Edge vertex prime labeling of graphs

M. Simaringa¹* and S. Muthukumaran²

Abstract
A bijective labeling \( f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, |V(G) \cup E(G)|\} \) is an edge vertex prime labeling if for any edge \( uv \in E(G) \), the numbers \( f(u), f(v) \) and \( f(uv) \) are pairwise relatively prime. A graph \( G \) which admits edge vertex prime labeling is called an edge vertex prime graph. In this paper, we have obtained some class of graphs such as \( p + q \) is odd for \( GΩW_n \), \( GΩf_n \), \( GΩF_n \), \( p + q \) is even for \( GΩP_n \), crown graph and union of cycles are edge vertex prime graph.

Keywords
Prime labeling, edge vertex prime labeling, relatively prime.

AMS Subject Classification
05C78.

1. Introduction

All our graphs are simple, finite and undirected and we follow Balakrishnan and Ranganathan [1] for standard notations and terminology. \( G = (V(G), E(G)) \), where \( V(G) \) is vertex set and \( E(G) \) is edge set of the graph. \( |V(G)| \) and \( |E(G)| \) are denoted by the number of vertices and edges respectively, which is order and size of \( G \). A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. See the dynamic graph labeling survey [2] by Gallian is regularly updated. Prime labeling is a type of graph labeling developed by Roger Entriger that was first formally introduced by Tout, Dabboucy and Howalla [7]. We define \(|n| = 1, 2, \ldots, n\) where \( n \) is a positive integers. Given a simple graph \( G \) of order \( n \), a prime labeling consists of labeling the vertices with integers from the set \([n]\) so that the labels of any pair of adjacent vertices are relatively prime.

Edge vertex prime labeling is a variation of prime labeling. A bijective function \( f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, |V(G) \cup E(G)|\} \) is said to be an edge vertex prime labeling if for any edge \( uv \in E(G) \), the numbers \( f(u), f(v) \) and \( f(uv) \) are pairwise relatively prime. Jagadhes and Baskar Babujee [3] introduced the concept of an edge vertex prime labeling and proved the existence of the same paths, cycles and star \( K_{1,n} \) in [4], if \( G_1(p_1, q_1) \) and \( G_2(p_2, q_2) \) are two connected graphs, then the graph obtained by superimposing any selected vertex of \( G_2 \) on any selected vertex of \( G_1 \) is denoted by \( G_1 \hat{G} G_2 \). The resultant graph \( G = G_1 \hat{G} G_2 \) contains \( p_1 + p_2 - 1 \) vertices \( q_1 + q_2 \) edges. In general, there are \( p_1 p_2 \) possibilities of getting \( G_1 \hat{G} G_2 \) from \( G_1 \) and \( G_2 \). In [4], they also proved that edge vertex prime labeling, for some class of graphs such as generalized star, generalized cycle star, \( p + q \) is odd for \( GΩK_{1,n} \), \( GΩf_n \), \( GΩF_n \). Parmer [5] proved that wheel \( W_n \), fan \( f_n \), friendship graph \( F_n \) are an edge vertex prime labeling. In [6], they also proved that \( K_{2,n} \), for all \( n \) and \( K_{1,n} \), for \( n = \{2, 3, \ldots, 29\} \) are edge vertex prime labeling. We [8] proved that triangular and rectangular book, butterfly graph, Drums graph \( D_n \), Jahangir graph \( J_{n,3} \) and \( J_{n,4} \) are edge vertex prime labeling.

A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. Fan graph \( f_n \), \( n \geq 2 \) obtained by joining all vertices of a path \( P_n \) to a further vertex called centre. That is, \( f_n = P_n + K_1 \). Friendship graph \( F_n \) is a graph which consists of \( n \)-triangles with a common vertex. The crown graph is obtained from a cycle \( C_n \) by attaching a pendant edge at each vertex of the \( n \)-cycle. Let \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) be two simple graphs. The graph \( G = (V(G), E(G)) \), where \( V = V_1 \cup V_2 \) and \( E = E_1 \cup E_2 \), is called the union of \( G_1 \) and \( G_2 \) is denoted by \( G_1 \hat{G} G_2 \).

In this paper, we established that \( p + q \) is odd for \( GΩW_n \),...
\(G \bar{O} f_n, G \bar{O} F_n, p + q\) is even for \(G \bar{O} P_n\), crown graph, union of cycles and some class of several graphs are edge vertex prime.

