On nano $\pi g^*s$-closed sets in nano topological spaces

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Abstract
In this paper, a new class of set called nano $\pi$ generalized star semi-closed sets in nano topological spaces is introduced and some of their basic properties are investigated. We shows that a new class of sets lies between the class of $\pi g$-closed sets and the class of $\pi g s$-closed sets. Further the notion of $\pi g^*s$-open sets, $\pi g^*s$-neighbourhoods, $\pi g^*s$-interior and $\pi g^*s$-closure are discussed. Several examples are also provided to illustrate the behaviour of new sets and functions.

Keywords
$\pi g^*s$-closed sets, $\pi g^*s$-open sets, $\pi g^*s$-neighbourhoods, $\pi g^*s$-interior, $\pi g^*s$-closure.

AMS Subject Classification
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1. Introduction
Levine [4] Introduced the class of g-closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology. Recently Lellis Thivagar introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X. The elements of nano topological space are called nano open sets. He has also defined nano closed sets, nano interior and nano closure of a set. He also introduced the weak forms of nano open sets. Bhuvaneswari [2] introduced nano g-closed sets and obtained some of the basic interesting results. Sathishmohan [9] et.al., studied the concept of $\theta g^*$-closed sets in topological spaces and investigate the composition of the functions between $\theta g^*$-continuous functions and continuous functions. In this paper, we define a new class of sets called nano $\pi$ generalized star semi-closed and its open sets in nano topological spaces and study the relationships with other nano sets.

2. Preliminaries

Definition 2.1. [5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

- The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(X)$ denotes the equivalence class determined by $X \in U$.

- The upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$
The boundary of the region of \( X \) with respect to \( R \) is the set of all objects, which can be classified neither as \( X \) nor as not \( X \) with respect to \( R \) and it is denoted by \( B_R(X) \). That is \( B_R(X) = U_R(X) - L_R(X) \).

**Definition 2.2.** [5] If \((U, R)\) is an approximation space and \( X, Y \subseteq U \), then

- \( L_R(X) \subseteq X \subseteq U_R(X) \).
- \( L_R(\emptyset) = U_R(\emptyset) = \emptyset \).
- \( L_R(U) = U_R(U) = U \).
- \( U_R(X \cup Y) = U_R(X) \cup U_R(Y) \).
- \( U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y) \).
- \( U_R(X \cup Y) \supseteq U_R(X) \cup U_R(Y) \).
- \( U_R(X \cap Y) = U_R(X) \cap U_R(Y) \).
- \( L_R(X) \subseteq L_R(Y) \) and \( U_R(X) \subseteq U_R(Y) \) whenever \( X \subseteq Y \).
- \( U_R(X^c) = [L_R(X)]^c \) and \( L_R(X^c) = [U_R(X)]^c \).
- \( U_R(U_R(X)) = L_R(U_R(X)) = U_R(X) \).
- \( L_R(L_R(X)) = U_R(L_R(X)) = L_R(X) \).

**Definition 2.3.** [5] Let \( U \) be the Universe and \( R \) be an equivalence relation on \( U \) and \( \tau_R(X) = \{ U, \emptyset, L_R(X), U_R(X), B_R(X) \} \). where \( X \subseteq U \). \( \tau_R(X) \) satisfies the following axioms:

- \( U \) and \( \emptyset \in \tau_R(X) \).
- The union of elements of any subcollection of \( \tau_R(X) \) is in \( \tau_R(X) \).
- The intersection of the elements of any finite subcollection of \( \tau_R(X) \) is in \( \tau_R(X) \).

We call \((U, \tau_R(X))\) is a nano topological space. The elements of \( \tau_R(X) \) are called a nano open sets and the complement of a nano open set is called nano closed sets.

Throughout this paper \((U, \tau_R(X))\) is a nano topological space with respect to \( X \) where \( X \subseteq U \). \( R \) is an equivalence relation on \( U \). \( U / R \) denotes the family of equivalence classes of \( U \) by \( R \).

**Remark 2.4.** [5] If \( \tau_R(X) \) is the nano topology on \( U \) with respect to \( X \). The set \( B = \{ U, L_R(X), B_R(X) \} \) is the basis for \( \tau_R(X) \).

