On bipolar fuzzy rough continuous functions

S. Anita Shanthi\textsuperscript{1,*} and M. Saranya\textsuperscript{2}

Abstract
The concept of bipolar fuzzy sets is an extension of fuzzy sets. Our aim is to define bipolar fuzzy rough topology by means of the lower and upper approximations on bipolar fuzzy sets, bipolar fuzzy rough image, bipolar fuzzy rough inverse image, bipolar fuzzy rough subspace and bipolar fuzzy rough continuous functions between these topologies. We construct some functions which are bipolar fuzzy rough continuous and further prove pasting lemma for bipolar fuzzy rough continuous mappings.

Keywords
Bipolar fuzzy rough image, bipolar fuzzy rough inverse image, bipolar fuzzy rough subspace, bipolar fuzzy rough continuous function.

AMS Subject Classification
03E72, 54A40.

1. Introduction

Pawlak [11, 12] proposed the theory of rough sets. Keyun Qin and Pei [7] successfully compared fuzzy rough set models and fuzzy topologies on a finite universe. Mathew and John [3] established and developed topological structures on rough sets. Rough topology in terms of rough sets was introduced by Lellis Thivagar et al. [8]. In [4, 5, 10], the concept of fuzzy rough sets were studied by replacing crisp binary relations with fuzzy relations on the universe.

The concept of bipolar fuzzy sets was introduced by Zhang [13] and Muthuraj [9]. Anita shanthi et al.\textsuperscript{2} proposed the notion of fuzzy rough continuous functions. They further introduced the concepts of Bipolar fuzzy rough set and bipolar fuzzy rough topology [1].

In this paper we aim to define bipolar fuzzy rough continuous function, construct some functions which are bipolar fuzzy rough continuous and further prove pasting lemma for bipolar fuzzy rough continuous mappings.

2. Bipolar fuzzy rough continuous function

In this section we define bipolar fuzzy rough image, bipolar fuzzy rough inverse image, bipolar fuzzy rough subspace, bipolar fuzzy rough continuous functions and also prove pasting lemma for bipolar fuzzy rough continuous mappings.

**Definition 2.1.** Let \((U, BFR(A), \tau)\) and \((U^*, BFR(B), \tau^*)\) be two bipolar fuzzy rough topological spaces. \(BFR(A) = (BFR(A), BFR(A)) \in \tau\) and \(BFR(B) = (BFR(B), BFR(B)) \in \tau^*\). If \(f^+: BFR^P(A) \rightarrow BFR^P(B), g^- : BFR^C(A) \rightarrow BFR^C(B), g^+ : BFR^D(A) \rightarrow BFR^D(B), \) then a bipolar fuzzy rough mapping \(h = ((f^-, f^+), (g^-, g^+)) : BFR(A) \rightarrow BFR(B)\) is defined as follows:

\[
h(BFR(A))(y) = \begin{cases} 
\bigvee_{x \in f^{-1}(y)} y, & \text{if } f^{-1}(y) \neq \emptyset \\
0, & \text{otherwise}
\end{cases}
\]

\(h(BFR(A))\) is called bipolar fuzzy rough image of the bipolar fuzzy rough set \(BFR(A)\).

**Example 2.2.** Let \((U = \{x_1, x_2, x_3\}, BFR(A), \tau)\) and \((U^* = \{y_1, y_2, y_3\}, BFR(B), \tau^*)\) be two bipolar fuzzy rough topological spaces. \(f^-, f^+, g^-, g^+\) are mappings defined as \(((f^-, f^+)(x_i), (g^-, g^+)(x_i)) = y_i\).
Consider $U = \{x_1, x_2, x_3\}$, $A = \{x_1/(-0.6,0.4), x_2/(-0.2,0.3), x_3/(-0.4,0.5)\}$ and
\[
    \begin{pmatrix}
        x_1 & x_2 & x_3 \\
        (1.1) & (-0.6,0.64) & (-0.6,0.54) \\
        (1.1) & (-0.6,0.64) & (-0.6,0.54) \\
        (1.1) & (-0.6,0.64) & (-0.6,0.54)
    \end{pmatrix}
\]

The negative upper approximations are
\[
    \mu_{BFR^A}(x_1) = -0.4, \quad \mu_{BFR^A}(x_2) = -0.4, \quad \mu_{BFR^A}(x_3) = -0.4,
\]
$BFR^A = \{x_1/ -0.4, x_2/ -0.4, x_3/ -0.4\}$.

