Malaya J. Mat. S(2)(2015) 348–354

Malaya Journal of Matematik

MJM
an international journal of mathematical sciences with computer applications...

www.malayajournal.org

ISSN : 2319-3786

Interval Valued Intuitionistic Triangular Hesitant Fuzzy Focal Elements And Their Corresponding Basic Probability Assignments Using Four New Operators

R. Irene Hepzibah[a], A. Nagoorgani[b] and G. Geethalakshmi[c,*]

[a] Department of Mathematics, AVC College of Engineering, Mayiladuthurai, Tamil Nadu, India.

[b] PG & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Trichy-20, Tamil Nadu, India .

[c] Department of Mathematics, K.S.K College of Engineering and Technology, Kumbakonam, Tamil Nadu, India.

Abstract

Hesitant fuzzy sets, permitting the membership of an element to be a set of several possible values can be used as an efficient mathematical tool for modeling peoples hesitancy in day to day life. The aim of this paper is to present the notion of interval-valued intuitionistic hesitant fuzzy set, which extends the hesitant fuzzy set to interval valued intuitionistic fuzzy environments and permits the membership of an element to be a set of several possible interval-valued intuitionistic fuzzy numbers and its application in Evidence Theory. In this paper, the Interval Valued Intuitionistic Hesitant Fuzzy Focal Elements (IVIHFFE) is extended to Interval Valued Intuitionistic Triangular Hesitant Fuzzy Focal Elements (IVITHFFE) and their corresponding Basic Probability Assignments (BPA) in Evidence theory. A series of aggregation operators for Interval Valued Intuitionistic Triangular Hesitant Fuzzy Focal Elements (IVITHFFE) are developed and the results with Interval Valued Intuitionistic Hesitant Fuzzy Focal Elements (IVIHFFE) are compared in this research work. Finally, a numerical example is provided to illustrate the application of the developed approach.

Keywords: Interval Valued Intuitionistic Hesitant Fuzzy Focal Elements (IVIHFFE), Interval Valued Intuitionistic Triangular Hesitant Fuzzy Focal Elements (IVITHFFE), Modified operators, Basic Probability Assignments (BPA).

1 Introduction

Hesitant Fuzzy Information Collection based on several types of sets such as Fuzzy sets, Fuzzy multi sets, Intuitionistic fuzzy sets, Interval Valued sets, Interval Valued Intuitionistic fuzzy sets, Type-2 Fuzzy sets, Type-n fuzzy sets. Hesitant fuzzy sets, in particular, Interval Valued Intuitionistic Hesitant fuzzy set, is a very useful tool to deal with the situations in which the experts hesitate between several possible interval-valued intuitionistic fuzzy numbers to assess the degree to which an alternative satisfies an attribute. Some set theoretic operations such as Union, intersection and complement on hesitant fuzzy sets have also been proposed by Torra [5]. Xia and Xu [6] made an investigation of hesitant fuzzy information techniques and their applications in decision Making. Zhiming Zhang [7] have recently proposed the concept of interval

*Corresponding author.

E-mail address: ireneraj74@gmail.com (R. Irene Hepzibah), ganijmc@yahoo.co.in (A. Nagoorgani), geethavish294@gmail.com (G. Geethalakshmi) .

Valued Intuitionistic triangular Hesitant Fuzzy sets, some basic properties and improved the operators for interval Valued Intuitionistic triangular Hesitant Fuzzy sets to solve multi-attribute group decision making. Dempster-Shafter Theory (DST) is a branch of mathematics that concerns Combination of empirical evidence in an alternative to traditional representation of uncertainty. In DST, the information given by sensors, observes or experts can be described by the focal elements on a frame of discernment and the corresponding Basic Probability Assignments (BPA).The determination of BPA is an important problem in the multi source information fusion. In this paper, the Interval Valued Intuitionistic Hesitant Fuzzy Focal Elements (IVIHFFE) is extended to Interval Valued Intuitionistic Triangular Hesitant Fuzzy Focal Elements (IVITHFFE) and their corresponding Basic Probability Assignments (BPA) in Evidence theory. The aim of this paper is to develop a series of aggregation operators for Interval Valued Intuitionistic Triangular Hesitant Fuzzy Focal Elements (IVITHFFE) and to compare the results with Interval Valued Intuitionistic Hesitant Fuzzy Focal Elements (IVIHFFE).

