The concept of an edge vertex prime labeling of a graph is a labeling of a graph where the labels of the vertices are integers such that each pair of labels from the label set is relatively prime. In this paper, we investigate several families of edge vertex prime labeling for triangular and rectangular book, Butterfly graph, Drums graph $D_n$, Jahangir graph $J_{n,3}$ and $J_{n,4}$.

**Keywords**
Prime labeling, edge vertex prime labeling, relatively prime, triangular and rectangular book, butterfly graph.

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## 1. Introduction

We begin with finite, connected and undirected graph $G = (V(G), E(G))$ without loops and multiple edges. Here elements of $V(G)$ and $E(G)$ are known as vertex set and edge set, respectively. For all other terminology and notations in graph theory we follow Balakrishnan and Ranganathan [1].

The concept of an edge vertex prime labeling of a graph is a variation of a prime labeling, which was developed by Roger Entringer and first introduced in [7] by Tout, Dabboucy and Howalla. For a simple graph $G$ with $n$ vertices in the vertex set $V(G)$, a prime labeling is an assignment of the integers 1 to $n$ as labels of the vertices such that each pair of labels from adjacent vertices is relatively prime. A graph that has such a labeling is called prime. Gallian’s dynamic graph labeling survey [2] contains a detailed list of graphs that have been proven to be prime. Recently, much focus has been on variations of prime labeling, such as an edge vertex prime labeling, which involves for any edge $e = xy$, they are pairwise relatively prime. We will give brief summary of definitions which are useful for the present investigations. Let $G = (V(G), E(G))$ be a graph with $p$ vertices and $q$ edges. A bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ is called an edge vertex prime labeling if for each edge $e = xy$, $gcd(f(x), f(y)) = 1$. A graph which admits an edge vertex prime labeling is called an edge vertex prime graph. Jagadesh and Baskar Babujee [3] was introduced the concept of an edge vertex prime labeling and they proved the existence of the same paths, cycles and star $K_{1,n}$. In [4], if $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are two connected graphs, then the graph obtained by superimposing any selected vertex of $G_2$ on any selected vertex of $G_1$ is denoted by $G_1 \hat{O} G_2$. The resultant graph $G = G_1 \hat{O} G_2$ contains $p_1 + p_2 - 1$ vertices $q_1 + q_2$ edges. In general, there are $p_1 p_2$ possibilities of getting $G_1 \hat{O} G_2$ from $G_1$ and $G_2$. In [4], they also proved that an edge vertex prime labeling, for some class of graphs such as generalized star, generalized cycle star, $p + q$ is odd for $G \hat{O} P_n$, $G \hat{O} C_n$, $G \hat{O} K_{1,n}$. Parmer [5] proved that wheel $W_n$, fan $f_n$, friendship graph $F_n$ are edge vertex prime labeling. In [6], they also proved that $K_{2,n}$, for all $n$ and $K_{3,n}$ for $n = \{2, 3, \ldots, 29\}$ are edge vertex prime labeling. The triangular book with $n$ pages is defined as $n$ copies of cycles $C_3$ sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B_{3,n}$. The rectangular book with $n$ pages is defined as $n$ copies of cycles $C_4$ sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B_{4,n}$. A shell $S_n$ is the graph obtained by taking $(n - 3)$ concurrent chords in a cycle $C_n$. The vertex at which all the chords are concurrent is called the apex. The shell is also called fan $f_{n-1}$. Parmer [5] proved that the fan graph $f_n$ is an edge vertex prime graph. In this case, we conclude that the shell $S_n$ that is, $f_{n-1}$ is an edge vertex prime graph.