The positive lower approximations are
\[
    \mu_{BFR^A}(x_1) = 0.3, \quad \mu_{BFR^A}(x_2) = 0.3, \quad \mu_{BFR^A}(x_3) = 0.46,
\]
$BFR^A = \{x_1/0.36, x_2/0.3, x_3/0.46\}$.

The positive upper approximations are
\[
    \mu_{BFR^A}(x_1) = 0.5, \quad \mu_{BFR^A}(x_2) = 0.6, \quad \mu_{BFR^A}(x_3) = 0.5,
\]
$BFR^A = \{x_1/0.5, x_2/0.5, x_3/0.5\}$.

Hence, $\tau = \{x_1/(-1.1), x_2/(-0.6,0.64), x_3/(-0.6,0.54), x_1/(0.0), x_2/(0.0), x_3/(0.0)\}$.

Now
\[
    f^{-}(BFR^A)\ y_1 = \cup_{x \in (f^{-})(1)} (y_1)\ \mu_{BFR^A}(x)
\]
\[
    = \cup_{x \in x_1} \mu_{BFR^A}(x) \Rightarrow x_1
\]
\[
    = \mu_{BFR^A}(x_1)
\]
\[
    = -0.4.
\]

Similarly, the other values are calculated.

Thus, $h(BFR(A)) = \{x_1/(-0.4,0.36), x_2/(-0.4,0.3), x_3/(-0.4,0.46), x_1/(-0.6,0.5), x_2/(-0.6,0.5), x_3/(-0.6,0.5)\}$ is the bipolar fuzzy rough image of the bipolar fuzzy rough set $BFR(A)$.

**Definition 2.3.** Let $(U, BFR(A), \tau)$ and $(U^*, BFR(B), \tau')$ be two bipolar fuzzy rough topological spaces.

$h = (f^{-}, f^{+}, (g^{-}, g^{+})) : BFR(A) \to BFR(B)$ where

- $f^{-} : BFR^A \to BFR^B$,
- $f^{+} : BFR^A \to BFR^B$,
- $g^{-} : BFR^B \to BFR^A$,
- $g^{+} : BFR^B \to BFR^A$ are bipolar fuzzy rough mappings.

Then for a bipolar fuzzy rough set $BFR(B) \in \tau$, $h^{-1}(BFR(B))$ is a bipolar fuzzy rough set in $\tau$ obtained as follows:

$h^{-1}(BFR(B))(x) = BFR(B)(h(x))$.

$h^{-1}(BFR(B))$ is called the bipolar fuzzy rough inverse image of the bipolar fuzzy rough set $BFR(B)$.

**Example 2.4.** Let $(U = \{x_1, x_2, x_3\}, BFR(A), \tau)$ be defined as in Example 2.2 and $(U^* = \{y_1, y_2, y_3\}, BFR(B), \tau')$ be two bipolar fuzzy rough topological spaces. $U^* = \{y_1, y_2, y_3\}$, $B = \{y_1/(-0.2,0.6), y_2/(-0.3,0.4), y_3/(-0.5,0.4)\}$ and

\[
    BFR = \begin{pmatrix}
        y_1/(-1.1) & (-0.3,0.6) & (-0.3,0.4) \\
        y_2/(-1.1) & (-1.1) & (-0.7,0.4) \\
        y_3/(-0.3,0.4) & (-0.7,0.4) & (-1.1)
    \end{pmatrix}
\]

Let $f^{-}, f^{+}, (g^{-}, g^{+})$ be functions defined as,

$h(x) = ((f^{-}, f^{+})(x_1), (g^{-}, g^{+})(x_2)) = y_1, 1, 2, 3$. Now,

\[
    (f^{-1})(BFR(B))(x_1) = BFR^B(y_1) = -0.5.
\]

Similarly, the other values are calculated.