**Definition 2.5.** [5] If \((U, \tau_R(X))\) is a nano topological space with respect to \( X \). Where \( X \subseteq G \) and if \( A \subseteq G \), then

- The nano interior of the set \( A \) is defined as the union of all nano open subsets contained in \( A \) and is denoted by \( Nint(A) \). \( Nint(A) \) is the largest nano open subset of \( A \).
- The nano closure of the set \( A \) is defined as the intersection of all nano closed sets containing \( A \) and is denoted by \( Ncl(A) \). \( Ncl(A) \) is the smallest nano closed set containing \( A \).

**Definition 2.6.** [10] A nano-subset of a nano topological spaces \( U \) is called nano-dense if \( Ncl(A) = U \).

**Definition 2.7.** [12] A nano topological space \((U, \tau_R(X))\) is said to be nano extremally disconnected, if the nano-closure of each nano-open set is nano-open.

**Definition 2.8.** [10] A space \( U \) is called nano-submaximal if each nano-dense subset of \( U \) is nano-open.

**Definition 2.9.** Let \((U, \tau_R(X))\) be a nano topological space and \( A \subseteq G \). Then \( A \) is said to be

- \( Nr\)-closed [5] if \( A = Ncl(Nint(A)) \).
- \( N\alpha\)-closed [5] if \( Nint(Ncl(Nint(A))) \subseteq A \).
- \( N\beta\)-closed [2] if \( Ncl(Nint(A)) \subseteq A \).
- \( N\pi\)-closed [1] if intersection of nano \( r \)-closed.
- \( Ng\)-closed [2] if \( Ncl(A) \subseteq G \) whenever \( A \subseteq G \) and \( G \) is nano-open in \( U \).
- \( Ng^*\)-closed [5] if \( Ncl(A) \subseteq G \) whenever \( A \subseteq G \) and \( G \) is nano g-open in \( U \).
- \( Ng^*\)-s-closed [4] if \( Nscl(A) \subseteq G \) whenever \( A \subseteq G \) and \( G \) is nano g-open in \( U \).
3. $N\pi g^s$-closed sets

Definition 3.1. A subset $A$ of a nano topological space $(U, \tau_\pi(X))$ is called nano $\pi$ generalized star semi-closed (briefly $N\pi g^s$-closed) set if $N\text{cl}(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $N\pi$-open.

Theorem 3.2. Every $N\pi$-closed set is $N\pi g^s$-closed.

Proof: Let $A$ be a $N\pi$-closed set in $U$. Let $G$ be a $N\pi g^s$-open set such that $A \subseteq G$. Since $A$ is $N\pi$-closed we have $N\text{cl}(A) = A \subseteq G$. But, $N\text{cl}(A) \subseteq N\text{cl}(A) \subseteq G$. Therefore $N\text{cl}(A) \subseteq G$. Hence $A$ is a $N\pi g^s$-closed set in $X$.

Remark 3.3. The converse of the above theorem need not be true as seen in the following example.

Example 3.4. Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\}, \{b, c, d\}\}$. Let $X=\{a, c\} \subseteq U$ and $\tau_\pi(X) = \{\phi, U, \{a\}, \{b, c, d\}\}$. Nano $g$-closed set = $\{\phi, U, \{a\}, \{b, c, d\}\}$ and $N\pi g^s$-closed set = $\{\phi, U, \{a\}, \{b, c, d\}\}$. Let $A=\{b,d\}$. The subset $A$ is $N\pi g^s$-closed but not $N\pi$-closed set.

Theorem 3.5. Every $N\pi g^s$-closed set is $N\pi g$s-closed.

Proof: Let $A$ be any $N\pi g^s$-closed set in $U$. Let $A \subseteq G$ and $G$ be $N\pi g$-open in $U$. Every nano open is $N\pi$-open and $N\pi g$-open and since $A$ is $N\pi g^s$-closed, we have $N\text{cl}(A) \subseteq G$. Therefore $A$ is $N\pi g$s-closed.

Remark 3.6. The converse of the above theorem need not be true as seen in the following example.