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**Abstract**
Let $G = (V(G), E(G))$ be a graph with $p$ vertices and $q$ edges. A bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ is said to be an edge vertex prime labeling, if for any edge $xy \in E(G)$, it is satisfies that $f(x), f(y)$ and $f(xy)$ are pairwise relatively prime. In this paper, we investigate several families of edge vertex prime labeling for triangular and rectangular book, Butterfly graph, Drums graph $D_n$, Jahangir graph $J_{n,3}$ and $J_{n,4}$. In [2], contains a detailed list of graphs that have been proven to be prime. Recently, much focus has been on variations of prime labeling, such as an edge vertex prime labeling, which involves for any edge $e = xy$, they are pairwise relatively prime. We will give brief summary of definitions which are useful for the present investigations. Let $G = (V(G), E(G))$ be a graph with $p$ vertices and $q$ edges. A bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ is called an edge vertex prime labeling if for each edge $e = xy$, $gcd(f(x), f(y)) = 1$, $gcd(f(x), f(xy)) = 1$, $gcd(f(y), f(xy)) = 1$. A graph which admits an edge vertex prime labeling is called an edge vertex prime graph. Jagadesh and Baskar Babujee [3] was introduced the concept of an edge vertex prime labeling and they proved the existence of the same paths, cycles and star $K_{1,n}$. In [4], if $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are two connected graphs, then the graph obtained by superimposing any selected vertex of $G_2$ on any selected vertex of $G_1$ is denoted by $G_1 \hat{O} G_2$. The resultant graph $G = G_1 \hat{O} G_2$ contains $p_1 + p_2 - 1$ vertices $q_1 + q_2$ edges. In general, there are $p_1 p_2$ possibilities of getting $G_1 \hat{O} G_2$ from $G_1$ and $G_2$. In [4], they also proved that an edge vertex prime labeling, for some class of graphs such as generalized star, generalized cycle star, $p + q$ is odd for $G \hat{O} P_n$, $G \hat{O} C_n$, $G \hat{O} K_{1,n}$. Parmer [5] proved that wheel $W_n$, fan $f_n$, friendship graph $F_n$ are edge vertex prime labeling. In [6], they also proved that $K_{2,n}$, for all $n$ and $K_{3,n}$ for $n = \{2, 3, \ldots, 29\}$ are edge vertex prime labeling. The triangular book with $n$ pages is defined as $n$ copies of cycles $C_3$ sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B_{3,n}$. The rectangular book with $n$ pages is defined as $n$ copies of cycles $C_4$ sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B_{4,n}$. A shell $S_n$ is the graph obtained by taking $(n - 3)$ concurrent chords in a cycle $C_n$. The vertex at which all the chords are concurrent is called the apex. The shell is also called fan $f_{n-1}$. Parmer [5] proved that the fan graph $f_n$ is an edge vertex prime graph. In this case, we conclude that the shell $S_n$ that is, $f_{n-1}$ is an edge vertex prime graph.
graph is a double shell in which each shell has any order. A butterfly graph is a bow graph with exactly two pendant edges at the apex. A multiple shell is a collection of edge disjoint shells that have their apex in common. Hence a double shell consists of two edge disjoint shells with a common apex.

The Drums graph $D_n$, $n \geq 3$ can be constructed by two cycles $2C_n$, $n \geq 3$ joining two paths $2P_n$, $n \geq 3$ with sharing a common vertex and it is denoted by $D_n = 2C_n + 2P_n$.

In this paper, we investigate triangular and rectangular book. Butterfly graph, multiple shell, Drums graph $D_n$, Jahnagir graph $I_{n,3}$, $I_{n,4}$ and that some classes of graphs are edge vertex prime labeling.

2. Main Results

**Theorem 2.1.** The triangular book $B_{3,n}$ admits edge vertex prime labeling for all $n$.

**Proof.** Let $B_{3,n}$ be a triangular book. Then $V(B_{3,n}) = \{x,y,z_i : 1 \leq i \leq n\}$ and $E(B_{3,n}) = \{x,y,z_i y,z_i x : 1 \leq i \leq n\}$. Also, $|V(B_{3,n})| = n+2$ and $|E(B_{3,n})| = 2n + 1$. Define a bijective function $f : V(B_{3,n}) \cup E(B_{3,n}) \rightarrow \{1,2,\ldots,3n+3\}$ by $f(x) = 1$, $f(xy) = 2$, $f(y) = 3$, for each $1 \leq i \leq [\frac{n}{2}]$, $f(z_{2i-1}) = 6i - 1$, $f(z_{2i}) = 6i + 1$, $f(xz_{2i-1}) = 6i$, $f(xz_{2i}) = 6i + 3$, $f(yz_{2i-1}) = 6i - 2$, $f(yz_{2i}) = 6i + 2$. Clearly, (i) $f(x), f(y)$ and $f(xy)$, (ii) $f(x), f(z_i)$ and $f(xz_i)$, (iii) $f(y), f(z_i)$ and $f(yz_i)$ are pairwise relatively prime. Therefore, for any edge $xy \in E(B_{3,n})$, the numbers $f(x), f(y)$ and $f(xy)$ are pairwise relatively prime. Hence the triangular book $B_{3,n}$ is an edge vertex prime labeling for all $n$. \hfill \Box