2. Main Results

Theorem 2.1. If \(G\) (\(G \neq P_n\) and \(W_k\)) has an edge vertex prime labeling with \(p + q\) is odd, then there exists a graph from the class \(G \bar{O} W_n\) that admits edge vertex prime labeling.

Proof. Let \(G(p,q)\) be an edge vertex prime labeling graph and \(G \neq P_n\) and \(W_k\), when \(p + q\) is odd, with bijective function \(f : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., p + q\}\) with the property that given any edge \(uv \in E(G)\), the numbers \(f(u), f(v)\) and \(f(uv)\) are pairwise relatively prime. Consider the graph \(W_n\) with vertex set \(\{w, w_i : 1 \leq i \leq n\}\) and edge set \(\{ww_i : 1 \leq i \leq n\}\). We superimpose one of the vertex say \(w\) of \(W_n\) on selected vertex \(v_1\) in \(G\). Now, we define new graph \(G_1 = G \bar{O} W_n\) with vertex set \(V_1 = V \cup \{w, w_i : 1 \leq i \leq n\}\) and edge set \(E_1 = E \cup \{ww_i : 1 \leq i \leq n\}\). Define a bijective function \(g : V_1 \cup E_1 \rightarrow \{1, 2, 3, ..., p + q, p + q + 1, ..., p + q + 3n + 1\}\) by \(g(v) = f(v)\) for all \(v \in V(G)\) and \(g(uv) = f(uv)\) for all \(uv \in E(G)\).

Consider \(G \bar{O} W_n\) the following cases.

Case(i). When \(n\) is even.

\[
g(w) = 1
\]

\[
g(w_i) = \begin{cases} p + q + 3i - 1; & \text{if } i \text{ is odd} \\ p + q + 3i - 2; & \text{if } i \text{ is even} \end{cases}
\]

\[
g(ww_i) = \begin{cases} p + q + 3i - 2; & \text{if } i \text{ is odd} \\ p + q + 3i - 1; & \text{if } i \text{ is even} \end{cases}
\]

\[
g(ww_{i+1}) = p + q + 3i, \forall i
\]

We have to prove that \(G_1\) is an edge vertex prime labeling. Already, \(G\) is an edge vertex prime labeling, it is enough to prove that for any edge \(uv \in E_1\) which is not in \(G\), the numbers \(g(u), g(v)\) and \(g(uv)\) are pairwise relatively prime.

(i) For any edge \(ww_i \in E_1 (1 \leq i \leq n)\),

\[
gcd(g(w), g(w_i)) = \begin{cases} gcd(1, p + q + 3i - 1); & \text{if } i \text{ is odd} \\ gcd(1, p + q + 3i - 2); & \text{if } i \text{ is even} \end{cases} = 1
\]

\[
gcd(g(w), g(ww_i)) = \begin{cases} gcd(1, p + q + 3i - 2); & \text{if } i \text{ is odd} \\ gcd(1, p + q + 3i - 1); & \text{if } i \text{ is even} \end{cases} = 1
\]

\[
gcd(g(w_i), g(ww_i)) = \begin{cases} gcd(p + q + 3i - 1, p + q + 3i - 2); & \text{if } i \text{ is odd} \\ gcd(p + q + 3i - 2, p + q + 3i - 1); & \text{if } i \text{ is even} \end{cases} = 1
\]

(ii) For any edge \(ww_{i+1} \in E_1 (1 \leq i \leq n)\),

\[
gcd(g(w), g(w_{i+1})) = \begin{cases} gcd(p + q + 3i - 1, p + q + 3i + 1); & \text{if } i \text{ is odd} \\ gcd(p + q + 3i - 2, p + q + 3i + 1); & \text{if } i \text{ is even} \end{cases} = 1
\]

\[
gcd(g(w_i), g(w_{i+1})) = \begin{cases} gcd(p + q + 3i - 1 + 1); & \text{if } i \text{ is odd} \\ gcd(p + q + 3i - 2 + 2); & \text{if } i \text{ is even} \end{cases} = 1
\]

\[
gcd(g(ww_i), g(ww_{i+1})) = \begin{cases} gcd(p + q + 3i - 1 + 1); & \text{if } i \text{ is odd} \\ gcd(p + q + 3i - 2 + 2); & \text{if } i \text{ is even} \end{cases} = 1
\]

