On ve-quasi and secured ve-quasi independent sets of a graph

D. K. Thakkar¹ and Neha P. Jamvecha²*

Abstract
In this paper, we have defined the concepts of ve-quasi independent set and secured ve-quasi independent set. In order to define these concepts we have used the concept of a vertex which m-dominates an edge. We prove a characterization of a maximal ve-quasi independent set. We also prove that the complement of a ve-quasi independent set is a ve-dominating set. We prove a necessary and sufficient condition under which a ve-quasi independent set is a secured ve-quasi independent set. Also we prove a necessary and sufficient condition under which the ve-quasi independence number and secured ve-quasi independence number decrease when a vertex is removed from the graph. Some examples have also been given.

Keywords
ve-quasi independent set, secured ve-quasi independent set, ve-quasi isolated vertex, ve-dominating set

AMS Subject Classification
05C69

1. Introduction
The domination related results have been appeared in several articles. The concepts of vertices dominate edges and edges dominate vertices are studied by several authors. The concept of a vertex-edge dominating set (ve-dominating set) is defined by E. Sampathkumar and S. S. Kamath in [2]. A vertex v ∈ V (G) m-dominates an edge x ∈ E (G) if x ∈ (N [v]). A set S ⊆ V (G) is a ve-dominating set if every edge in G is m-dominated by a vertex in S [2]. We introduce the concept of ve-quasi independent sets using the concept of ve-dominination in graphs. We call a set S of vertices to be a ve-quasi independent set if whenever u, v ∈ S are adjacent vertices, there is a vertex x in V (G) \ S which m-dominates the edge uv in G. We also introduce the concept of secured ve-quasi independent set. A ve-quasi independent set S is a secured ve-quasi independent set if for each v ∈ S, there is a vertex u in V (G) \ S which is adjacent to v such that (S \ {v}) \ {u} is a ve-quasi independent set.

We also introduce maximal ve-quasi independent set and maximum ve-quasi independent set as well as maximal secured ve-quasi independent set and maximum secured ve-quasi independent set.

2. Preliminaries and Notations
If G is a graph then E (G) denotes the edge set and V (G) denotes the vertex set of the graph. If v is a vertex of G then G \ v denotes the subgraph of G obtained by removing the vertex v and all the edges incident to v. N (v) denotes the set of vertices which are adjacent to v. N [v] = N (v) \ {v}. If G is a graph then β0 (G) denotes the independence number of a graph G. If G is a graph then the induced subgraph denoted as (S) is the graph whose vertex set is S and whose edge set consists of all the edges that have both end points in S.

3. Main Results

Definition 3.1. Let G be a graph. A set S of vertices is said to be a ve-quasi independent set if whenever u, v ∈ S are adjacent vertices, N (u) \ N (v) \ (V (G) \ S) ≠ φ. i.e. u and v have a common neighbor in V (G \ S) if u and v are adjacent vertices of S.
We can also characterize a ve-quasi independent set as follows:
A subset $S$ of $V(G)$ is a ve-quasi independent set if and only if whenever $u,v \in S$ are adjacent vertices, there is a vertex $x$ in $V(G) \setminus S$ which m-dominates the edge $uv$ in $G$.

**Theorem 3.2.** Let $G$ be a graph and $S \subset V(G)$ then $S$ is a ve-quasi independent set if and only if $V(G \setminus S)$ is a ve-dominating set.

**Proof.** First suppose that $S$ is a ve-quasi independent set. Let $e = uv$ be any edge of $G$. If $u \in V(G \setminus S)$ or $v \in V(G \setminus S)$, then $e$ is m-dominated by some vertex of $V(G \setminus S)$. Suppose $u \notin V(G \setminus S)$ and $v \notin V(G \setminus S)$. Then $u$ and $v$ are adjacent vertices of $S$. Since $S$ is a ve-quasi independent set, there is some vertex $x$ in $N(u) \cap N(v) \cap (V(G) \setminus S)$. Then $x \in V(G \setminus S)$ and $e$ is m-dominated by $x$. Thus, we have proved that any edge of $G$ is m-dominated by some vertex of $V(G \setminus S)$. Therefore, $V(G \setminus S)$ is a ve-dominating set.