Example 3.7. Let $U=\{a, b, c, d\}$ with $U/R=\{\{a, c\}, \{b\}, \{d\}\}$. Let $X=\{a, b\} \subseteq U$ and $\tau_\pi(X) = \{\phi, U, \{a\}, \{a, c\}, \{a, c\}\}$. Nano $g$-closed set = $\{\phi, U, \{a\}, \{a, c\}\}$ and $N\pi g^s$-closed set = $\{\phi, U, \{a\}, \{a, c\}\}$. Let $A=\{b, d\}$. The subset $A$ is $N\pi g^s$-closed set but not in $N\pi g$s-closed set. The subset $B$ is in $N\pi g$s-closed set but not in $N\pi g^s$-closed set. Therefore $N\pi g$s-closed sets and $N\pi g^s$-closed sets are independent of each other.

Theorem 3.12. Union of any two $N\pi g^s$-closed sets is $N\pi g^s$-closed.

Proof: Let $A$ and $B$ be any two $N\pi g^s$-closed sets in $U$, such that $A \subseteq G$ and $B \subseteq G$ where $G$ is $N\pi g$-open in $U$ and so $A \cup B \subseteq G$. Since $A$ and $B$ are $N\pi g^s$-closed, we have $A \subseteq N\text{cl}(A)$ and $B \subseteq N\text{cl}(B)$. Therefore $A \cup B \subseteq N\text{cl}(A) \cup N\text{cl}(B)$ and $N\pi g^s$-closed.
Theorem 3.16. Let A be an N\(g\)s-closed set in U, \(\tau_U(\mathcal{X})\).

Proof: Let \(A\) and \(B\) be any two \(N\(g\)s-closed sets in \(U\), such that \(A \subseteq G\) and \(B \subseteq G\) where \(G\) is \(N\(g\)s-open in U and so \(A \subseteq B \subseteq G\). Since \(A\) and \(B\) are \(N\(g\)s-closed, we have \(A \subseteq \text{Nscl}(A)\) and \(B \subseteq \text{Nscl}(A)\) and hence \(A \cap B \subseteq \text{Nscl}(A) \cap \text{Nscl}(B) \subseteq \text{Nscl}(A \cap B)\). This shows \(B\) is a \(N\(g\)s-closed set in \(U/\tau_U(\mathcal{X})\)).

Example 3.15. Let \(U = \{a, b, c, d\}\) with \(U/R = \{a, b\}\). Let \(X = \{a, c\} \subseteq U\) and \(\tau_U(\mathcal{X}) = \{\phi, U, \{a, c\}, \{a, c, d\}\}\). Then \(N\(g\)s-closed=\(\{\phi, U, \{a\}, \{a, c\}, \{a, c, d\}\}\). Let \(A = \{a\}\) is \(N\(g\)s-closed set and \(B = \{b, c, d\}\) is \(N\(g\)s-closed set, then \(A \cup B = \{a\} \cup \{b\} = \{a\} \cup \{b\} = \{a\}\) is also a \(N\(g\)s-closed set.

Theorem 3.14. Intersection of any two \(N\(g\)s-closed subsets is \(N\(g\)s-closed.

Proof: Let \(A\) and \(B\) be any two \(N\(g\)s-closed subsets in \(U\), such that \(A \subseteq G\) and \(B \subseteq G\). Hence \(A \cap B \subseteq G\). Therefore \(N\(g\)s-closed=\(\{\phi, U, \{a\}, \{a, c\}, \{a, c, d\}\}\).

Theorem 3.17. If \(A\) is a \(N\(g\)s-closed subset of \(U\) such that \(A \subseteq B \subseteq \text{Nscl}(A)\), then \(B\) is a \(N\(g\)s-closed set in \(U\).

Proof: Let \(A \subseteq B \subseteq \text{Nscl}(A)\) and \(A \subseteq G\), then \(N\(g\)s-closed=\(\{\phi, U, \{a\}, \{a, c\}, \{a, c, d\}\}\) and \(B \subseteq \text{Nscl}(A)\). Since \(B \subseteq \text{Nscl}(A)\), we have \(B \subseteq \text{Nscl}(A)\). Now \(\text{Nscl}(B) \subseteq \text{Nscl}(\text{Nscl}(A)) = \text{Nscl}(A) \subseteq G\). Therefore \(B\) is a \(N\(g\)s-closed set in \(U\).