**Theorem 2.2.** The rectangular book $B_{4,n}$ admits edge vertex prime labeling for all $n$.

**Proof.** Let $B_{4,n}$ be a rectangular book. Then $V(B_{4,n}) = \{x_i, y_i : 1 \leq i \leq n\}$ and $E(B_{4,n}) = \{x_i y_i, y_i x_i : 1 \leq i \leq n\} \cup \{xy\}$. Also, $|V(B_{4,n})| = 2n + 2$ and $|E(B_{4,n})| = 3n + 1$. Define a bijective function $f : V(B_{4,n}) \cup E(B_{4,n}) \rightarrow \{1,2,\ldots,5n+3\}$ by $f(x) = 1$, $f(y) = 5$, $f(xy) = 4$, $f(x_1) = 2$, $f(x_1 y_1) = 3$, $f(y_1) = 6$, $f(y_1 x_1) = 7$, $f(x_1 x_1 y_1) = 8$. For each $1 \leq i \leq [\frac{n}{2}]$, $f(x_{2i}) = 10i - 1$, $f(y_{2i}) = 10i + 3$, $f(x_{2i} y_{2i}) = 10i + 1$, $f(x_{2i+1}) = 10i + 2$. For each $1 \leq i \leq [\frac{n-2}{2}]$, $f(x_{2i+1}) = 10i + 5$, $f(y_{2i+1}) = 10i + 7$, $f(x_{2i+1} y_{2i+1}) = 10i + 6$, $f(x_{2i+2} y_{2i+2}) = 10i + 8$. Clearly, (i) $f(x), f(y)$ and $f(xy)$, (ii) $f(x), f(x_i)$ and $f(x_i y_i)$, (iii) $f(y), f(y_i)$ and $f(y_i x_i)$, (iv) $f(x), f(y_i)$ and $f(xy_i)$ are pairwise relatively prime. Therefore, for any edge $xy \in E(B_{4,n})$, the numbers $f(x), f(y)$ and $f(xy)$ are pairwise relatively prime. Hence the rectangular book $B_{4,n}$ is an edge vertex prime labeling. \hfill \Box

**Theorem 2.3.** The butterfly graph with shell is an edge vertex prime labeling.

**Proof.** Let $G$ be a butterfly graph with shells of order $m$ and $n$ excluding the apex. Without loss of generality, assume that $m \leq n$. Then $V(G) = \{w_0, w_1, w_2, u_i : 1 \leq i \leq m\}$ and $E(G) = \{w_0 w_1, w_0 w_2, w_0 u_i : 1 \leq i \leq m, w_i u_{i+1} : 1 \leq i \leq m - 1, w_0 v_j : m + 1 \leq j \leq m + n - 1\}$. Also, $|V(G)| = m + n + 3$ and $|E(G)| = 2m + 2n$. Define a bijective function $f : V(G) \cup E(G) \rightarrow \{1,2,\ldots,3m + 3n + 3\}$ by $f(w_0) = 1$, $f(u_i) = \begin{cases} 3i : & i \text{ is odd} \\ 3i - 1 : & i \text{ is even} \end{cases}$ $f(u_{i+1}) = 3i + 1, i = 1, 2, 3, \ldots, m - 1$. Consider the following cases.

**Case 1a.** $m$ is even.

$f(w_0) = \begin{cases} 3i - 1 : & i = 1, 3, \ldots, m - 1 \\ 3i : & i = 2, 4, \ldots, m \end{cases}$

**Case 1b.** $m$ is odd.

$f(w_0) = \begin{cases} 3i - 1 : & i = 1, 3, 5, \ldots, m \\ 3i : & i = 2, 4, 6, \ldots, m - 1 \end{cases}$

**Case 2a.** Both $m, n$ is even.

$f(v_j) = \begin{cases} 3j - 2 : & j = m + 1, m + 3, \ldots, m + n - 1 \\ 3j - 1 : & j = m + 2, m + 4, \ldots, m + n \end{cases}$