Conversely, suppose that $V(G \setminus S)$ is a ve-dominating set. Suppose $u,v$ are adjacent vertices of $S$. Now, $e = uv$ is an edge of $G$ and $V(G \setminus S)$ is a ve-dominating set of $G$. Therefore, there is a vertex $x$ in $V(G \setminus S)$ which m-dominates $e$. This means that $u$ is adjacent to $x$ or $v$ is also adjacent to $x$. Therefore, $x \in N(u) \cap N(v) \cap (V(G) \setminus S)$. Hence, $S$ is a ve-quasi independent set.

**Remark 3.3.** (i) Every independent set is a ve-quasi independent set but the converse is not true in general.

(ii) A ve-quasi independence is a hereditary property.

**Example 3.4.** Consider the graph $C_3$ with vertices $\{v_1, v_2, v_3\}$

\[ \text{Figure 1. C}_3 \]

Let $S = \{v_2, v_3\}$. Then $S$ is a ve-quasi independent set but it is not an independent set.

**Definition 3.5.** Let $G$ be a graph. $S \subset V(G)$ and $v \in S$. Then $v$ is said to be a ve-quasi isolated vertex of $S$ if whenever $u$ is adjacent to $v$, $N(v) \cap N(u) \cap (V(G) \setminus S) \neq \emptyset$.

**Proposition 3.6.** Let $G$ be a graph and $S \subset V(G)$. Then $S$ is a ve-quasi independent set if and only if every vertex of $S$ is a ve-quasi isolated vertex of $S$.

**Proof.** First suppose that $S$ is a ve-quasi independent set. From the definition, it is clear that each vertex of $S$ is a ve-quasi isolated vertex of $S$.

Conversely, suppose each vertex of $S$ is a ve-quasi isolated vertex of $S$. Let $u,v$ be adjacent vertices of $S$. Now, $v$ is a ve-quasi isolated vertex of $S$ and $u$ is adjacent to $v$. Therefore, $N(v) \cap N(u) \cap (V(G) \setminus S) \neq \emptyset$. This proves that $S$ is a ve-quasi independent set.

**Definition 3.7.** Let $G$ be a graph and $S \subset V(G)$ be a ve-quasi independent set then $S$ is said to be a maximum ve-quasi independent set if its cardinality is maximum among all ve-quasi independent subsets of $G$.

The cardinality of a maximum ve-quasi independent set is called the ve-quasi independence number of the graph $G$ and it is denoted as $\beta_q(G)$.

Note that for any graph $G$, $\beta_0(G) \leq \beta_q(G)$.

**Example 3.8.** Consider the figure 1

Here, $S = \{v_2, v_3\}$ is a maximum ve-quasi independent set and $\beta_q(G) = 2$. Also $\beta_0(G) = 1$.

Thus for this graph, $\beta_0(G) < \beta_q(G)$.

**Example 3.9.** Consider the graph $G$ with vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$

\[ \text{Figure 2. G} \]

Obviously, $\beta_0(G) = 3$. Let $S = \{v_1, v_2, v_6\}$. Then $S$ is a maximum ve-quasi independent set of $G$ and therefore, $\beta_q(G) = 3$. Thus for this graph, $\beta_0(G) = \beta_q(G)$.

**Proposition 3.10.** Let $G$ be a graph and $v \in V(G)$. Then $\beta_q(G \setminus v) \leq \beta_q(G)$.

**Proof.** Let $S$ be a maximum ve-quasi independent subset of $G \setminus v$. Obviously, $S$ is a ve-quasi independent subset of $G$. Therefore, $\beta_q(G) \geq |S| = \beta_q(G \setminus v)$.

Therefore, $\beta_q(G \setminus v) \leq \beta_q(G)$.

**Theorem 3.11.** Let $G$ be a graph and $v \in V$. Then $\beta_q(G \setminus v) < \beta_q(G)$ if and only if for every maximum ve-quasi independent subset $S$ of $G$ not containing $v$, there are adjacent vertices $x$ and $y$ of $S$ such that $N(x) \cap N(y) \cap (V(G) \setminus S) = \{v\}$.

**Proof.** First suppose that $\beta_q(G \setminus v) < \beta_q(G)$. Let $S$ be a maximum ve-quasi independent subset of $G$ not containing $v$. Since $\beta_q(G \setminus v) < \beta_q(G)$, $S$ can not be a ve-quasi independent subset of $G \setminus v$. Therefore, there are adjacent vertices $x$ and...
y of \( S \) such that \( N(x) \cap N(y) \cap (V(G) \setminus S) = \emptyset \). However, \( N(x) \cap N(y) \cap (V(G) \setminus S) \neq \emptyset \) because \( S \) is a ve-quasi independent subset of \( G \). Therefore, \( N(x) \cap N(y) \cap (V(G) \setminus S) = \{v\} \). Thus, the condition is satisfied.