Theorem 3.18. For each \(\{a\} \subseteq U\), either \(\{a\}\) is \(N\(g\)s-closed set or \(\{a\}\) is \(N\(g\)s-closed set in \(\tau_U(\mathcal{X})\).

Proof: Suppose \(\{a\}\) is nano closed in \(U\). Then \(\{a\}\) is \(N\(g\)s-closed in \(U\) and hence \(\{a\}\) is \(N\(g\)s-closed in \(U\).

Corollary 3.19. Let \(A\) be a \(N\(g\)s-closed set and suppose that \(F\) is a nano closed set. Then \(A \cap F\) is a \(N\(g\)s-closed set.

Example 3.20. Let \(U = \{a, b, c, d\}\) with \(U/R = \{\{a, c\}, \{b\}\}\). \(X = \{a, b\} \subseteq U\) and \(\tau_U(\mathcal{X}) = \{\phi, U, \{a, c\}, \{a, b\}\}\). Then \(N\(g\)s-closed=\(\{\phi, U, \{a, c\}, \{a, b\}\}\). Let \(A = \{a, c\}\) and \(F = \{b, c\}\). Then \(A \cap F = \{a, c\}\) is a \(N\(g\)s-closed set.

Definition 3.21. A subset \(A\) of a nano topological space \((U, \tau_U(\mathcal{X}))\) is called nano \(\pi\) generalized star semi-open (briefly nano \(\pi\) s-open) if \(A^c\) is a \(N\(g\)s-closed.

Theorem 3.22. A subset \(A\) \(\subseteq U\) is \(N\(g\)s-closed iff \(F \subseteq NInt(A)\) whenever \(F\) is a \(N\(g\)s-closed set and \(F \subseteq A\).

Definition 3.41. Let \((U, \tau_U(\mathcal{X}))\) be a nano topological space and \(A \subseteq U\) then nano \(\pi\) s-interior is defined as \(N\(g\)s-int(A) = \phi\). Clearly \(N\(g\)s-int(A)\) is the largest nano \(\pi\) s-open set over \(U\) which is contained in \(A\).

Example 3.42. Let \((U, \tau_U(\mathcal{X}))\) be a nano topological space and \(A \subseteq U\) then nano \(\pi\) s-star-closed is defined as \(N\(g\)s-cl(A) = \{F: F\) is nano \(\pi\) s-closed, \(A \subseteq F\}\). Clearly \(N\(g\)s-cl(A)\) is the smallest nano \(\pi\) s-closed set over \(U\) which contains \(A\).

Lemma 4.3. Let \(A\) and \(B\) be any two subsets of \(U\) in a nano topological spaces \((U, \tau_U(\mathcal{X}))\) and the following are true

(i) \(N\(g\)s-int(A) \subseteq C\)
(ii) \(A \subseteq B\) \(\Rightarrow\) \(N\(g\)s-int(B) \subseteq N\(g\)s-int(A)\)
(iii) \(N\(g\)s-int(A) \cup N\(g\)s-int(B) \subseteq N\(g\)s-int(A \cup B)\)
(iv) \(N\(g\)s-int(A) \cap N\(g\)s-int(B) \subseteq N\(g\)s-int(A \cap B)\)

Lemma 4.4. For a subset \(A\) of \(U\)

(i) \(\pi\) s-cl(A) \(\subseteq\) Ncl(A)
(ii) Ncl(A) \(\subseteq\) \(\pi\) s-cl(A)

Lemma 4.5. A subset \(A\) of \(U\) is nano \(\pi\) generalized star semi-closed if and only if \(A = \text{Nscl}(A)\).

Theorem 4.6. Let \(A\) and \(B\) be two subsets of nano topological space \((U, \tau_U(\mathcal{X}))\). Then

(i) \(N\(g\)s-int(U) = U\) and \(N\(g\)s-int(\(\phi\)) = \(\phi\)
(ii) \(N\(g\)s-int(A) \subseteq A\)
(iii) If \(B\) is any \(N\(g\)s-closed set contained in \(A\), then \(B\) \(\subseteq\) \(A\)
Let A and B be subsets of \((U, \tau)\). Then nano \(A\) is nano \(B\) if for every \(x \in A\), there is a nano \(B\)-neighborhood of \(x\) that is contained in \(A\).