$h^{-1}(BFR(B)) = \{x_1/(-0.5,0.4), x_2/(-0.3,0.4), x_3/(-0.3,0.4)\}$.

**Definition 2.5.** Let $(U, BFR(A), \tau)$ and $(U^*, BFR(B), \tau')$ be bipolar fuzzy rough topological spaces, where $\tau = \{U, \phi, BFR(A), \tau' = \{U^*, \phi, BFR(B)\}$ and $h = ((f^{-}, f^{+}, (g^{-}, g^{+})) : BFR(A) \to BFR(B)$.

Then $h$ is said to be bipolar fuzzy rough continuous if the inverse image under $h$ of any $BFR(B) \in \tau'$ is a bipolar fuzzy rough set $BFR(A) \in \tau$, i.e. $h^{-1}(BFR(B)) \in \tau$, whenever $BFR(B) \in \tau'$.

**Example 2.6.** Let $(U = \{x_1, x_2, x_3\}, BFR(A), \tau)$ and $(U^* = \{y_1, y_2, y_3\}, BFR(B), \tau')$ be two bipolar fuzzy rough topological spaces. Consider $U = \{x_1, x_2, x_3\}$, the bipolar fuzzy subset $A$ of $U$ defined as

\[
    A = \{x_1/(-0.15,0.14), x_2/(-0.12,0.17), x_3/(-0.19,0.15)\}
\]

and the bipolar fuzzy relation $BFR$ defined on $U \times U$ as

\[
    BFR = \begin{pmatrix}
        x_1 & x_2 & x_3 \\
        (-1.1) & (-0.19,0.3) & (-0.19,0.23) \\
        (-1.1) & (-0.19,0.3) & (-0.19,0.23) \\
        (-0.2,0.23) & (-0.2,0.23) & (-1.1)
    \end{pmatrix}
\]

Now, $\tau = \{x_1/(-1.1), x_2/(-0.19,0.3), x_3/(-0.19,0.23), x_3/(-0.19,0.14), x_2/(-0.15,0.17), x_1/(-0.19,0.15), x_1/(-0.19,0.14), x_2/(-0.15,0.17), x_3/(-0.19,0.15), x_1/(-0.19,0.17), x_2/(-0.19,0.17), x_3/(-0.19,0.17)\}$.

$BFR(A) = \{x_1/(-0.19,0.14), x_2/(-0.15,0.17), x_3/(-0.19,0.15)\}$. 

Similarly, the other values are calculated.
Consider $U^* = \{y_1, y_2, y_3\}$, $B = \{(1/0.8, 0.76), (−0.75, 0.68), (−0.6, 0.61)\}$, $\mathcal{B}_F^{\prime} = \{\lambda_1, \lambda_2, \lambda_3\}$, $y_1 = (−1.1, −0.75, 0.8), y_2 = (−0.75, 0.8, 1), y_3 = (−0.75, 0.7, 0.7), y_3 = (−0.75, 0.7, 0.7)$.

Let $h((f^−, f^+), (g^−, g^+)) : BFR(A) \rightarrow BFR(B)$, where $f^− : BFR^O(A) \rightarrow BFR^O(B)$, $f^+ : BFR^O(A) \rightarrow BFR^O(B)$, $g^− : BFR^O(A) \rightarrow BFR^O(B)$, $g^+ : BFR^O(A) \rightarrow BFR^O(B)$ are bipolar fuzzy rough mappings. Then the following statements are equivalent:

(i) The bipolar fuzzy rough function $h(f, g) : BFR(A) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous.

(ii) The inverse image of every bipolar fuzzy rough closed set is bipolar fuzzy rough closed.

(iii) For each bipolar fuzzy rough point neighborhood of $h(x(BFR(A)))$ under $h$ is a bipolar fuzzy rough neighborhood of $x(BFR(A))$.

(iv) For each bipolar fuzzy rough point $x(BFR(A))$ in $BFR(A)$ and each bipolar fuzzy rough neighborhood $BFR(B)$ of $h(x(BFR(A)))$, there is a bipolar fuzzy rough neighborhood $BFR(C)$ of $x(BFR(A))$ such that $h(BFR(C)) \subseteq BFR(B)$.