Conversely, suppose the condition is satisfied.

Suppose, \( \beta_q(G \setminus v) < \beta_q(G) \). Therefore, \( \beta_q(G \setminus v) = \beta_q(G) \).

Let \( S \) be a maximum ve-quasi independent subset of \( G \setminus v \). Then \( S \) is also a maximum ve-quasi independent subset of \( G \) not containing \( v \). Let \( x \) and \( y \) be adjacent vertices of \( S \). Since \( S \) is a ve-quasi independent subset of \( G \setminus v \), \( x \) and \( y \) have a common neighbor in \( V(G) \setminus S \) say \( u \). Therefore, \( u \in N(x) \cap N(y) \cap (V(G) \setminus S) \) and \( u \neq v \). Therefore, \( N(x) \cap N(y) \cap (V(G) \setminus S) \neq \{v\} \) for any two adjacent vertices \( x \) and \( y \) of \( S \) which contradicts the given condition.

Therefore, \( \beta_q(G \setminus v) \leq \beta_q(G) \).

**Definition 3.12.** Let \( G \) be a graph and \( S \subseteq V(G) \) be a ve-quasi independent set. Then \( S \) is said to be a maximal ve-quasi independent set if \( S \) is not properly contained in any ve-quasi independent subset of \( G \).

We may note that a ve-quasi independent set \( S \) is a maximal ve-quasi independent set if and only if for each \( v \in V(G) \setminus S \), \( S \cup \{v\} \) is not a ve-quasi independent set.

**Theorem 3.13.** Let \( G \) be a graph and \( S \subseteq V(G) \) be a ve-quasi independent set then \( S \) is a maximal ve-quasi independent set if and only if for each \( v \in V(G) \setminus S \), one of the following two conditions is satisfied.

(i) There are adjacent vertices \( x \) and \( y \) of \( S \) such that \( v \) is the only common neighbor of \( x \) and \( y \) in \( V(G) \setminus S \).

(ii) There is a vertex \( x \) in \( S \) adjacent to \( v \) such that \( x \) and \( v \) do not have a common neighbor in \( V(G) \setminus S \).

**Proof.** Suppose, \( S \) is a maximal ve-quasi independent set. Let \( v \in V(G) \setminus S \). Now, \( S \cup \{v\} \) is not a ve-quasi independent set. Therefore, there are adjacent vertices \( x \) and \( y \) of \( S \cup \{v\} \) such that \( x \) and \( y \) do not have a common neighbor in \( V(G) \setminus (S \cup \{v\}) \).

**Case (i):** \( x \neq v \) and \( y \neq v \)

Then \( x, y \in S \). Now \( x \) and \( y \) do not have a common neighbor in \( V(G) \setminus (S \cup \{v\}) \). However \( x \) and \( y \) have a common neighbor in \( V(G) \setminus S \). Therefore, \( v \) is the only common neighbor of \( x \) and \( y \) in \( V(G) \setminus S \). Thus condition (i) is satisfied.

**Case (ii):** \( x = v \) or \( y = v \)

We may assume that \( y = v \). Then \( x \) and \( v \) are adjacent vertices and they do not have any common neighbor in \( V(G) \setminus (S \cup \{v\}) \). Therefore, \( x \) and \( v \) do not have a common neighbor in \( V(G) \setminus S \). Thus condition (ii) is satisfied.

Conversely, suppose \( S \) is ve-quasi independent set for which condition (i) and (ii) are satisfied for each \( v \in V(G) \setminus S \). Let \( v \in V(G) \setminus S \). Suppose condition (i) is satisfied. Then \( x \) and \( y \) are two vertices of \( S \cup \{v\} \) which are adjacent and they do not have a common neighbor in \( v \in V(G) \setminus (S \cup \{v\}) \). Suppose condition (ii) is satisfied. Let \( x \) be a vertex of \( S \) such that \( x \) is adjacent to \( v \) and \( x \). And \( v \) do not have a common vertex in \( v \in V(G) \setminus S \). Then \( x \) and \( v \) are adjacent vertices of \( S \cup \{v\} \) such that they do not have a common neighbor in \( v \in V(G) \setminus (S \cup \{v\}) \).

