_B(y,q),\nu_B(z,q)\}$ for all $x$ and $y$ in $R$ and $q$ in $Q$. And $\nu_A(xyz,q) \geq \min\{\nu_A(x,q),\nu_A(y,q)\}$ which implies that $1 - \nu_B(xyz,q) \geq \min\{(1 - \nu_B(x,q)),(1 - \nu_B(y,q)),(1 - \nu_B(z,q))\} \neq \max\{\nu_B(x,q),\nu_B(y,q),\nu_B(z,q)\}$. Therefore $\nu_B(xyz,q) \leq \max\{\nu_B(x,q),\nu_B(y,q),\nu_B(z,q)\}$ for all $x, y$ and $z$ in $R$. Hence $B = A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $R$.

Remark 3.1. The converse of the above theorem is not true. It is shown by the following example: Consider the hemiring $Z_5 = \{0,1,2,3,4\}$ with addition modulo and multiplicative modulo operation and $Q = \{p\}$. Then $A = \{(0,0.7,0.2),\mu_A(x,q)\}$, $(1,0.5,0.1),\mu_A(x,q)$, $(2,0.5,0.4),\mu_A(x,q)$, $(3,0,5,0.1),\mu_A(x,q)$, $(4,0.5,0.4),\mu_A(x,q)$ is not an intuitionistic $Q$-fuzzy ternary subhemiring of $Z_5$, but $A = \{(0,0.7,0.3),\mu_A(x,q)\}$, $(1,0.5,0.5),\mu_A(x,q)$, $(2,0,5,0.5),\mu_A(x,q)$, $(3,0,5,0.5),\mu_A(x,q)$, $(4,0,5,0.5),\mu_A(x,q)$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $Z_5$.

Theorem 3.5. If $A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $(R,+)$, then $A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $R$.

Proof. Let $A$ be an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $R$. That is $A = \{(x,q),\mu_A(x,q)\}$, for all $x \in R$ and $q \in Q$. Let $A = B = \{(x,q),\mu_B(x,q),\nu_B(x,q)\}$, where $\mu_B(x,q) = 1 - \nu_B(x,q), \nu_B(x,q) = \nu_A(x,q)$. Clearly, $\nu_B(x+y,q) \leq \max\{\nu_B(x,q),\nu_B(y,q)\}$ and $\nu_B(xyz,q) \leq \max\{\nu_B(x,q),\nu_B(y,q),\nu_B(z,q)\}$ for all $x,y$ and $z$ in $R$. Since $A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $R$, we have, $\nu_A(x+y,q) \leq \max\{\nu_A(x,q),\nu_A(y,q)\}$ for all $x$ and $y$ in $R$, which implies that $1 - \nu_B(x+y,q) \leq \max\{(1 - \nu_B(x,q))\},(1 - \nu_B(y,q))\}$ which implies that $\nu_B(x+y,q) \geq 1 - \min\{(1 - \nu_B(x,q))\},(1 - \nu_B(y,q))\} = \min\{\nu_B(x,q),\nu_B(y,q)\}$, Therefore, $\nu_B(x+y,q) \geq \min\{\nu_B(x,q),\nu_B(y,q)\}$ for all $x$ and $y$ in $R$ and $q$ in $Q$. And $\nu_A(xyz,q) \leq \max\{\nu_A(x,q),\nu_A(y,q),\nu_A(z,q)\}$ which implies that $1 - \nu_B(xyz,q) \leq \max\{(1 - \nu_B(x,q)),(1 - \nu_B(y,q)),(1 - \nu_B(z,q))\}$ which implies that $\nu_B(xyz,q) \geq 1 - \min\{(1 - \nu_B(x,q)),(1 - \nu_B(y,q)),(1 - \nu_B(z,q))\} \neq \min\{\nu_B(x,q),\nu_B(y,q),\nu_B(z,q)\}$, Therefore $\nu_B(xyz,q) \geq \min\{\nu_B(x,q),\nu_B(y,q),\nu_B(z,q)\}$, for all $x,y$ and $z$ in $R$. Hence $B = A$ is an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $R$.

Remark 3.2. The converse of the above theorem is not true. It is shown by the following example: Consider the hemiring $Z_5 = \{0,1,2,3,4\}$ with addition modulo and multiplicative modulo operation and $Q = \{p\}$. Then $A = \{(0,0.5,0.1,\mu_A(x,q)\},(1,0,6,0.4),\mu_A(x,q),\nu_B(x,q)\}$, $(2,0,5,0.4),\mu_A(x,q)$, $(3,0,6,0.4),\mu_A(x,q)$, $(4,0,5,0.4),\mu_A(x,q)$ is not an intuitionistic $Q$-fuzzy ternary subhemiring of $Z_5$, but $A = \{(0,0.9,0.1),\mu_A(x,q)\},(1,0,6,0.4),\mu_A(x,q),\nu_B(x,q)\}$, $(2,0,6,0.4),\mu_A(x,q),\nu_B(x,q)\}$, $(3,0,6,0.4),\mu_A(x,q),\nu_B(x,q)\}$, $(4,0,6,0.4),\mu_A(x,q),\nu_B(x,q)\}$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $Z_5$.

In The Following Theorem ô Is The Composition Operation of Functions:

Theorem 3.6. Let $A$ be an intuitionistic $Q$-fuzzy ternary subhemiring of a hemiring $H$ and $f$ is an isomorphism from a hemiring $R$ onto $H$. Then $A \circ f$ is an intuitionistic $Q$-fuzzy ternary subhemiring of $R$.