2 Preliminaries

2.1 Dempster-Shafter Theory (DST) [3]

Some basic concepts of D-S theory are briefly introduced as follows

2.1.1 Frame of Discernment

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ be a set called frame of discernment, if it contains mutually exclusive and exhaustive events.

2.1.2 Basic Probability Assignment (BPA)

A function $m : 2^{\Theta} \to [0,1]$ is called a Basic Probability Assignment(BPA) on Θ if it satisfies the following three properties (i) $m(\Theta) = 0$ (ii) $m(A) \geq 0$ (iii) $\sum\limits_{A \subseteq \Theta} m(A) = 1$, $A \in 2^{\Theta}$ is called a focal element of $'m'$ satisfies $m(A) \geq 0$.

2.1.3 Belief Function

From the BPA, a function $Bel(A) : 2^{\Theta} \to [0,1]$ is defined as $Bel(A) = \sum\limits_{B \subseteq A} m(B)$.

2.1.4 Plausibility Function

From the BPA, a function $Pls(A) : 2^{\Theta} \to [0,1]$ is defined as $Pls(A) = \sum\limits_{B \cap A \neq \phi} m(B)$.

2.1.5 Dempster Rule of Combination

Let m_1 and m_2 be two mass functions defined on the same frame of discernment, Θ and then a combined BPA can be obtained by using Dempster's combination rule, the combined BPA is defined as follows

$$m = \begin{cases} \dfrac{\sum\limits_{B \cap = A} m_1(B)m_2(C)}{1 - \sum\limits_{B \cap C} m_1(B)m_2(C)} & , \forall \phi \subseteq \Theta \\ 0 & \text{, otehrwise} \end{cases}$$

2.2 Interval Valued Intuitioninistic Hesitant Fuzzy Sets [3]

Definition 2.2.1. *[1] Given a fixed set $X = \{x_1, x_2, \ldots, x_n\}$, an intuitionistic fuzzy set (IFS) is defined as $\tilde{A}^I = (\langle x_i, \mu_{\tilde{A}^I}(x_i), \nu_{\tilde{A}^I}(x_i)\rangle / x_i \in X)$ which assigns to each element x_i, a membership degree $\mu_{\tilde{A}^I}(x_i)$ and a non-membership degree $\nu_{\tilde{A}^I}(x_i)$ under the condition $0 \leq \mu_{\tilde{A}^I}(x_i) + \nu_{\tilde{A}^I}(x_i) \leq 1$, for all $x_i \in X$.*

Definition 2.2.2. *[4] Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$ and $X(=\phi)$ be a given set. An IFS A in X is defined as $\tilde{A}^I = (\langle x_i, \mu_{\tilde{A}^I}(x_i), \nu_{\tilde{A}^I}(x_i)\rangle / x_i \in X)$, where $\mu_{\tilde{A}^I} : X \to D[0,1]$, $\nu_{\tilde{A}^I} : X \to D[0,1]$ with the condition $0 \leq \sup(\mu_{\tilde{A}^I}(x_i)) + \sup(\nu_{\tilde{A}^I}(x_i)) \leq 1$ for any $x \in X$.*

Definition 2.2.3. *[8] A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$*

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & , a_1 \leq x \leq a_2 \\ \dfrac{x-a_3}{a_2-a_3} & , a_2 \leq x \leq a_3 \\ 0 & , otherwise \end{cases} \quad and \quad \nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{a_2-x}{a_2-a_1} & , a_1' \leq x \leq a_2 \\ \dfrac{x-a_2}{a_3-a_2} & , a_2 \leq x \leq a_3' \\ 1 & , otherwise \end{cases}$$

where and $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$ or $\mu_{\tilde{A}^I}(x) = \nu_{\tilde{A}^I}(x)$, for all $x \in R$. This TIFN is denoted by $\tilde{A}^I = \{(a_1, a_2, a_3), (a_1', a_2, a_3')\}$

Definition 2.2.4. *[8] A trapezoidal intuitionistic fuzzy number (TRIFN) $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$*