\[
gcd(g(ww_i), g(ww_{i+1})) = \begin{cases} gcd(p + q + 3i - 1 + 1); & \text{if } i \text{ is odd} \\ gcd(p + q + 3i - 2 + 2); & \text{if } i \text{ is even} \end{cases} = 1
\]
\[ \text{gcd}(w_i, g(w_{i+1})) = \begin{cases} 
\gcd(p + q + 3i - 1, p + q + 3i - 2); & i \text{ is odd} \\
\gcd(p + q + 3i - 2, p + q + 3i - 1); & i \text{ is even} \\
\gcd(p + q + 3i - 3, p + q + 3n); & i = n 
\end{cases} \]

\[ \text{gcd}(g(w_i), g(w_{i+1})) = \begin{cases} 
\gcd(p + q + 3i + 1, p + q + 3i); & i = 1, 3, 5, ..., n - 2 \\
\gcd(p + q + 3i + 2, p + q + 3i); & i = 2, 4, 6, ..., n - 3 \\
\gcd(p + q + 3i, p + q + 3i + 2); & i = n - 1 
\end{cases} \]

Therefore, for any edge \( uv \in E_1 \) which is not in \( G \), the numbers \( g(u), g(v) \) and \( g(uv) \) are pairwise relatively prime. Hence \( G \text{OW}_n \) is an edge vertex prime labeling. 

\[ \text{Theorem 2.2.} \text{ If } G(p, q) \text{ has an edge vertex prime labeling with } p + q \text{ is odd, then there exists a graph from the class } G \text{O}f_{n} \text{ that admits edge vertex prime labeling.} \]

Next, consider a graph \( G = W_4 \text{O}W_5 \). Then \( V(G) = \{u, u, v, v : 1 \leq i \leq 4, 1 \leq j \leq 5\} \) and \( E(G) = \{uu_i, uv_j : 1 \leq i \leq 4, 1 \leq j \leq 5\} \). Also, \( |V(G)| = 10 \) and \( |E(G)| = 18 \). Define a bijective function \( f : V(G) \cup E(G) \rightarrow \{1, 2, ..., 28\} \) by \( f(u) = 1, f(u_1) = 3, f(u_2) = 5, f(u_3) = 9, f(u_4) = 11, f(uu_1) = 2, f(uu_2) = 6, f(uu_3) = 8, f(uu_4) = 12, f(uv_1) = 4, f(uv_2) = 7, f(uv_3) = 10, f(uv_4) = 13, f(v_1) = 15, f(v_2) = 17, f(v_3) = 19, f(v_4) = 21, f(v_5) = 23, f(v_1v_2) = 16, f(v_1v_3) = 18, f(v_1v_4) = 20, f(v_1v_5) = 22, f(v_2v_3) = 14, f(vu_1) = 28, f(vu_2) = 24, f(vu_3) = 25, f(vu_4) = 26, f(uv) = 27 \). Clearly, for any edge \( uv \in E(G) \), the numbers \( f(u), f(v) \) and \( f(uv) \) are pairwise relatively prime. Hence \( G = W_4 \text{O}W_5 \) is an edge vertex prime labeling.

\[ \text{Theorem 2.2.} \quad \text{If } G(p, q) \text{ has an edge vertex prime labeling with } p + q \text{ is odd, then there exists a graph from the class } G \text{O}f_{n} \text{ that admits edge vertex prime labeling.} \]

\[ \text{Proof.} \text{ Let } G(p, q) \text{ be an edge vertex prime labeling graph when } p + q \text{ is odd with bijective function } f : V(G) \cup E(G) \rightarrow \{1, 2, ..., p + q\} \text{ with the property that given any edge } uv \in E(G), \text{ the numbers } f(u), f(v) \text{ and } f(uv) \text{ are pairwise relatively prime. Consider the graph } f_n \text{ with vertex set } \{w, w_i : 1 \leq i \leq n\} \text{ and edge set } \{ww_i : 1 \leq i \leq n\} \text{. We superimpose one of the vertex say, } w \text{ of } f_n \text{ on selected vertex } v_1 \text{ in } G. \text{ Now, we define new graph } G_1 = G \text{O}f_{n} \text{ with vertex set } V_1 = V(G) \cup \{w, w_i : 1 \leq i \leq n\} \text{ and edge set } E_1 = E(G) \cup \{ww_i : 1 \leq i \leq n\} \text{. Define a bijective function } g : V_1 \cup E_1 \rightarrow \{1, 2, ..., p + q, p + q + 1, ..., p + q + 3n\} \text{ by } g(v) = f(v) \text{ for all } v \in V(G) \text{ and } g(uv) = f(uv) \text{ for all } uv \in E(G). \]
We have to prove that $G_1$ is an edge vertex prime labeling. Hence there exists a graph from the class $G\widehat{OF}_n$ that admits edge vertex prime labeling.