(v) $h(BFR(A)) \subseteq h(BFR(A))$.

Proof. (i)$\Rightarrow$ (ii). Let $(f, g) : BFR(A) \rightarrow BFR(B)$ be bipolar fuzzy rough continuous and $BFR(B) \in \tau$. Then $h^−(BFR(B)) \in \tau$, $BFR(B) \in \tau$.

Again $h^−(BFR(B)) = [h^−(BFR(B))]^\tau$ and $h^−(BFR(B))$ be bipolar fuzzy rough continuous, $h^−(BFR(B))$ is bipolar fuzzy rough open in $(U, BFR(A), \tau)$. Hence $h^−(BFR(B))$ is bipolar fuzzy rough closed in $(U, BFR(A), \tau)$. i.e., $h^−(BFR(B))$ is bipolar fuzzy rough closed in $(U, BFR(A), \tau)$.

(ii)$\Rightarrow$ (iii). Let $BFR(C)$ be a bipolar fuzzy rough neighborhood of $h(x(BFR(A)))$. Then there is a bipolar fuzzy rough open set $BFR(B) \in \tau$ such that $h(x(BFR(A))) \subseteq BFR(B) \subseteq BFR(C)$.

Now $x(BFR(A)) \in h^−(h(x(BFR(A)))) \subseteq h^−(BFR(B)) \subseteq h^−(BFR(C))$,

i.e., $x(BFR(A)) \in h^−(BFR(B)) \subseteq h^−(BFR(C))$,

where $h^−(BFR(B))$ is bipolar fuzzy rough open in $(U, BFR(A), \tau)$.

(iii)$\Rightarrow$ (iv). Let $x(BFR(A)) \in BFR(A)$ and $BFR(B)$ be bipolar fuzzy rough neighborhoods of $h(x(BFR(A)))$. Then $h^−(BFR(B))$ is a bipolar fuzzy rough neighborhood of $h(BFR(A))$. Thus there exists a bipolar fuzzy rough open set $BFR(C)$ in $BFR(A)$ such that

$x(BFR(A)) \in BFR(C) \subseteq h^−(BFR(B)) \subseteq h^−(BFR(C))$.

i.e., $h(BFR(C)) \subseteq BFR(B) \subseteq BFR(C)$.

(iv)$\Rightarrow$ (v). Since $h^−(BFR(A))$ is bipolar fuzzy rough closed in $BFR(B)$, $h^−(h(BFR(A)))$ is bipolar fuzzy rough closed in $BFR(A)$, Thus $h^−(h(BFR(A))) = h^−(h^−(h(BFR(A))))$.

Now, $BFR(A) \subseteq h^−(h(BFR(A))) \subseteq h^−(h(BFR(A)))$, as $h(BFR(A)) \subseteq h(BFR(A))$.

$\Rightarrow BFR(A) \subseteq h^−(h(BFR(A))) = h^−(h(BFR(A)))$

$\Rightarrow BFR(A) \subseteq h^−(BFR(A))$

$\Rightarrow BFR(A) \subseteq h^−(BFR(A))$

i.e., $h(BFR(A)) \subseteq h(BFR(A))$.

(v)$\Rightarrow$ (i). Let $BFR(B)$ be bipolar fuzzy rough closed in
Let $BFR(B)$ and let $BFR(A) \in h^{-1}(BFR(B))$. By assumption $h^{-1}([BFR(B)]) \subseteq [h^{-1}(BFR(B))].$

Then $h^{-1}(BFR(B)) \subset BFR(B) \Rightarrow h^{-1}(BFR(B)) \subset BFR(B)$

Therefore, $h^{-1}(BFR(B))$ is bipolar fuzzy rough open in $BFR(A)$ whenever $BFR(B)$ is bipolar fuzzy rough open in $BFR(B).$ Let $BFR(C)$ be bipolar fuzzy rough open set of $BFR(B)$.