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & , x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & , a_1 \leq x \leq a_2 \\ 1 & , a_2 \leq x \leq a_3 \\ \dfrac{x-a_4}{a_3-a_4} & , a_3 \leq x \leq a_4 \\ 0 & , a_4 \leq x \end{cases} \quad and \quad \nu_{\tilde{A}^I}(x) = \begin{cases} 0 & , x < a_1' \\ \dfrac{x-a_1'}{a_2'-a_1'} & , a_1' \leq x \leq a_2' \\ 1 & , a_2' \leq x \leq a_3' \\ \dfrac{x-a_4'}{a_3'-a_4'} & , a_3' \leq x \leq a_4' \\ 0 & , a_4' \leq x \end{cases}$$

where and $a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_3' \leq a_4 \leq a_4'$. This TRIFN is denoted by $\tilde{A}^I = \{(a_1, a_2, a_3, a_4), (a_1', a_2', a_3', a_4')\}$

Definition 2.2.5. *[1] A (α, β)- cut set of a intuitionistic fuzzy number is defined as $A^I_{\alpha,\beta}\{x/\mu_{\tilde{A}^I}(x) \geq \alpha, \nu_{\tilde{A}^I}(x) \leq \beta\}$, where $0 \leq \alpha \leq 1$; $0 \leq \beta \leq 1$ and $0 \leq \alpha + \beta + 1$.*

Definition 2.2.6 (Set Operations on intuitionistic Fuzzy Sets (IFS)). *[4] Let IFS (X) denotes the family of IFS defined on the universe X, and let $\alpha, \beta \in$ IFS (X) be given as $\alpha = (\mu_\alpha, \gamma_\alpha)$, $\beta = (\mu_\beta, \gamma_\beta)$ then four set operations are defined as follows:*

(i) $\alpha @ \beta = \left(\dfrac{\mu_\alpha + \mu_\beta}{2}, \dfrac{\gamma_\alpha + \gamma_\beta}{2}\right)$

(ii) $\alpha \$ \beta = \left(\sqrt{\mu_\alpha \mu_\beta}, \sqrt{\gamma_\alpha \gamma_\beta}\right)$

(iii) $\alpha \neq \beta = \left(\dfrac{2\mu_\alpha \mu_\beta}{\mu_\alpha + \mu_\beta}, \dfrac{2\gamma_\alpha \gamma_\beta}{\gamma_\alpha + \gamma_\beta}\right)$

(iv) $\alpha * \beta = \left(\dfrac{\mu_\alpha + \mu_\beta}{2(\mu_\alpha \mu_\beta + 1)}, \dfrac{\gamma_\alpha + \gamma_\beta}{2(\gamma_\alpha \gamma_\beta + 1)}\right)$

Definition 2.2.7 (Internal Valued intuitionistic Fuzzy Sets). *[1] An Interval Valued Intuitionistic Fuzzy Sets (IVIFS) α in the finite universe X is expressed by the form $\alpha = \{\langle x, \mu_\alpha(x), \gamma_\alpha(x)\rangle / x \in X\}$ where $\mu_\alpha(x) = \lfloor \mu_\alpha^L(x), \mu_\alpha^R(x) \rfloor \in [I]$ is called membership interval of element to IVIFS α, where $\gamma_\alpha(x) = \lfloor \gamma_\alpha^L(x), \gamma_\alpha^R(x) \rfloor \in [I]$ is called non-membership interval of that element to IVIFS with the condition $0 \leq \mu_\alpha^R(x) + \gamma_\alpha^R(x) \leq 1$ must hold for any $x \in X$. For convenience, the lower and upper bounds of $\mu_\alpha(x)$ and $\gamma_\alpha(x)$ are denoted by, $\gamma_\alpha^L(x), \gamma_\alpha^R(x)$ respectively. Thus, the IVIFS α may be concisely expressed as $\alpha = (\mu_\alpha, \gamma_\alpha) = \{\langle x, [\mu_\alpha^L, \mu_\alpha^R], [\gamma_\alpha^L, \gamma_\alpha^R]\rangle / x \in X\}$, where $0 \leq \mu_\alpha^R + \gamma_\alpha^R \leq 1$.*