From both the above cases it follows that \( S \) is a maximal ve-quasi independent set. □

Obviously, every maximum ve-quasi independent set is a maximal ve-quasi independent set. However, the converse is not true.

**Example 3.14.** Consider the following graph \( G \) with vertices \( \{v_1, v_2, v_3, v_4, v_5\} \)

![Figure 3. G](image)

Let \( T = \{v_1, v_2, v_3, v_4\} \). Then \( T \) is a maximum ve-quasi independent set of \( G \). Let \( S = \{v_1, v_3, v_5\} \). Then \( S \) is a maximal ve-quasi independent set and \( |S| < |T| \). Therefore, \( S \) is a maximal ve-quasi independent set which is not a maximum ve-quasi independent set.

**Definition 3.15.** Let \( G \) be a graph and \( S \subseteq V(G) \) be a ve-quasi independent set. Then \( S \) is said to be a secured ve-quasi independent set if for each \( v \in S \), there is \( u \in V(G) \setminus S \) which is adjacent to \( v \) such that \( (S \setminus \{v\}) \cup \{u\} \) is a ve-quasi independent set.

**Example 3.16.** Consider the following graph \( G \) with vertices \( \{v_1, v_2, v_3, v_4\} \)

![Figure 4. G](image)

Let \( S = \{v_1, v_3\} \). Then \( S \) is a secured ve-quasi independent set of \( G \).

**Example 3.17.** Consider the figure 3

Let \( S = \{v_1, v_2, v_3, v_4\} \). Then \( S \) is not a secured ve-quasi independent set of \( G \).

**Definition 3.18.** Let \( G \) be a graph and \( S \subseteq V(G) \) be a secured ve-quasi independent set then \( S \) is said to be a maximum secured ve-quasi independent set if its cardinality is maximum among all secured ve-quasi independent subsets of \( G \).
The cardinality of a maximum secured ve-quasi independent set is called the secured ve-quasi independence number of the graph $G$ and it is denoted as $\beta_{sq}(G)$.

**Proposition 3.19.** Let $G$ be a graph and suppose $\{M_1, M_2, \ldots, M_k\}$, $k \geq 2$ is the set of all maximum ve-quasi independent sets of $G$. Suppose at least one of them is a secured ve-quasi independent set then $M_1 \cap M_2 \cap \ldots \cap M_k = \emptyset$.

**Proof.** Suppose, $M_1 \cap M_2 \cap \ldots \cap M_k \neq \emptyset$. Let $v \in M_1 \cap M_2 \cap \ldots \cap M_k$. Suppose for some $j$ ($j \in \{1, 2, \ldots, k\}$), $M_j$ is a secured ve-quasi independent set. Then $v \in M_j$. There is a neighbor $u$ of $v$ such that $u \notin M_j$ and $N = (M_j \setminus \{v\}) \cup \{u\}$ is a ve-quasi independent set. Now, $|N| = |M_j|$. Therefore, $M$ is a maximum ve-quasi independent set and therefore $N \in \{M_1, M_2, \ldots, M_k\}$ and therefore $v \in N$ which is a contradiction. Thus, $M_1 \cap M_2 \cap \ldots \cap M_k = \emptyset$. \hfill $\Box$

Now, we give a necessary and sufficient condition under which a ve-quasi independent set is a secured ve-quasi independent set.

**Theorem 3.20.** Let $G$ be a graph and $S \subseteq V(G)$ be a ve-quasi independent set. Then $S$ is a secured ve-quasi independent set if and only if for each $v \in S$ there is a neighbor $u$ of $v$ in $V(G) \setminus S$, for each $x, y \in (S \setminus \{v\}) \cup \{u\}$ one of the following two conditions is satisfied

(i) $v$ is a common neighbor of $x$ and $y$.

(ii) There is a common neighbor of $x$ and $y$ in $V(G) \setminus S$ which is different from $u$.

**Proof.** Suppose $S$ is a secured ve-quasi independent set. Let $v \in S$. Then there is a neighbor $u$ of $v$ in $V(G) \setminus S$ such that $(S \setminus \{v\}) \cup \{u\}$ is a ve-quasi independent set.

