On the anti fuzzy subsemirings under $t$-norms

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Abstract
In this paper we introduce anti $T$-fuzzy subsemirings and anti $T$-product of two fuzzy sets which can be regarded as a generalization of anti fuzzy subgroups under $t$-norms.

Keywords
Anti-$T$-fuzzy subsemiring, anti-$T$-product and homomorphism.

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1. Introduction

After an introduction of fuzzy sets by L.A. Zadeh [4] several researchers explored on the generalization of the notion of fuzzy set. Since the concept of fuzzy subgroups was introduced by Rosenfeld [1], it has been studying by several authors in [2, 5-10]. Recently, Biswas [8] has proposed the concept of anti fuzzy subgroups. We will generalize this concept to that of anti fuzzy subsemirings under $t$-norms and investigate some of their properties. We will also study the problems of the anti products of anti fuzzy subsemirings under $t$-norms.

Throughout this paper, let $R$ be a ring, $I = [0, 1]$. We will denote a $t$-norm by $T$ and refer for its properties to [7, 10].

2. Preliminaries

Definition 2.1. Let $T_1$ and $T_2$ be $t$-norms and $f : I \to I$ an order-preserving bijection. We say that $T_2$ is the conjugate of $T_1$, written as $T_1^f$, if $T_1(a, b) = T_2(a, b) = 1 - T_1(1 - a, 1 - b), \ \forall a, b \in I.$

and that $T_2$ dominates $T_1$, written as $T_2 >> T_1$ or $T_1 << T_2$, if $T_2(T_1(a, b), T_1(c, d)) >> T_1(T_2(a, c), T_2(b, d)) \ \forall a, b, c, d \in I.$

Definition 2.2. Let $X$ be an ordinary set. By a fuzzy subsets $u$ of $X$, we mean a function $u : X \to I$ with $u(x)$ as the grade of membership for $\forall x \in X$.

Definition 2.3. Let $R$ be a semiring. A fuzzy subset $A$ of $R$ is said to be an anti-fuzzy subsemiring of $R$ if it satisfies the following conditions.
(i) $\mu_A(x + y) \leq \mu_A(x) \vee \mu_A(y)$
(ii) $\mu_A(x y) \leq \mu_A(x) \mu_A(y)$, for all $x$ and $y$ in $R$.

Definition 2.4. A fuzzy subsemiring of $R$ under a $t$-norm $T$ (called $T$-fuzzy subsemiring of $R$, for short) is a fuzzy subset $u$ of $R$ satisfying
(i) $u(x + y) \geq T(u(x), u(y)), \ \forall x, y \in R.$
(ii) $u(xy) \geq T(u(x), u(y)), \ \forall x, y \in R.$

3. Anti Fuzzy Subsemiring under $t$-norms

Definition 3.1. A fuzzy subset $u$ of $R$ is called anti-fuzzy subsemiring of $R$ under a $t$-norm $T$ (called anti-$T$-fuzzy subsemiring of $R$, for short) if
(i) $u(x + y) \leq T(u(x), u(y))$
(ii) $u(xy) \leq T(u(x), u(y))$
where $T$ is the conjugate of $T, \forall x, y \in R.$

Proposition 3.2. A fuzzy subset $u$ of $R$ is an anti-$T$-fuzzy subsemiring of $R$ if its complement $u^c$, defined by $u^c(x) = 1 - u(x), \ \forall x \in R$, is a $T$-fuzzy subsemiring of $R$.

Proof. Let $u$ be an anti-$T$-fuzzy subsemiring of $R$.
Then (i) $u(x + y) \leq T(u(x), u(y))$
(ii) $u(xy) \leq T(u(x), u(y))$
Now $u^c(x + y) = 1 - u(x + y)$
$\geq 1 - T(u(x), u(y))$
\[= 1 - [1 - T(1 - u(x), 1 - u(y))]\]
\[= T(u(x), u(y))\]
and \(u^c(xy) = 1 - u(xy)\)
\[\geq 1 - T(u(x), u(y))\]
\[= 1 - [1 - T(1 - u(x), 1 - u(y))]\]
\[= T(u(x), u(y))\]
Hence \(u^c\) is a \(T\)-fuzzy subsemiring of \(R\).

Proposition 3.3. Let \(T\) be a \(t\)-norm satisfying \(T(a, b) < 1, \forall a, b \in (0, 1)\). If \(u\) is an anti \(T\)-fuzzy subsemiring of \(R\), then \(L(u) = \{x \in R; u(x) < 1\}\) is a subsemiring of \(R\).

Proof. Let \(u\) be an anti-\(T\)-fuzzy subsemiring of \(R\) and \(L(u) = \{x \in R; u(x) < 1\}\). Let \(x, y \in L(u)\). \(\therefore u(x) < 1, u(y) < 1\).
\[
\therefore u(x+y) \leq T(u(x), u(y)) = 1 - T(1 - u(x); 1 - u(y)) < 1 \quad \therefore T(1 - u(x), 1 - u(y)) < 1.
\]
\[
\therefore x+y \in L(u), \forall x, y \in L(u).
\]
\[
\therefore L(u) \text{ is a subsemiring of } R.
\]

Definition 3.4. Let \(X\) and \(Y\) be ordinary sets and \(h : X \to Y\) be a mapping. If \(u\) is a fuzzy subsets of \(X\), then the fuzzy subset \(h(u)\) of \(Y\) defined by
\[
[h(u)](y) = \begin{cases} 
\inf \{x \in h^{-1}(y) \mid u(x)\} & \text{if } y \in h(x) \\
0 & \text{otherwise}
\end{cases}
\]
is called the image of \(u\) under \(h\).