Definition 2.2.8 (Hesitant Fuzzy Set). *Let X be a fixed set. A Hesitant Fuzzy Sets (HFS) on X is in terms of a function that when applied that to X returns a subset of $[0,1]$ the Hesitant fuzzy set is expressed by mathematical symbol $k = \{\langle x, h_k(x)\rangle / x \in\}$ where $h_k(x)$ is a set of some values in $[0,1]$ denoting the possible membership degree of the element $x \in X$ to the set 'k'. For convenience 'Said Bromi' and 'Florentin Smarandache' called $h = h_k(x)$. A Hesitant Fuzzy Elements (HFE) and 'H' be the set of all HFEs.*

Definition 2.2.9 (Interval Valued Intuitionistic Hesitant Fuzzy Sets). *Let X be a fixed set, an Interval Valued Intuitionistic Hesitant Fuzzy Sets (IVIHFS) on X is given in terms of a function that when applied to X returns a subset of Ω. The IVIHFS is expressed by a mathematical symbol $k = \{\langle x, h_k(x)\rangle / x \in\}$. Where $h_{\tilde{k}}(x)$ is a set of some IVIFNs in x, denoting the possible membership degree intervals and non membership degree intervals of the element $x \in X$ to the set $\tilde{k}$. An interval valued intuitionistic hesitant fuzzy element (IVIHFE) is denoted by $\tilde{h} = h_{\tilde{k}}(x)$ and $\tilde{h}$ be the set of all IVIHFEs. If $\alpha \in \tilde{h}$ then and IVIF can be denoted by $\alpha = (\mu_\alpha, \gamma_\alpha) = \left[\mu_\alpha^L, \mu_\alpha^R\right], \left[\gamma_\alpha^L, \gamma_\alpha^R\right]$ for any $\alpha \in \tilde{h}$ if α is a real number in $[0,1]$ then $\tilde{h}$ reduces to a hesitant fuzzy element (HFE) [4].*

Definition 2.2.10 (Interval Valued Intuitionistic Triangular Hesitant Fuzzy Sets). *Let X be a fixed set, an Interval Valued Intuitionistic triangular Hesitant Fuzzy Sets (IVITHFS) on X is given in terms of a function that when applied to X returns a subset of Ω. The IVITHFS is expressed by a mathematical symbol $\tilde{k} = \left\{\langle x, h_{\tilde{k}}(x)\rangle / x \in\right\}$. Where $h_{\tilde{k}}(x)$ is a set of some IVIFNs in x, denoting the possible membership degree intervals and non membership degree intervals of the element $x \in X$ to the set $\tilde{k}$. An interval valued intuitionistic triangular hesitant fuzzy element (IVITHFE) is denoted by $\tilde{h} = h_{\tilde{k}}(x)$ and $\tilde{h}$ be the set of all IVIHFEs. If $\alpha \in \tilde{h}$ then and IVIF can be denoted by $\alpha = (\mu_\alpha, \gamma_\alpha) = \left[\mu_\alpha^L, \mu_\alpha^R\right], \left[\gamma_\alpha^L, \gamma_\alpha^R\right]$ for any $\alpha \in \tilde{h}$ if α if α is a real number in $[0,1]$ then $\tilde{h}$ reduces to a hesitant fuzzy element (HFE) [4].*

3 Four New Operators on IVITHFEs

Let $\tilde{h}_1$ and $\tilde{h}_2$ $\in$IVITHFEs (x) we propose the following operators on IVITHFES as follows

(i) $\tilde{h}_1 @ \tilde{h}_2 = \left\{\left[\left(\dfrac{\mu_{\alpha_1}^L + \mu_{\alpha_2}^L}{2}, \dfrac{\mu_{\alpha_1}^R + \mu_{\alpha_2}^R}{2}\right), \left(\dfrac{\gamma_{\alpha_1}^L + \gamma_{\alpha_2}^L}{2}, \dfrac{\gamma_{\alpha_1}^R + \gamma_{\alpha_2}^R}{2}\right)\right] \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\right\}$

(ii) $\tilde{h}_1 \$ \tilde{h}_2 = \left\{\left(\left[\sqrt{\mu_{\alpha_1}^L \mu_{\alpha_2}^L}, \sqrt{\mu_{\alpha_1}^R \mu_{\alpha_2}^R}\right], \left[\sqrt{\gamma_{\alpha_1}^L \gamma_{\alpha_2}^L}, \sqrt{\gamma_{\alpha_1}^R \gamma_{\alpha_2}^R}\right]\right) \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\right\}$