$\Rightarrow [BFR(C)]\,^{c}$ is bipolar fuzzy rough open in $BFR(C)$.

$\Rightarrow h^{-1}(BFR(C))\,^{c}$ is bipolar fuzzy rough open in $BFR(A)$.

$\Rightarrow h^{-1}(BFR(C))$ is bipolar fuzzy rough open in $BFR(A)$, whenever $BFR(C)$ is bipolar fuzzy rough open in $BFR(B).$

Therefore $h$ is bipolar fuzzy rough continuous.

**Theorem 2.10.** Let $(U, BFR(A), \tau)$ and $(U^*, BFR(B), \tau')$ be two bipolar fuzzy rough topological spaces. If $h : BFR(A) \rightarrow BFR(B)$ maps all of $BFR(A)$ into a single bipolar fuzzy rough point(constant) $x$ for $BFR(B)$ of $BFR(B).$ Then $h$ is bipolar fuzzy rough continuous.

**Proof.** Let $h : BFR(A) \rightarrow BFR(B)$ be bipolar fuzzy rough mapping such that $f(xBFR(A)) = xBFR(B)$ for every $xBFR(A) \in BFR(A).$ Consider $BFR(B)$ a bipolar fuzzy rough open set in $BFR(A)$.

$f^{-1}(BFR(B)) = \begin{cases} \emptyset, & \text{if } xBFR(B) \notin BFR(B) \\ U, & \text{if } xBFR(B) \in BFR(B). \end{cases}$

Then $U$ and $\emptyset$ are bipolar fuzzy rough open in $BFR(A).$ Therefore $f^{-1}(BFR(B))$ is bipolar fuzzy rough open in

$BFR(A)$, whenever $BFR(C)$ is bipolar fuzzy rough open in $BFR(B).$ Therefore any constant function is bipolar fuzzy rough continuous.

**Theorem 2.11.** Let $(U, BFR(A), \tau)$ be a bipolar fuzzy rough topological space. If $BFR(B)$ is a bipolar fuzzy rough subspace of $BFR(A)$, the inclusion function $i : BFR(B) \rightarrow BFR(A)$ is bipolar fuzzy rough continuous.

**Proof.** Let $BFR(B)$ be bipolar fuzzy rough subspace of $BFR(A)$ and $i : BFR(B) \rightarrow BFR(A).$ Since $BFR(B)$ is a bipolar fuzzy rough subspace of $BFR(A)$, for any bipolar fuzzy rough open set $BFR(V)$ of $BFR(A)$, $i^{-1}(BFR(V)) = BFR(V) \cap BFR(B)$ is bipolar fuzzy rough open in bipolar fuzzy rough subspace topology of $BFR(B)$, whenever $BFR(V)$ is bipolar fuzzy rough open in $BFR(B).$ Therefore, every inclusion map is bipolar fuzzy rough continuous.

**Theorem 2.12.** Let $(U, BFR(A), \tau)$, $(U^*, BFR(B), \tau')$ and $(U^{**}, BFR(C), \tau'')$ be bipolar fuzzy rough topological spaces. If $h : BFR(A) \rightarrow BFR(B)$ and $j : BFR(B) \rightarrow BFR(C)$ are bipolar fuzzy rough continuous, then the map $j \circ h : BFR(A) \rightarrow BFR(C)$ is bipolar fuzzy rough continuous.

**Proof.** Let $BFR(D)$ be bipolar fuzzy rough open in $BFR(C).$ As $j$ is bipolar fuzzy rough continuous, $j^{-1}(BFR(D))$ is bipolar fuzzy rough open in $BFR(B).$ As $h$ is bipolar fuzzy rough continuous,

$h^{-1}(j^{-1}(BFR(D))) = (j \circ h)^{-1}(BFR(D))$ is bipolar fuzzy rough open in $BFR(A)$, whenever $BFR(D)$ is bipolar fuzzy rough open in $BFR(C).$ Therefore, composition of two bipolar fuzzy rough continuous functions is bipolar fuzzy rough continuous.