Let $x, y \in (S \setminus \{v\}) \cup \{u\}$. Since $(S \setminus \{v\}) \cup \{u\}$ is ve-quasi independent set, $x$ and $y$ have a common neighbor $w$ outside $(S \setminus \{v\}) \cup \{u\}$. If $w = v$ then condition (i) is satisfied. If $w \neq v$ then $x$ and $y$ have a common neighbor outside $(S \setminus \{v\}) \cup \{u\}$ which is different from $u$. Thus condition (ii) is satisfied.

Conversely, suppose for each $v \in S$ there is a neighbor $u$ of $v$ in $V(G) \setminus S$ such that (i) or (ii) is satisfied. Let $v \in S$ and $u \in V(G) \setminus S$ be a neighbor of $v$ such that (i) or (ii) is satisfied. Now, we prove that $(S \setminus \{v\}) \cup \{u\}$ is a ve-quasi independent set. Let $x, y \in (S \setminus \{v\}) \cup \{u\}$. Suppose condition (i) is satisfied then $v$ is a common neighbor of $x$ and $y$ outside $(S \setminus \{v\}) \cup \{u\}$. Suppose condition (ii) is satisfied. Thus there is a common neighbor $w$ of $x$ and $y$ outside $(S \setminus \{v\}) \cup \{u\}$ which is different from $u$.

This proves that $(S \setminus \{v\}) \cup \{u\}$ is a ve-quasi independent set. Thus the theorem is proved. \hfill $\Box$

Note that for any graph $G$, $\beta_{sq}(G) \leq \beta_q(G)$.

**Corollary 3.21.** Let $G$ be a graph. Then

(i) If $\beta_q(G) = 1$ then $\beta_{sq}(G) = \beta_q(G)$.

(ii) If $\beta_q(G) \geq 2$ and if the intersection of all maximum ve-quasi independent sets of $G$ is non-empty then $\beta_{sq}(G) < \beta_q(G)$.

**Proof.** (i) If $\beta_q(G) = 1$ and $\beta_{sq}(G) \leq 1$ it follows that $\beta_{sq}(G) = 1 = \beta_q(G)$.

(ii) Suppose, $\beta_q(G) \geq 2$ and suppose the intersection of all maximum ve-quasi independent sets of $G$ is non-empty then none of these maximum ve-quasi independent sets can be a secured ve-quasi independent set. (by the above property). Therefore, the cardinality of any maximum secured ve-quasi independent set is strictly less than $\beta_q(G)$. Therefore, $\beta_{sq}(G) < \beta_q(G)$.

**Example 3.22.** Consider the figure 3

In this graph, $S = \{v_1, v_2, v_3, v_4\}$ is a maximum ve-quasi independent set and therefore $\beta_q(G) = 4$. Let $T = \{v_1, v_3, v_4\}$. Then $T$ is a maximum secured ve-quasi independent set of $G$ and $|T| = 3$. Thus for this graph $\beta_{sq}(G) < \beta_q(G)$.

**Example 3.23.** Consider the figure 1

Here, $S_1 = \{v_1, v_2\}, S_2 = \{v_2, v_3\}$ and $S_3 = \{v_1, v_3\}$ are all the maximum ve-quasi independent sets of $C_3$ and $S_1 \cap S_2 \cap S_3 = \emptyset$. Also $\beta_q(C_3) = 2$ and the above sets are also maximum secured ve-quasi independent sets and therefore $\beta_{sq}(C_3) = 2$. Thus for this graph, $\beta_{sq}(C_3) < \beta_q(C_3)$ although $\beta_q(C_3) \geq 2$.

**Example 3.24.** Consider the graph $K_2$ with vertices $\{v_1, v_2\}$

![Figure 5. $P_2$](image)

Here, $\beta_q(K_2) = 1$ and $\beta_{sq}(K_2) = 1$.

**Proposition 3.25.** Let $G$ be a graph and $v \in V(G)$. Then $\beta_{sq}(G \setminus v) \leq \beta_{sq}(G)$.

**Proof.** Let $S$ be a maximum secured ve-quasi independent set of $G \setminus v$. Let $u \in S$. Then there is a vertex $u'$ of $G \setminus v$ such that $u' \notin S$. $u'$ is adjacent to $u$ and $(S \setminus \{u\}) \cup \{u'\}$ is a ve-quasi independent set of $G \setminus v$. Note that $(S \setminus \{u\}) \cup \{u'\}$ is also a ve-quasi independent set of $G$ and $u' \in V(G) \setminus S$. Thus we have proved that for each $u$ in $S$ there is a neighbor $u'$ of $u$ in $V(G) \setminus S$ such that $(S \setminus \{u\}) \cup \{u'\}$ is a ve-quasi independent set of $G$. Thus, $S$ is a secured ve-quasi independent set of $G$ also. Therefore, $\beta_{sq}(G) \geq |S| = \beta_{sq}(G \setminus v)$.