**Theorem 4.10.** Let A be a subset of a nano topological space \((U, \tau)\). Then nano \(A\) is nano \(B\) if for every \(x \in A\), there is a nano \(B\)-neighborhood of \(x\) that is contained in \(A\).

**Proof:** Let \(A\) and \(B\) be subsets of \((U, \tau)\). Then nano \(A\) is nano \(B\) if for every \(x \in A\), there is a nano \(B\)-neighborhood of \(x\) that is contained in \(A\).

**Theorem 4.14.** A subset \(A\) of \((U, \tau)\) is nano \(\tau\)-closed if and only if \(A\) is \(\tau\)-closed and for every \(x \in \partial A\), there exists an open \(\tau\)-neighborhood of \(x\) that intersects \(A\) in a set of \(\tau\)-closures.

**Proof:** Let \(A\) be a subset of \((U, \tau)\). Then \(A\) is nano \(\tau\)-closed if and only if \(A\) is \(\tau\)-closed and for every \(x \in \partial A\), there exists an open \(\tau\)-neighborhood of \(x\) that intersects \(A\) in a set of \(\tau\)-closures.

**Remark 4.15.** Containment relation in the above theorem may be proper as seen from the following example.

**Example 4.16.** Let \(U = \{a,b,c,d\}\) with \(U/R = \{x,y\}\). Let \(X = \{a,b\}\) be \(\tau\)-closed. Then \(\tau\)-closed \(X\) is \(\tau\)-closed. Hence \(\tau\)-closed \(X\) is \(\tau\)-closed.

**Example 4.17.** Let \(U = \{a,b,c,d\}\) with \(U/R = \{x,y\}\). Let \(X = \{a,b\}\) be \(\tau\)-closed. Then \(\tau\)-closed \(X\) is \(\tau\)-closed. Hence \(\tau\)-closed \(X\) is \(\tau\)-closed.
5. Nano $\pi g^*$-s-neighbourhoods

Definition 5.1. A subset $M_x \subseteq U$ is called a nano $\pi g^*$-s-neighbourhood ($N\pi g^*$-s-nghd) of a point $x \in U$ if and only if there exists a $A \in N\pi g^*$sO($U, X$) such that $x \in A \subseteq M_x$ and a point $x$ is called $N\pi g^*$-s-nghd point of the set $A$.

Definition 5.2. The family of all $N\pi g^*$-s-nghd of the point $x \in U$ is called $N\pi g^*$-s-nghd system of $U$ and is denoted by $N\pi g^*$-s-nghd $x$.

Example 5.3. Let $U=\{a, b, c, d\}$ with $U/R=\{\{a, c\}, \{b\}, \{d\}\}$. Let $X=\{a, b\} \subseteq U$ and $\tau_R(X)=\{\phi, U, \{a\}, \{b, c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Then $N\pi g^*$-s-nghds$(a)=\{\phi, U, \{a\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}\}$, $N\pi g^*$-s-nghds$(b)=\{\phi, U, \{a\}, \{b\}, \{b, c\}, \{b, a, c\}\}$, $N\pi g^*$-s-nghds$(c)=\{\phi, U, \{c\}, \{a\}, \{b\}, \{a, c\}, \{a, c, d\}\}$, $N\pi g^*$-s-nghds$(d)=\{\phi, U, \{b\}, \{d\}, \{c, d\}\}$. Let $A=\{a, b\}$, $B=\{c, d\}$, $C=\{a, b, c\}$, $D=\{a, c, d\}$. Then $\{a, b\} \in \mathcal{N}\pi g^*$-s-nghd$(a)$ and $\{c, d\} \in \mathcal{N}\pi g^*$-s-nghd$(d)$.

Lemma 5.4. An arbitrary union of $N\pi g^*$-s-nghds of a point $x$ is again a $N\pi g^*$-s-nghd of $x$.