**Case 2b.** $m$ is even and $n$ is odd.

$f(v_j) = \begin{cases} 3j + 1 : & j = m + 1, m + 3, \ldots, m + n - 1 \\ 3j + 2 : & j = m + 2, m + 4, \ldots, m + n \end{cases}$

**Case 3a.** Both $m, n$ is even and both $m, n$ is odd.

$f(v_j) = \begin{cases} 3j - 1 : & j = m + 1, m + 3, \ldots, m + n - 1 \\ 3j - 2 : & j = m + 2, m + 4, \ldots, m + n \end{cases}$

**Case 3b.** $m$ is odd and $n$ is even or $m$ is even and $n$ is odd.

$f(v_j) = \begin{cases} 3j + 2 : & j = m + 1, m + 3, \ldots, m + n - 1 \\ 3j + 1 : & j = m + 2, m + 4, \ldots, m + n \end{cases}$

**Case 4a.** $3m + 3n - 1$ is even.

$f(w_1) = 3m + 3n, f(w_1 w_2) = 3m + 3n + 1, f(w_2) = 3m + 3n + 2, f(w_2 w_2) = 3m + 3n + 3$.

**Case 4b.** $3m + 3n - 1$ is odd.

$f(w_1) = 3m + 3n + 1, f(w_1 w_2) = 3m + 3n, f(w_2) = 3m + 3n + 3, f(w_2 w_2) = 3m + 3n + 2$.

Clearly, (i) $f(w), f(u_i)$ and $f(w_0 u_i)$, (ii) $f(u_i), f(u_{i+1})$ and $f(u_i u_{i+1})$, (iii) $f(w), f(v_j)$ and $f(w_0 v_j)$, (iv) $f(v_j), f(v_{j+1})$ and $f(v_j v_{j+1})$, (v) $f(w), f(w_0)$ and $f(w_0 w_2)$, (vi) $f(w), f(w_2)$ and $f(w_0 w_2)$ are pairwise relatively prime. Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the butterfly graph with shell is an edge vertex prime labeling. \hfill \Box
Corollary 2.4. A multiple shell is an edge vertex prime graph.

Theorem 2.5. The Drums graph \( D_n \), \( n \geq 3 \) is an edge vertex prime labeling.

Proof. Let \( D_n \) be the Drums graph. Then \( V(D_n) = \{u_i: 1 \leq i \leq 4n - 3\} \) and \( E(D_n) = \{uu_{i+1}: 1 \leq i < n - 1\} \cup \{uu_n\} \cup \{uu_{i+1}: n + 1 \leq i \leq 2n - 2\} \cup \{uu_{i+2}\} \cup \{uu_{i+1}: 2n \leq i \leq 3n - 3\} \cup \{uu_{i+3}: 3n - 1 \leq i \leq 4n - 2\} \cup \{uu_{i+4}\} \cup \{uu_{i+3}: 4n \leq i \leq 5n - 3\} \cup \{uu_{i+4}: 5n - 1 \leq i \leq 6n - 2\} \cup \{uu_{i+5}\}. \]

Define a bijective function \( f: V(D_n) \cup E(D_n) \rightarrow \{1, 2, \ldots, 8n - 4\} \). Clearly, for any edge \( uv \in E(D_n) \), the numbers \( f(u), f(v) \) and \( f(uv) \) are pairwise relatively prime. Hence joining the path \( P_2 \) of two copies of cycle \( C_n \) admits edge vertex prime labeling.

Theorem 2.6. The following graphs are edge vertex prime labeling.

(a) \( K_{1,m} \) for all \( m, n \geq 1 \), (b) \( K_{1,m} + K_1 \) for all \( m \geq 1 \), (c) \( K_{m,n} \) for all \( m, n \geq 1 \).

Proof. (a) Without loss of generality, we assume that \( m \leq n \). Now, let the vertex set of \( K_{1,m} \) be \( L = \{u\} \) and \( M = \{v_1: 1 \leq i \leq m\} \) and edge set \( N = \{uv_i: 1 \leq i \leq m\} \) and let the vertex set of \( K_{m,n} \) be \( \{v_1, \ldots, v_n\} \). Then label the vertices and edges of the sets \( L, M \) and \( N \) as \( f(u) = 1 \) and \( f(v_1) = 2i + 1 \) for \( 1 \leq i \leq m \), \( f(uv_i) = 2i \) for \( 1 \leq i \leq m \), \( f(v) = p \), where \( p \) is the greatest prime number in the set \( \{2n + 2, 2m + 3, \ldots, 2m + n, 2n + n + 1\} \). Hence, we can easily verify that the considered graph is an edge vertex prime labeling.