**Theorem 2.4.** If $G(p,q)$ has an edge vertex prime labeling with $p+q$ is odd, then there exists a graph from the class $G\widehat{OF}_n$ that admits edge vertex prime labeling.

**Proof.** Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is odd with bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph $F_n$ with vertex set $\{w_i : 1 \leq i \leq 2n\}$ and edge set $\{(w_i, w_{i+1}) : 1 \leq i \leq n\}$. We superimpose one of the vertex say $w$ of $F_n$ on selected vertex $v_1$ in $G$. Now, we define new graph $G_1 = G\widehat{OF}_n$ with vertex set $V_1 = V\cup\{w_i : 1 \leq i \leq 2n\}$ and edge set $E_1 = E\cup\{(w_i, w_{i+1}) : 1 \leq i \leq n\}$. Define a bijective function $f : V_1 \rightarrow \{1, 2, 3, \ldots, p+q+1, p+q+2, \ldots, p+q+5n+1\}$ by $f(v) = f(v')$ for all $v \in V(G)$ and $f(uv) = f(uv')$ for all $uv \in E(G)$, 

$$g(w) = \begin{cases} p+q+5i-3; & \text{if } i \text{ is odd} \\
\text{even} & \text{if } i \text{ is even} 
\end{cases}$$

$$g(w_{2i-1}) = \begin{cases} p+q+5i-4; & \text{if } i \text{ is odd} \\
\text{even} & \text{if } i \text{ is even} 
\end{cases}$$

$$g(w_{2i}) = \begin{cases} p+q+5i-1; & \text{if } i \text{ is odd} \\
\text{even} & \text{if } i \text{ is even} 
\end{cases}$$

$$g(w_{2i-1}) = \begin{cases} p+q+5i-3; & \text{if } i \text{ is odd} \\
\text{even} & \text{if } i \text{ is even} 
\end{cases}$$

$$g(w_{2i}) = \begin{cases} p+q+5i-1; & \text{if } i \text{ is odd} \\
\text{even} & \text{if } i \text{ is even} 
\end{cases}$$

Therefore, for any edge $uv \in E_1$ which is not in $G$, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. Hence there exists a graph from the class $G\widehat{OF}_n$ admits edge vertex prime labeling. 

The planter graph $R_n$ ($n \geq 3$) can be constructed by joining fan graph $f_n$ ($n \geq 2$) and cycle $C_n$, ($n \geq 3$) with sharing a common vertex, where $n$ is any positive integer, that is $R_n = f_n\widehat{OC}_n$.

**Corollary 2.3.** The planter graph $R_n$ ($n \geq 3$) admits edge vertex prime labeling graph, where $n$ is any positive integer.
Theorem 2.5. If $G(p,q)$ has an edge vertex prime labeling with $p+q$ is even, then there exists a graph from the class $G_{OP}$ that admits edge vertex prime labeling.

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is even with bijective function $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph $P_n$ with vertex set $\{w_i : 1 \leq i \leq n\}$ and edge set $\{w_{i\ell}: 1 \leq i \leq n-1\}$. We superimpose one of the vertex say $w_1$ of $P_n$ on selected vertex $v_1$ in $G$. Now, we define a new graph $G_1 = G_{OP}$ with vertex set $V_1 = V \cup \{w_i : 2 \leq i \leq n\}$ and edge set $E_1 = E \cup \{w_{i\ell}: 1 \leq i \leq n-1\}$. Define a bijection function $g : V_1 \cup E_1 \rightarrow \{1, 2, 3, ..., p+q+p+q+1, ..., p+q+2n-2\}$ by $g(v) = f(v)$, for all $v \in V$ and $g(uv) = f(uv)$, for all $uv \in E(G)$. For $g(w_1) = p$, $g(w_i) = p+q+2n+1-2i$ for $2 \leq i \leq n$, $g(w_{i\ell}) = p+2n-2i$ for $1 \leq i \leq n-1$. For any edge $w_{i\ell}w_{i+1} \in E_1 (2 \leq i \leq n-1)$, 

$$gcd(g(w_1),g(w_2)) = gcd(1,p+q+2n-3) = 1, \quad gcd(g(w_1),g(w_1w_2)) = gcd(1,p+q+2n-2) = 1,$$