Definition 3.5. If \(u\) is a fuzzy subset of \(Y\), then the fuzzy subset \(h^{-1}(u)\) of \(X\) defined by
\[
[h^{-1}(u)](x) = u(h(x)), \forall x \in X
\]
is called the pre-image of \(u\) under \(h\).

Proposition 3.6. Let \(h\) be a homomorphism of semiring \(R_1\) into semiring \(R_2\). If \(T\) is a continuous \(t\)-norm and \(u\) is an anti \(T\)-fuzzy subsemiring of \(R_1\) then \(h(u)\), the image of \(u\) under \(h\), is an anti \(T\)-fuzzy subsemiring of \(R_2\).

Proposition 3.7. Let \(h\) be a homomorphism of semiring \(R_1\) into semiring \(R_2\). If \(u\) is an anti-\(T\)-fuzzy subsemiring of \(R_2\), then \(h^{-1}(u)\), the pre-image of \(u\) under \(h\), is an anti \(T\)-fuzzy subsemiring of \(R_1\).

Proof. Let \(h : R_1 \to R_2\) be a homomorphism and \(u\) be an anti \(T\)-fuzzy subsemiring of \(R_2\).
\[
[h^{-1}(u)](x_1 + x_2) = u[h(x_1 + x_2)]
\]
\[
\leq T(u[h(x_1)], u[h(x_2)])
\]
\[
\leq T([h^{-1}(u)](x_1), [h^{-1}(u)](x_2))
\]
\[
[h^{-1}(u)](x_1x_2) = u[h(x_1)x_2]
\]
\[
\leq T(u[h(x_1)], u[h(x_2)])
\]
\[
\leq T([h^{-1}(u)](x_1), [h^{-1}(u)](x_2)).
\]
Hence \(h^{-1}(u)\) is an anti \(T\)-fuzzy subsemiring of \(R_1\).

4. Anti Products under \(t\)-norms of Anti \(T\)-fuzzy Subsemirings

Definition 4.1. Let \(u\) and \(v\) be fuzzy subsets of \(R\). The anti product of \(u\) and \(v\) under a \(t\)-norm \(T\) (called anti \(T\)-product of \(u\) and \(v\) for short), written as \([u,v]_T\), is a fuzzy subset of \(R\) defined by
\[
[u,v]_T(x) = T(u(x), v(x)), \forall x \in R.
\]
Based on the properties of anti \(T\)-fuzzy subsemirings, we have the following properties of anti \(T\)-products.

Lemma 4.2. If \(T_2 \gg T_1\), then \(T_2 << T_1\).

Proof. It is obvious.

Proposition 4.3. Let \(T_1\) and \(T_2\) be \(t\)-norms and \(T_2 \gg T_1\). If \(u\) and \(v\) are anti \(T_1\)-fuzzy subsemirings of \(R\) then \([u,v]_{T_2}\), the anti \(T_2\)-product of \(u\) and \(v\) is also an anti \(T_1\)-fuzzy subsemiring of \(R\).

Proof. \([u,v]_{T_2}(x+y) = T_2(u(x+y), v(x+y))\)
\[
\leq T_2(T_1(u(x), u(y)), T_1(v(x), v(y)))
\]
\[
\leq T_1(T_2(u(x), v(x)), T_2(u(y), v(y)))
\]
\[
= T_1([u,v]_{T_2}(x), [u,v]_{T_2}(y))
\]
\[
[u,v]_{T_2}(xy) = T_2(u(x)y), v(xy))\]
\[
\leq T_2(T_1(u(x), u(y)), T_1(v(x), v(y)))
\]
\[
\leq T_1(T_2(u(x), v(x)), T_2(u(y), v(y)))
\]
\[
= T_1([u,v]_{T_2}(x), [u,v]_{T_2}(y)).
\]
Hence \(T_2\) product of \(u\) and \(v\) is also anti \(T_1\)-fuzzy subsemiring of \(R\).

Proposition 4.4. Let \(T_1, T_2\) be \(t\)-norms and \(T_2 \gg T_1\) and \(u\) and \(v\) are anti \(T_1\)-fuzzy subsemirings of \(R\). If \(h\) is homomorphism of \(R_1 \rightarrow R_2\) then \(h^{-1}([u,v]_{T_2})\) is also an anti \(T_1\)-fuzzy subsemiring of \(R_1\).

Proof. \(h^{-1}([u,v]_{T_2})(x+y) = [u,v]_{T_2}(h(x+y))\)
\[
= [u,v]_{T_2}(h(x) + h(y))
\]
\[
= T_2(u(h(x)), v(h(x)) + h(y)))
\]
\[
\leq T_2(T_1(u(h(x)), u(h(y)))).
\]
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### References