Proof: Let $\{A_i\}_{i \in I}$ be an arbitrary collection of $N\pi g^*$-s-nghds of a point $x \in U$. We have to prove that $\bigcup A_i$ for $\lambda \in I$ (where $I$ denotes index set) also a $N\pi g^*$-s-nghd of $x$. For all $\lambda \in I$, there exists $N\pi g^*$-s-open $M_{\lambda}$ such that $x \in M_\lambda \subseteq A_\lambda \subseteq \bigcup A_i$, i.e., $x \in M_{\lambda}$ and $M_\lambda \subseteq U \subseteq A_\lambda$ therefore $A_\lambda$ for $\lambda \in I$, is a $N\pi g^*$-s-nghd of $x$. That is an arbitrary union of $N\pi g^*$-s-nghds of $x$ is again a $N\pi g^*$-s-nghd of $x$.

But intersection of $N\pi g^*$-s-nghds of a point is not a $N\pi g^*$-s-nghd of that point in general.

Example 5.5. Let $U=\{a, b, c, d\}$ with $U/R=\{\{a, b\}, \{c\}, \{d\}\}$. Let $X=\{a, c\} \subseteq U$ and $\tau_R(X)=\{\phi, U, \{a\}, \{a, c\}, \{a, b\}, \{a, c, d\}\}$ be nano topological on $U$. Now $\mathcal{N}\pi g^*$-sO($U, X$) = $\{\phi, U, \{a\}, \{a, c\}, \{a, b\}, \{a, c, d\}\}$. Clearly $\{a\}$ and $\{b, c\}$ are $\mathcal{N}\pi g^*$-s-nghd of $b \in U$ but $\{a\} \cap \{b, c\} = \{b\}$ is not a $\mathcal{N}\pi g^*$-s-nghd of $b$.

Theorem 5.6. The $\mathcal{N}\pi g^*$-s-nghd system $\mathcal{N}\pi g^*$-s-nghd($x$) of a point $x \in U$ satisfies the following properties:

(a) if $\lambda \in \mathcal{N}\pi g^*$-s-N($x$) then $x \in \lambda$
(b) if $\lambda \in \mathcal{N}\pi g^*$-s-N($x$) and $N \subseteq M$ then $M \in \mathcal{N}\pi g^*$-s-N($x$)
(c) if $\lambda \in \mathcal{N}\pi g^*$-s-N($x$) then there exists an $A \in \mathcal{N}\pi g^*$-s-N($x$) such that $G \cap N \subseteq \mathcal{N}\pi g^*$-s-N($y$), for all $y \in G$.

Proof: (a) Let $\lambda \in \mathcal{N}\pi g^*$-s-N($x$) implies $\lambda$ is the $\mathcal{N}\pi g^*$-s-nghd of $x$, therefore $x \in \lambda$.

(b) Let $\lambda \in \mathcal{N}\pi g^*$-s-N($x$) and $N \subseteq M$. Therefore there exists $G \in \mathcal{N}\pi g^*$-sO($U, X$) such that $x \in G \subseteq N \subseteq M$ implies $M$ is a $\mathcal{N}\pi g^*$-s-nghd of $x$ and hence $M \in \mathcal{N}\pi g^*$-s-N($x$).

(c) Let $\lambda \in \mathcal{N}\pi g^*$-s-N($x$) implies $G \in \mathcal{N}\pi g^*$-sO($X$) such that $x \in G \subseteq N \subseteq \mathcal{N}\pi g^*$-s-nghd of each of its points implies for all $y \in G$, $G$ is the $\mathcal{N}\pi g^*$-s-nghd of $y$ and hence $G \subseteq \mathcal{N}\pi g^*$-s-N($y$) for all $y \in G$.

Theorem 5.7. If $A$ is a subset of $(U, \tau_R(X))$. Then $\mathcal{N}\pi g^*$-s-int($A$) = $\bigcup \{B : B$ is a $\mathcal{N}\pi g^*$-s-open, $B \subseteq A\}$.

Proof: let $A$ be a subset of $(U, \tau_R(X))$.

$x \in \mathcal{N}\pi g^*$-s-int($A$)