$$gcd(g(w_2),g(w_1w_2)) = gcd( p+q+2n-3, p+q+2n-2) = 1, \quad gcd(g(w_1),g(w_{i\ell})) = gcd(p+q+2n+1-2i, p+q+2n-2i-1) = 1, \quad gcd(g(w_i),g(w_{i\ell})) = gcd(p+q+2n+1-2i, p+q+2n-2i) = 1,$$

$$gcd(g(w_{i\ell}),g_{i\ell+1})) = gcd(p+q+2n-2i-1, p+q+2n-2i) = 1. \quad For any edge $uv \in E_1$, the numbers $g(u),g(v)$ and $g(uv)$ are pairwise relatively prime. Hence $G_{OP}$ admits edge vertex prime labeling.

\[\square\]

Corollary 2.6. The graph $C_iO_{K_1,m}O_{P_n}$ is an edge vertex prime labeling.

Proof. Let $G = C_iO_{K_1,m}O_{P_n}$ be a graph. Then $V(G) = \{u_i : 1 \leq i \leq l\} \cup \{v_j : 1 \leq j \leq m\}$ and $E(G) = \{u_iu_{i+1} : 1 \leq i \leq l-1\} \cup \{u_iu_l\} \cup \{u_iw_k : 1 \leq k \leq n\} \cup \{u_{i\ell}w_{i\ell+1} : 1 \leq k \leq n-2\}$. Also, $W(G) = l+m+n-1$ and $E(G) = l+m+n-1$. We superimpose two of the vertices say, $v$ of $K_{1,m}$ and $w_n$ of $P_n$ on selected vertex $u_1$ in $C_i$. Define a bijection function $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., 2(l+m+n-1)\}$ by $f(u_1) = f(v) = f(w_n) = 1$, $f(u_i) = 2i-1$ for $2 \leq i \leq l$, $f(u_{i\ell}) = 2\ell-1$ for $1 \leq \ell \leq l-1$, $f(u_{i\ell}) = 2\ell+1$ for $1 \leq \ell \leq m$, $f(v_1v_j) = 2\ell+2j-1$ for $1 \leq j \leq m$, $f(w_kv_j) = 2\ell+2j+k-1$ for $1 \leq k \leq n-1$, $f(w_kw_{k+1}) = 2\ell+2j+k$ for $1 \leq k \leq n-1$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = C_iO_{K_1,m}O_{P_n}$ admits edge vertex prime labeling.

\[\square\]

Parmer [5] proved that $f_m$ is an edge vertex prime labeling. Jagadeesh, Baskar Babu [4] proved that if $G$ has an edge vertex prime labeling, then there exist a graph from the class $G_{OP}$ admits edge vertex prime labeling. An Umbrella graph $U(m,n)$ is the graph obtained by joining a path $P_n$ with the central vertex of a fan $f_m$.

Corollary 2.7. The Umbrella graph $U(m,n)$ is an edge vertex prime labeling.

Theorem 2.8. If $G$ has an edge vertex prime labeling with $p+q$ is even, then there exists a graph from the class $GOC_3$ that admits edge vertex prime labeling.

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is even with bijection function $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph $C_3$ with vertex set $\{w_1, w_2, w_3\}$ and edge set $\{w_1w_2, w_2w_3, w_3w_1\}$. We superimpose one of the vertex say $w_1$ of $C_3$ on selected vertex $v_1$ in $G$. Now, we define new graph $G_1 = GOC_3$ with vertex set $V_1 = V \cup \{w_i : 2 \leq i \leq n\}$ and edge set $E_1 = E \cup \{w_{i\ell} : 1 \leq i \leq n-1\}$. Define a bijection function $g : V_1 \cup E_1 \rightarrow \{1, 2, 3, ..., p+q, p+q+1, ..., p+q+2n-2\}$ by $g(v) = f(v)$, for all $v \in V$ and $g(uv) = f(uv)$, for all $uv \in E(G)$. For $g(w_1) = 1$, $g(w_i) = p+q+2n+1-2i+1$ for $2 \leq i \leq n$, $g(w_{i\ell}) = p+q+2n-2i$ for $1 \leq i \leq n-1$. For any edge $w_{i\ell}w_{i+1} \in E_1 (2 \leq i \leq n-1)$, 

$$gcd(g(w_1),g(w_2)) = gcd(1,p+q+2n-3) = 1, \quad gcd(g(w_1),g(w_1w_2)) = gcd(1,p+q+2n-2) = 1,$$