(iii) $\tilde{h}_1 \neq \tilde{h}_2 = \left\{\left(\left[\dfrac{2\mu_{\alpha_1}^L \mu_{\alpha_2}^L}{\mu_{\alpha_1}^L + \mu_{\alpha_2}^L}, \dfrac{2\mu_{\alpha_1}^R \mu_{\alpha_2}^R}{\mu_{\alpha_1}^R + \mu_{\alpha_2}^R}\right], \left[\dfrac{2\gamma_{\alpha_1}^L \gamma_{\alpha_2}^L}{\gamma_{\alpha_1}^L + \gamma_{\alpha_2}^L}, \dfrac{2\gamma_{\alpha_1}^R \gamma_{\alpha_2}^R}{\gamma_{\alpha_1}^R + \gamma_{\alpha_2}^R}\right]\right) \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\right\}$

(iv) $\tilde{h}_1 * \tilde{h}_2 = \left\{\left(\left[\dfrac{\mu_{\alpha_1}^L + \mu_{\alpha_2}^L}{2(\mu_{\alpha_1}^L \mu_{\alpha_2}^L + 1)}, \dfrac{\mu_{\alpha_1}^R + \mu_{\alpha_2}^R}{2(\mu_{\alpha_1}^R \mu_{\alpha_2}^R + 1)}\right], \left[\dfrac{\gamma_{\alpha_1}^L + \gamma_{\alpha_2}^L}{2(\gamma_{\alpha_1}^L \gamma_{\alpha_2}^L + 1)}, \dfrac{\gamma_{\alpha_1}^R + \gamma_{\alpha_2}^R}{2(\gamma_{\alpha_1}^R \gamma_{\alpha_2}^R + 1)}\right]\right) \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\right\}$

4 Algebraic Combination of Focal Elements [2]

Let X_1 and X_2 be two variables whose values are represented by Dempster- Shafer structure with focal elements $A_1, A_2, A_3, \ldots, A_n$ and $B_1, B_2, B_3 \ldots, B_m$ which are considered as intervals and their corresponding Basic Probability Assignments (BPA) are as follows:

$$m(A_i) = a_i \quad \text{and} \quad m(B_i) = b_j, \qquad i = 1, 2, 3, \ldots, n \quad \& \quad j = 1, 2, 3, \ldots, m$$

where $\sum_{i=1}^{n} a_i = 1$ and $\sum_{i=1}^{m} b_j = 1$

Initially we combine all the fuzzy focal elements using fuzzy arithmetic which will produce nm number of fuzzy focal elements and there after the corresponding basic probability assignments of resulting fuzzy focal elements will be calculated as follows

4.1 Addition of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i + B_j) = \frac{m(A_i) + m(B_j)}{\sum_i \sum_j (m(A_i) + m(B_j))} \tag{4.1}$$

4.2 Subtraction of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i - B_j) = \frac{m(A_i)(1 - m(B_j))}{\sum_i \sum_j (m(A_i)(1 - m(B_j)))} \tag{4.2}$$

4.3 Multiplication of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i \cdot B_j) = \frac{m(A_i) \cdot m(B_j)}{\sum_i \sum_j (m(A_i) \cdot m(B_j))} \tag{4.3}$$

4.4 Division of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i / B_j) = \frac{m(A_i) / m(B_j)}{\sum_i \sum_j (m(A_i) / m(B_j))} \tag{4.4}$$

Finally, we arrange all the focal elements in increasing order of the left end point.

5 Numerical Example

Suppose Basic Probability Assignment of two parameters is assigned by an expert and are given in the following tables.

Interval valued Intutionisic Hesitant FFE		BPA
Membership Interval Value	Non-Membership Interval Value	
[16.5, 28.5]	[18.75, 26.25]	0.05
[31.5, 43.5]	[33.75, 41.25]	0.1

Table 1: BPA of first parameter

Interval valued Intutionisic Hesitant FFE		BPA
Membership Interval Value	Non-Membership Interval Value	
[6.4, 9.6]	[7,9]	0.3
[11,19]	[12.5, 17.5]	0.6

Table 2: BPA of second parameter

6 Addition of Focal Elements

Addition of focal elements using IVITHFE arithmetic gives four focal elements. The corresponding BPA of resulting focal elements are calculated using (4.1) and arranging all the focal elements in increasing order of the left end point are given in the following table.