**Theorem 3.26.** Let $G$ be a graph and $v \in V$. Then $\beta_{sq}(G \setminus v) < \beta_{sq}(G)$ if and only if for every maximum secured ve-quasi independent set $S$ of $G$ not containing $v$ at least one of the following two conditions holds

(i) There are adjacent vertices $x$ and $y$ of $S$ such that $v$ is the only common neighbor of $x$ and $y$ in $V(G) \setminus S$.

(ii) If $\beta_q(G) \geq 2$ and if the intersection of all maximum ve-quasi independent sets of $G$ is non-empty then $\beta_{sq}(G) < \beta_q(G)$.
(ii) There is a vertex $u$ in $S$ such that for every $u'$ in $V(G \setminus v) \setminus S$, $(S \setminus \{u\}) \cup \{u'\}$ is not a ve-quasi independent set of $G \setminus v$.

Proof. First suppose that $\beta_{sq}(G \setminus v) < \beta_{sq}(G)$.

Let $S$ be a maximum secured ve-quasi independent set of $G$ not containing $v$. Since $|S| > \beta_{sq}(G \setminus v)$, $S$ can not be a secured ve-quasi independent set of $G \setminus v$. Then one of the following two possibilities arises.

**Case (i):** $S$ is not a ve-quasi independent set of $G \setminus v$.

In this case, there are adjacent vertices $x$ and $y$ of $S$ such that $x$ and $y$ have no common neighbor in $(V(G \setminus v) \setminus S)$. However, $x$ and $y$ have a common neighbor in $V(G) \setminus S$. Therefore, $v$ is the only common neighbor of $x$ and $y$ in $V(G) \setminus S$.

**Case (ii):** $S$ is not a secured ve-quasi independent set of $G \setminus v$.

Therefore, there is a vertex $u$ of $S$ such that for every neighbor $u'$ of $u$ in $(V(G \setminus v) \setminus S)$, $(S \setminus \{u\}) \cup \{u'\}$ is not a ve-quasi independent set of $G \setminus v$.

Thus, condition (i) or (ii) is satisfied.

Conversely, suppose for any maximum secured ve-quasi independent set $S$ of $G$ not containing $v$, (i) or (ii) is satisfied. Suppose, $\beta_{sq}(G \setminus v) = \beta_{sq}(G)$. Let $T$ be a maximum secured ve-quasi independent set of $G \setminus v$. Now, $T$ is a secured ve-quasi independent set of $G$ also. Since $|T| = \beta_{sq}(G \setminus v) = \beta_{sq}(G)$, $T$ is a maximum secured ve-quasi independent set of $G$ not containing $v$. Note that for any two adjacent vertices $x$ and $y$ of $S$, $x$ and $y$ have a common neighbor in $(V(G \setminus v) \setminus S)$. Thus condition (i) is violated. Since $S$ is a secured ve-quasi independent set of $G \setminus v$, for each $u \in S$ there is a neighbor $u'$ of $u$ in $(V(G \setminus v) \setminus S)$ such that $(S \setminus \{u\}) \cup \{u'\}$ is a ve-quasi independent set of $G \setminus v$. Therefore, condition (ii) is also violated. Thus if we assume that $\beta_{sq}(G \setminus v) = \beta_{sq}(G)$ then both the conditions (i) and (ii) are violated for some maximum secured ve-quasi independent set $S$ of $G$ not containing $v$. This is a contradiction.

Therefore, $\beta_{sq}(G \setminus v) < \beta_{sq}(G)$.

**Example 3.27.** Consider the figure 3

Here, $S = \{v_1, v_3, v_5\}$ is a secured ve-quasi independent set of $G$. Therefore $\beta_{sq}(G) = 3$. Consider the subgraph $G' \setminus \{v_3\}$. Then $T = \{v_1, v_3\}$ is a secured ve-quasi independent set of $G' \setminus \{v_3\}$. Therefore $\beta_{sq}(G' \setminus v_3) = 2$. Thus, $\beta_{sq}(G' \setminus v_3) < \beta_{sq}(G)$.

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