**A-perfect lattice**

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**Abstract**
In the Paper [3] authors define the concept of A-perfect Group. Inspired by [3], we give a new concept of A-perfect lattice. If \( g \in L \) and \( \alpha \in A \), then the element \([g;\alpha] = g^{-1}\alpha(g)\) is an auto commutator of \( g \) and \( \alpha \), if is taken to be an inner automorphism, then the auto commutator sublattice is the derived sublattice \( L' \) of \( L \). A lattice \( L \) is said to be perfect if \( L = L' \). Here, the perception of A-perfect lattices would be introduced. A lattice \( L \) would be known as A-perfect, if \( L = K(L) \).

**Keywords**
Perfect lattice, A-perfect group, finite abelian group.

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**2. Some Important Results**

**Theorem 2.1.** Let \( H \) and \( T \) be two lattices. Suppose, the following conditions are satisfied:

(i) \( K(H) \times K(T) \subseteq K(H \times T) \);

(ii) \( H \) and \( T \) are such that \( (|H|; |T|) = 1 \).

Then, \( K(H) \times K(T) = K(H \times T) \).

**Proof.** (i) For \( \alpha \in \text{Aut}(H) \) and \( \beta \in \text{Aut}(T) \) we define the automorphism of lattice \( H \times T \), given by

\[
(\alpha \times \beta)(h, t) = \alpha(h)\beta(t) \quad \forall h \in H, t \in T.
\]

It is easy to check that \([h;\alpha];[t;\beta] = [(h, t);\alpha \times \beta]\). This implies the result.

(ii) It is sufficient to prove \( K(H \times T) \subseteq K(H) \times K(T) \). It is easy to check that \( \hat{\lambda}/H \in \text{Aut}(H) \) and \( \hat{\lambda}/T \in \text{Aut}(T) \), for all \( \hat{\lambda} \in \text{Aut}(H \times T) \). Now

\[
[(h;\hat{\lambda}H);\hat{\lambda}] = ([h;\hat{\lambda}];[t;\hat{\lambda}/T]), \forall h \in H, t \in T, \text{Aut}(H \times T).
\]

This implies the result.

**Theorem 2.2.** For all nonnegative integers \( n > m_1 \geq m_2 \geq \cdots \geq m_k \).

**Corollary 2.3.** If \( G \) is a finite abelian group of odd order, then \( G \) is A-perfect.
Proof. $L$ is a direct product of finitely many $Z_{p^i}$, where $p$ is an odd prime number and $i \geq 1$. Hence, the result is true due to previous theorem. $\square$

Theorem 2.4. For all nonnegative integers $n > m_1 \geq m_2 \geq \cdots \geq m_k$,

$$K(Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) = Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}.$$  

Theorem 2.5. For all nonnegative integers $n > m_1 \geq m_2 \geq \cdots \geq m_k$,

$$K(Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) = Z_{2^n} \times Z_{2^{m_1}} \times Z_{2^{m_2}} \times \cdots \times Z_{2^{m_k}}.$$  

Proof. We define the automorphisms $\alpha, \alpha', \beta_1, \ldots, \beta_k$ of the lattice $L$ given by:

$$\begin{align*}
\alpha(a, b, c_1, \ldots, c_k) &= (a + b, c_1, \ldots, c_k) \\
\alpha'(a, b, c_1, \ldots, c_k) &= (a, a + b, c_1, \ldots, c_k) \\
\beta_1(a, b, c_1, \ldots, c_k) &= (a, a + b + c_1, c_2, \ldots, c_k) \\
\vdots & \\
\beta_k(a, b, c_1, \ldots, c_k) &= (a, b, c_1, \ldots, a + c_k).
\end{align*}$$

for all $a, b \in \{0, 1, 2, \ldots, 2^n - 1\}$ and $c_i \in \{0, 1, 2, \ldots, 2^m - 1\}$, $1 \leq i \leq k$. Clearly,

$$\begin{align*}
(a, 0, \ldots, 0) &= [(0, a, 0, \ldots, 0), \alpha], (0, b, 0, \ldots, 0) \\
&= [(b, 0, \ldots, 0), \alpha'] \\
(0, 0, c_1, 0, \ldots, 0) &= [(c_1, 0, 0, \ldots, 0), \beta_1] \\
(0, 0, 0, c_2, 0, \ldots, 0) &= [(c_2, 0, 0, \ldots, 0), \beta_2] \\
&\quad \vdots \\
(0, 0, \ldots, 0, c_2) &= [(c_k, 0, 0, \ldots, 0), \beta_k].
\end{align*}$$

These imply that

$$K(Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) \supseteq Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}.$$  

$\square$

Theorem 2.6. A finite abelian lattice $L$ is $A$-perfect if and only if

$$L \approx Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}} \times M.$$  

for some nonnegative integers $n > m_1 \geq m_2 \geq \cdots \geq m_k$, where $M$ is a finite lattice of odd order.

Proof. The necessary condition follows from Theorem 2.5. Now, for the reverse conclusion, we assume that $L$ is not a product of $Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}} \times M$, so it is $Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}} \times N$ where $N$ is a finite abelian lattice of odd order. Theorem 2.1 implies that

$$K(Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}} \times N) = K(Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) \times K(N).$$  

Now, the lattice $L$ is not $A$-perfect due to previous. It completes the proof. $\square$

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References


References