Hesitant Operators									BPA
S.No	$\tilde{h}_1 @ \tilde{h}_2$		$\tilde{h}_1 \$ \tilde{h}_2$		$\tilde{h}_1 \neq \tilde{h}_2$		$\tilde{h}_1 * \tilde{h}_2$		
1	[11.45, 19.1]	[12.9, 17.6]	[10.3, 16.5]	[11.5, 15.4]	[20.9, 14.4]	[10.2, 13.4]	[0.1, 0,06]	[0.09,0.07]	0.0864
2	[13.8, 23.8]	[15.6, 21.9]	[13.5, 23.3]	[15.3, 21.4]	[3.2, 22.8]	[15, 21]	[0.08, 0,04]	[0.07,0.05]	0.1605
3	[19, 26.6]	[20.4,25.1]	[14.2, 20.4]	[15.4, 19.3]	[10.6, 15.7]	[11.6, 14.8]	[0.09, 0,06]	[0.08,0.07]	0.0988
4	[21.3, 31.3]	[23.1, 29.4]	[18.6, 28.7]	[20.5,26.9]	[16.3, 26.4]	[18.2, 24.6]	[0.06, 0,04]	[0.05,0.04]	0.1728

Table 3: BPA of resulting focal elements using algebraic addition

6.1 Subtraction of Focal Elements

Subtraction of focal elements using IVITHFE arithmetic gives four focal elements. The corresponding BPA of resulting focal elements are calculated using (4.2) and arranging all the focal elements in increasing order of the left end point are given in the following table

Hesitant Operators									BPA
S.No	$\tilde{h}_1 @ \tilde{h}_2$		$\tilde{h}_1 \$ \tilde{h}_2$		$\tilde{h}_1 \neq \tilde{h}_2$		$\tilde{h}_1 * \tilde{h}_2$		
1	[11.45, 19.1]	[12.9, 17.6]	[10.3, 16.5]	[11.5, 15.4]	[20.9, 14.4]	[10.2, 13.4]	[0.1, 0,06]	[0.09,0.07]	0.05
2	[13.8, 23.8]	[15.6, 21.9]	[13.5, 23.3]	[15.3, 21.4]	[3.2, 22.8]	[15, 21]	[0.08, 0,04]	[0.07,0.05]	0.0286
3	[19, 26.6]	[20.4,25.1]	[14.2, 20.4]	[15.4, 19.3]	[10.6, 15.7]	[11.6, 14.8]	[0.09, 0,06]	[0.08,0.07]	0.1
4	[21.3, 31.3]	[23.1, 29.4]	[18.6, 28.7]	[20.5,26.9]	[16.3, 26.4]	[18.2, 24.6]	[0.06, 0,04]	[0.05,0.04]	0.0571

Table 4: BPA of resulting focal elements using algebraic subtraction

6.2 Multiplication of Focal Elements

Multiplication of focal elements using IVITHFE arithmetic gives four focal elements. The corresponding BPA of resulting focal elements are calculated using (4.3) and arranging all the focal elements in increasing order of the left end point are given in the following table

Hesitant Operators									BPA
S.No	$\tilde{h}_1 @ \tilde{h}_2$		$\tilde{h}_1 \$ \tilde{h}_2$		$\tilde{h}_1 \neq \tilde{h}_2$		$\tilde{h}_1 * \tilde{h}_2$		
1	[11.45, 19.1]	[12.9, 17.6]	[10.3, 16.5]	[11.5, 15.4]	[20.9, 14.4]	[10.2, 13.4]	[0.1, 0,06]	[0.09,0.07]	0.0429
2	[13.8, 23.8]	[15.6, 21.9]	[13.5, 23.3]	[15.3, 21.4]	[3.2, 22.8]	[15, 21]	[0.08, 0,04]	[0.07,0.05]	0.0857
3	[19, 26.6]	[20.4,25.1]	[14.2, 20.4]	[15.4, 19.3]	[10.6, 15.7]	[11.6, 14.8]	[0.09, 0,06]	[0.08,0.07]	0.0857
4	[21.3, 31.3]	[23.1, 29.4]	[18.6, 28.7]	[20.5,26.9]	[16.3, 26.4]	[18.2, 24.6]	[0.06, 0,04]	[0.05,0.04]	0.1714