(b) Let \( G = K_{1,m} + K_1 \) be a graph. Then \( V(G) = \{u, v, v_1: 1 \leq i \leq m\} \) and \( E(G) = \{uu, uv_i, v_1: 1 \leq i \leq m\} \). Also, \( |V(G)| = m + 2 \) and \( |E(G)| = 2m \). Define a bijective function \( f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, 3m + 2\} \) by \( f(u) = 1 \), \( f(uv_i) = 2i + 1 \) for \( 1 \leq i \leq m \), \( f(v) = p \), where \( p \) is choose the greatest prime number in the set \( \{2m + 2, 2m + 3, \ldots, 3m + 2\} \) and label the edge set \( \{v_1: 1 \leq i \leq m\} \) by remaining labels. Hence, we can easily verify that the considered graph is an edge vertex prime labeling.

(c) Let \( G = \overline{K_{m,n}} \) for all \( m, n \geq 1 \). Without loss of generality, assume that \( m \leq n \). Now, let we label the vertices of \( \overline{K_{m,n}} \) as \( 1, 2, 3, \ldots, m \) and label the vertices of \( \overline{K_{m,n}} \) as \( m + 1, m + 2, \ldots, m + n \). Hence, we can easily verify that the considered graph is an edge vertex prime labeling.

Theorem 2.7. The graph \( G \) obtained by joining the path \( P_2 \) of two copies of cycle \( C_n \) is an edge vertex prime labeling, where \( n \equiv 0, 2 \) (mod \( 2 \)).

Proof. Let \( \{v_1, v_2, \ldots, v_n\} \) and \( \{v_{n+1}, v_{n+2}, \ldots, v_{2n}\} \) are the vertices of first and second cycle \( C_n \), respectively. Then \( V(G) = \{v_i: 1 \leq i \leq 2n\} \) and \( E(G) = \{v_iv_{i+1}: 1 \leq i < n - 1\} \cup \{v_iv_{i+1}: n + 1 \leq i \leq 2n - 1\} \cup \{v_{i+1}v_{2n}\} \cup \{v_1v_{n+1}\} \). Also, \( |V(G)| = 2n \) and \( |E(G)| = 2n + 1 \). Define a bijective function \( f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, 4n + 1\} \) by \( f(v_i) = 2i - 1 \) for \( 1 \leq i \leq 2n \), \( f(v_{i+1}) = 2i \) for \( 1 \leq i \leq n - 1 \), \( f(v_{i+1}) = 2n \), \( f(v_{i+1}) = 2i \) for \( n + 1 \leq i \leq 2n - 1 \), \( f(v_{i+1}) = 4n \), \( f(v_{i+1}) = 4n + 1 \). Clearly, for any edge \( uv \in E(G) \), the numbers \( f(u), f(v) \) and \( f(uv) \) are pairwise relatively prime. Hence joining the path \( P_2 \) of two copies of cycle \( C_n \) admits edge vertex prime labeling.
Theorem 2.10. The graph $J_{n, 3}$, $n \geq 1$ is an edge vertex prime labeling.

Proof. Let $G$ be a $J_{n, 3}$ graph. Then $V(G) = \{u, v_i : 1 \leq i \leq 3n\}$ and $E(G) = \{v_i v_{i+1} : 1 \leq i \leq 3n - 1\} \cup \{uv_1, uv_{3n+1}, uv_{2n+1}\}$. Also, $|V(G)| = 3n + 1$ and $|E(G)| = 3n + 3$. Define a bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, 6n + 4\}$ by $f(v_i) = 2i - 1$ for $1 \leq i \leq 3n$, $f(v_{i+1}) = 2i + 1$ for $1 \leq i \leq 3n - 1$, $f(v_{3n+1}) = 6n$, $f(u) = 6n + 1$, $f(uw_1) = 6n + 2$, $f(w_1) = 6n + 3$, $f(w_1v_1) = 6n + 4$, $f(uw_2) = 6n + 5$, $f(w_2) = 6n + 6$, $f(w_2v_{3n+1}) = 6n + 7$, $f(v_{3n+1}) = 6n + 8$, $f(w_3) = 6n + 9$, $f(w_3v_{2n+1}) = 6n + 10$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the graph $G$ is an edge vertex prime labeling.