Proof. Let $x$ and $y$ in $R$ and $A$ be an intuitionistic $Q$-fuzzy ternary subhemiring of $H$. Then we have $\mu_A(f(x,y,q)) = \mu_A(f(x,q)) + \mu_A(f(y,q)) \geq \min\{\mu_A(f(x,q)),\mu_A(f(y,q))\}$ (as $A$ is an IFTSHR of $H$) $\geq \min\{\mu_A(f(x,q)),\mu_A(f(y,q))\}$ which implies that $\mu_A(f(x+y,q)) \geq \min\{\mu_A(f(x,q)),\mu_A(f(y,q))\}$, for all $x$ and $y$ in $R$ and $q$ in $Q$. And $\mu_A(f(xyz,q)) = \mu_A(f(xy,q)) = \mu_A(f(x,q)) \mu_A(f(y,q))$, as $f$ is an isomorphism $\geq \min\{\mu_A(f(x,q)),\mu_A(f(y,q)),\mu_A(f(z,q))\}$, as $A$ is an IFTSHR of $H$ $\geq \min\{\mu_A(f(x,q)),\mu_A(f(y,q))\}$ which implies that $\mu_A(f(xyz,q)) \geq \min\{\mu_A(f(x,q)),\mu_A(f(y,q))\}$.
\(f(x, y), (\mu \circ f)(y, q), (\mu \circ f)(z, q)\), for all \(x, y, z\) in \(R\). We have \((\nu \circ f)(x + y, q) = \nu_A(f(x + y, q)) = \nu_A(f(y, q) + f(x, q))\), as \(f\) is an isomorphism \(\leq \) \(\max\ \{\nu_A(f(x, q)), \nu_A(f(y, q))\} \leq \max\ \{\nu_A \circ f(x, q), (\nu_A \circ f)(y, q)\}\) which implies that \((\nu \circ f)(x + y, q) \leq \max\ \{\nu_A \circ f(x, q), (\nu_A \circ f)(y, q)\}\) for all \(x, y\) in \(R\). And \((\nu \circ f)(xyz, q) = \nu_A(f(xyz, q)) = \nu_A(f(x, q)f(y, q)f(z, q))\), as \(f\) is an isomorphism \(\leq \max\ \{\nu_A(f(x, q)), \nu_A(f(y, q)), \nu_A(f(z, q))\}\) which implies that \((\nu \circ f)(xyz, q) \leq \max\ \{\nu_A \circ f(x, q), (\nu_A \circ f)(y, q), (\nu_A \circ f)(z, q)\}\) for all \(x, y, z\) in \(R\). Therefore \((\nu \circ f)\) is an intuitionistic \(Q\)-fuzzy ternary subhemiring of a hemiring \(R\).

**Theorem 3.7.** Let \(A\) be an intuitionistic \(Q\)-fuzzy ternary subhemiring of a hemiring \(H\) and \(f\) is an anti-isomorphism from a hemiring \(H\) onto \(H\). Then \(A \circ f\) is an intuitionistic \(Q\)-fuzzy ternary subhemiring of \(R\).

**Proof.** Let \(x\) and \(y\) in \(R\) and \(A\) be an intuitionistic \(Q\)-fuzzy ternary subhemiring \(H\). Then we have \((\mu \circ f)(x + y, q) = \mu_A(f(x + y, q)) = \mu_A(f(y, q) + f(x, q))\), as \(f\) is an anti-homomorphism \(\geq \) \(\min\ \{\mu_A(f(y, q)), \mu_A(f(x, q))\}\) as \(A\) is an IFTSHR of \(H\). \(\geq \) \(\min\ \{\mu_A \circ f(x, q), (\mu_A \circ f)(y, q)\}\) which implies that \((\mu \circ f)(x + y, q) \geq \min\ \{\mu_A \circ f(x, q), (\mu_A \circ f)(y, q)\}\) for all \(x, y\) in \(R\).

And \((\mu \circ f)(xyz, q) = \mu_A(f(xyz, q)) = \mu_A(f(x, q)f(y, q)f(z, q))\), as \(f\) is an anti-isomorphism \(\geq \) \(\min\ \{\mu_A \circ f(x, q), (\mu_A \circ f)(y, q), (\mu_A \circ f)(z, q)\}\) which implies that \((\mu \circ f)(xyz, q) \geq \min\ \{\mu_A \circ f(x, q), (\mu_A \circ f)(y, q), (\mu_A \circ f)(z, q)\}\) for all \(x, y, z\) in \(R\). We have \((\nu \circ f)(x + y, q) = \nu_A(f(x + y, q)) = \nu_A(f(y, q) + f(x, q))\), as \(f\) is an anti-isomorphism \(\leq \) \(\max\ \{\nu_A(f(x, q)), \nu_A(f(y, q))\}\) which implies that \((\nu \circ f)(x + y, q) \leq \max\ \{\nu_A(f(x, q)), \nu_A(f(y, q))\}\) for all \(x, y\) in \(R\). And \((\nu \circ f)(xyz, q) = \nu_A(f(xyz, q)) = \nu_A(f(x, q)f(y, q)f(z, q))\), as \(f\) is an anti-isomorphism \(\leq \) \(\max\ \{\nu_A(f(x, q)), \nu_A(f(y, q)), \nu_A(f(z, q))\}\) which implies that \((\nu \circ f)(xyz, q) \leq \max\ \{\nu_A \circ f(x, q), (\nu_A \circ f)(y, q), (\nu_A \circ f)(z, q)\}\) for all \(x, y, z\) in \(R\) and \(q\) in \(Q\). Therefore \((\nu \circ f)\) is an intuitionistic \(Q\)-fuzzy ternary subhemiring of a hemiring \(R\).

**References**


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