Table 5: BPA of resulting focal elements using algebraic multiplication

6.3 Division of Focal Elements

Division of focal elements using IVITHFE arithmetic gives four focal elements. The corresponding BPA of resulting focal elements are calculated using (4.4) and arranging all the focal elements in increasing order of the left end point are given in the following table

Hesitant Operators									BPA
S.No	$\tilde{h}_1 @ \tilde{h}_2$		$\tilde{h}_1 \$ \tilde{h}_2$		$\tilde{h}_1 \neq \tilde{h}_2$		$\tilde{h}_1 * \tilde{h}_2$		
1	[11.45, 19.1]	[12.9, 17.6]	[10.3, 16.5]	[11.5, 15.4]	[20.9, 14.4]	[10.2, 13.4]	[0.1, 0,06]	[0.09,0.07]	0.0317
2	[13.8, 23.8]	[15.6, 21.9]	[13.5, 23.3]	[15.3, 21.4]	[3.2, 22.8]	[15, 21]	[0.08, 0,04]	[0.07,0.05]	0.0159
3	[19, 26.6]	[20.4,25.1]	[14.2, 20.4]	[15.4, 19.3]	[10.6, 15.7]	[11.6, 14.8]	[0.09, 0,06]	[0.08,0.07]	0.0635
4	[21.3, 31.3]	[23.1, 29.4]	[18.6, 28.7]	[20.5,26.9]	[16.3, 26.4]	[18.2, 24.6]	[0.06, 0,04]	[0.05,0.04]	0.0317

Table 6: BPA of resulting focal elements using algebraic division

7 Conclusion

In this paper we have considered the interval valued intuitionistic triangular hesitant fuzzy focal elements and their corresponding Basic Probability Assignments (BPA) of two variables under four new aggregate operators on hesitant fuzzy sets. From Table 6, it is observed that out of four new aggregate operators, the operation

$\tilde{h}_1 * \tilde{h}_2$ have the significance of simple calculation and high accuracy. Moreover among the four arithmetic operations on fuzzy focal elements, the more accurate Basic Probability Assignments were obtained under division of fuzzy focal elements using IVITHFFEs.

References

[1] Atanassov K T, Intuitionistic Fuzzy Sets, *Fuzzy sets and systems* 20(1), 1986, pp.87-96.

[2] R.Irene Hebzibah , A.Nagoorgani and G.Geethalakshmi Modified Arithmetic Operations of Focal Elements and their corresponding Basic Probability Assignments in Evidence Theory, *Elixir.Appl.Math*. 74(2014) 26798-26984.

[3] Said Broumi, Florentin Smarandache, New operations over Interval valued Intuitionistic Hesitant Fuzzy Set, *Mathematics and Statistics* 2(2):62-71,2014.

[4] G.S.Thakur Rekha Thakur Ravi singh, New Hesitant Fuzzy Operators, *science Direct, fuzzy inf.Eng*. (2014) 6: 379-392.

[5] Torra V, Narukawa Y , On Hesitant Fuzzy Sets and Decision, *The 18th IEEE international conference on Fuzzy Systems, Jeju Island ,Korea* ,2009,pp. 1378-1382.

[6] Zadeh,L.A., The concept of a Linguistic variable and its applications to approximate Reasoning parts I, II and III, *Information sciences*, 8 (1975)199-249; 8(1975)301-357; 9(1976),43-80.

[7] Zeshui Xu, Meimei Xia, Distance and similarity measures for hesitant fuzzy sets, *ELSEVIER, Information sciences*, 181 (2011) 2128-2138.

[8] Zhiming Zhang , Interval Valued Intuitionistic Hesitant Fuzzy Aggregation operators and their applications in Group Decision Making, *Journal of Applied Mathematics* 3(2013), Article ID 670285, 33 pages, http://dx.doi.org/10.1155/2013/670285.

Received: July 10, 2015; *Accepted*: September 13, 2015

UNIVERSITY PRESS

Website: http://www.malayajournal.org/