Consider the following cases.

Case (i). When $n = 1 \mod 3$.

$f(uv_1) = 6n + 4$, $f(uv_{n+1}) = 6n + 2$, $f(uv_{2n+1}) = 6n + 3$.

Case (ii). When $n = 2 \mod 3$.

$f(uv_1) = 6n + 3$, $f(uv_{n+1}) = 6n + 2$, $f(uv_{2n+1}) = 6n + 4$.

Next, we prove the property of an edge vertex prime labeling. For each $1 \leq i \leq 3n - 1$,

$\text{gcd}(f(v_i), f(v_{i+1})) = \text{gcd}(2i - 1, 2i + 1) = 1$,

$\text{gcd}(f(v_i), f(v_1v_{i+1})) = \text{gcd}(2i - 1, 2i) = 1$,

$\text{gcd}(f(v_{i+1}), f(v_1v_{i+1})) = \text{gcd}(2i + 1, 2i) = 1$,

$\text{gcd}(f(v_1), f(v_{3n})) = \text{gcd}(6n - 1, n) = 1$,

$\text{gcd}(f(v_1), f(v_{1v_{3n}})) = \text{gcd}(6n, 1, n) = 1$.

Verification of Case (i).

$\text{gcd}(f(u), f(v_1)) = \text{gcd}(6n, 1) = 1$,

$\text{gcd}(f(u), f(uv_1)) = \text{gcd}(6n + 1, 6n) = 1$,

$\text{gcd}(f(u), f(uv_{n+1})) = \text{gcd}(6n, 1 + 6n + 2) = 1$,

$\text{gcd}(f(u), f(uv_{2n+1})) = \text{gcd}(2n + 1, 6n + 2) = 1$,

$\text{gcd}(f(u), f(w_1v_1)) = \text{gcd}(6n + 1, 4n + 1) = 1$,

$\text{gcd}(f(u), f(w_2v_{3n+1})) = \text{gcd}(6n, 1, 4n) = 1$,

$\text{gcd}(f(u), f(w_2v_{2n+1})) = \text{gcd}(6n + 1, 6n + 3) = 1$,

Similarly the other case (ii) are verified. Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $J_{n, 3}, n \geq 1$ has an edge vertex prime labeling.

Theorem 2.11. The graph $G$ is obtained by subdividing the edges which are all incident with the centre vertex of $J_{n, 3}$ is an edge vertex prime labeling, where $n$ is congruent to 0 modulo 3.

Proof. Let $G$ be a graph which is obtained by subdividing the edges which are all adjacent with the centre vertex of $J_{n, 3}$, where $n \equiv 0 \pmod{3}$. Then $V(G) = \{u, v_i, w_j : 1 \leq i \leq 3n, 1 \leq j \leq 3\}$ and $E(G) = \{v_i v_{i+3}, v_1 v_{3n+1} : 1 \leq i \leq 3n - 1\} \cup \{uv_1, uv_{3n+1}, uv_{2n+1}\}$. Here $|V(G)| = 3n + 4$ and $|E(G)| = 3n + 6$. Define a bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, 6n + 10\}$ by $f(v_i) = 2i - 1$ for $1 \leq i \leq 3n$, $f(v_{i+3}) = 2i + 1$ for $1 \leq i \leq 3n - 1$, $f(v_{3n+1}) = 6n$, $f(u) = 6n + 1$, $f(uw_1) = 6n + 2$, $f(w_1) = 6n + 3$, $f(w_1v_1) = 6n + 4$, $f(uw_2) = 6n + 5$, $f(w_2) = 6n + 6$, $f(w_2v_{3n+1}) = 6n + 7$, $f(v_{3n+1}) = 6n + 8$, $f(w_3) = 6n + 9$, $f(w_3v_{2n+1}) = 6n + 10$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the graph $G$ is an edge vertex prime labeling.

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