$$gcd(g(w_2),g(w_1w_2)) = gcd( p+q+2n-3, p+q+2n-2) = 1, \quad gcd(g(w_1),g(w_{i\ell})) = gcd(p+q+2n+1-2i, p+q+2n-2i-1) = 1, \quad gcd(g(w_i),g(w_{i\ell})) = gcd(p+q+2n+1-2i, p+q+2n-2i) = 1,$$

$$gcd(g(w_{i\ell}),g_{i\ell+1})) = gcd(p+q+2n-2i-1, p+q+2n-2i-1) = 1. \quad For any edge $uv \in E_1$, the numbers $g(u),g(v)$ and $g(uv)$ are pairwise relatively prime. Hence $GOC_3$ admits edge vertex prime labeling.

\[\square\]
Theorem 2.9. The crown graph $C_nK_1$ is an edge vertex prime labeling, where $n$ is a positive integer.

Proof. Let $G = C_nK_1$ be a graph. The degree of the vertices of a crown graph is either 3 or 1. Consider $u_1, u_2, ..., u_n$ be the vertices with degree 3 and $v_1, v_2, ..., v_n$ be the vertices with degree 1. The edges of the crown graph are $\{u_iv_i : 1 \leq i \leq n\} \cup \{u_{i+1}u_i : 1 \leq i \leq n-1\} \cup \{u_1u_n\}$. Here $|V(G)| = 2n$ and $|E(G)| = 2n$. Define a bijection function $f : V(G) \cup E(G) \to \{1, 2, ..., 2n\}$. For any edge $u_iv_i, f(u_iv_i) = 2i$ for $i = 1, 2, ..., n$. When $n$ is even, for each $i \leq k \leq \frac{n}{2} - 3$, $f(u_{3i}) = 12i - 1$, $f(v_{3i}) = 12i - 3$. For $i \leq k \leq \frac{n}{2} - 2$, $f(u_{3i}) = 12i - 1$, $f(v_{3i}) = 12i - 3$. For $i \leq k \leq \frac{n}{2}$, $f(u_{3i}) = 12i - 1$, $f(v_{3i}) = 12i - 3$. For $i > \frac{n}{2}$, $f(u_{3i}) = 12i - 1$, $f(v_{3i}) = 12i - 3$.

Therefore, for any edge $uv \in E(G)$, the numbers $f(u)$, $f(v)$ and $f(uv)$ are pairwise relatively prime. Hence, the crown graph $C_nK_1$ is an edge vertex prime labeling.

Theorem 2.10. The graph $C_n \cup C_n \cup C_n$, $n \geq 3$ and $n \equiv 0 \pmod{3}$ is an edge vertex prime labeling.

Proof. Let $G = C_n \cup C_n \cup C_n$, $n \geq 3$ and $n \equiv 0 \pmod{3}$ be a graph. Then $V(G) = \{v_i : 1 \leq i \leq 3n\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq n-1\}$ $\cup \{v_{i+1}v_{i+2} : 2n+1 \leq i \leq 3n-1\}$. Also, $|V(G)| = 3n$ and $|E(G)| = 3n$. Define a bijection function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., 6n\}$ as follows.

Case 1. $n \equiv 0 \pmod{3}$. For any edge $uv \in E(G)$, the numbers $f(u)$, $f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = C_n \cup C_n \cup C_n$, $n \geq 3$ and $n \equiv 0 \pmod{3}$ is an edge vertex prime labeling.

Theorem 2.11. The graph $C_n \cup C_n \cup ... \cup C_n$, $n \equiv 0 \pmod{5}$ is an edge vertex prime labeling.

Proof. Let $G = C_n \cup C_n \cup ... \cup C_n$, $n \equiv 0 \pmod{5}$ be a graph. Then $V(G) = \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 5\}$ and $E(G) = \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 4\} \cup \{v_{5j}v_{5j+1} : 1 \leq i \leq m\}$. Also, $|V(G)| = 5m$ and $|E(G)| = 5m$. Define a bijection function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., 10m\}$ by $f(v_{ij}) = 10(i-1)+j+2$ for $1 \leq i \leq m, 1 \leq j \leq 4$, $f(v_{5j}) = 10(i-1)+2j$ for $1 \leq i \leq m, 1 \leq j \leq 5$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u)$, $f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = C_n \cup C_n \cup ... \cup C_n$, $n \equiv 0 \pmod{5}$ is an edge vertex prime labeling.

References