**Theorem 2.13.** Let $(U, BFR(A), \tau)$ and $(U^*, BFR(B), \tau')$ be bipolar fuzzy rough topological spaces. If $h : BFR(A) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous and if $BFR(B)$ is a subspace of $BFR(A)$, then the restricted function $h/_{BFR(B)} : BFR(B) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous.

**Proof.** Let $i : BFR(B) \rightarrow BFR(A)$ be the inclusion map of $BFR(B)$ into $BFR(A).$ $i$ being the inclusion map is bipolar fuzzy rough continuous. Given $h : BFR(A) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous. Therefore it follows that

$h/_{BFR(B)} : BFR(B) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous.

**Theorem 2.14.** Let $(U, BFR(A), \tau)$ and $(U^*, BFR(B), \tau')$ be bipolar fuzzy rough topological spaces. If $h : BFR(A) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous and $BFR(B)$ is a bipolar fuzzy rough subspace of $BFR(B)$ containing the image set $h(BFR(A))$, then the function $j : BFR(A) \rightarrow BFR(B)$ obtained by restricting the range of $h$, is bipolar fuzzy rough continuous. If $BFR(C)$ has $BFR(B)$ as a subspace, then the function $k : BFR(A) \rightarrow BFR(C)$ obtained by expanding the range of $h$ is bipolar fuzzy rough continuous.

**Proof.** Let $h : BFR(A) \rightarrow BFR(B)$ be bipolar fuzzy rough continuous. If $h(BFR(A)) \subseteq BFR(B) \subset BFR(A)$, we show that the function $j : BFR(A) \rightarrow BFR(B)$ obtained from $h$ is bipolar fuzzy rough continuous. Let $BFR(F)$ be bipolar fuzzy rough open in $BFR(B)$.

$BFR(F) = BFR(B) \cap BFR(E)$, for some bipolar fuzzy rough open set $BFR(E)$ of $(U^*, BFR(B), \tau')$. Since $h^{-1}(BFR(E))$ is bipolar fuzzy rough open in $(U, BFR(A), \tau)$, $h^{-1}(BFR(E)) = j^{-1}(BFR(F))$. Because $BFR(B)$ contains the entire image set $h(BFR(A))$. $j^{-1}(BFR(F))$ is bipolar fuzzy rough open in $(U, BFR(A), \tau)$ whenever $BFR(F)$ is bipolar fuzzy rough open in $BFR(B)$. Therefore $j : BFR(A) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous. $h : BFR(A) \rightarrow BFR(B)$ is bipolar fuzzy rough continuous and the inclusion map $i : BFR(B) \rightarrow BFR(C)$ being the composition of two bipolar fuzzy rough continuous functions is bipolar fuzzy rough continuous.

**Theorem 2.15.** (Pastning lemma) Let $(U, BFR(A), \tau)$ and $(U^*, BFR(B), \tau')$ be bipolar fuzzy rough topological spaces. If $f : BFR(A) \rightarrow BFR(C)$ and $g : BFR(B) \rightarrow BFR(C)$ are bipolar fuzzy rough continuous mappings, then $f$ and $g$ combine to give a bipolar fuzzy rough
continuous function \( h : (BFR(A), BFR(B)) \rightarrow BFR(C) \), defined by setting
\[
h(BFR(A), BFR(B))(y) = \begin{cases} f(BFR(A))(y) & \text{if } BFR(C) = f(BFR(A)) \\ g(BFR(B))(y) & \text{if } BFR(C) = g(BFR(B)). \end{cases}
\]

**Proof.** Consider \( BFR(C) \) a bipolar fuzzy rough open set in \( \tau \). Then \( h^{-1}(BFR(C)) = f^{-1}(BFR(C)) \cup g^{-1}(BFR(C)) \). \( f : BFR(A) \rightarrow BFR(C) \) is bipolar fuzzy rough continuous and \( BFR(C) \) is bipolar fuzzy rough open in \( \tau \). But \( BFR(A) \) is bipolar fuzzy rough open in \( \tau \). Therefore \( f^{-1}(BFR(C)) \) is bipolar fuzzy rough open in \( \tau \). \( g : BFR(B) \rightarrow BFR(C) \) is bipolar fuzzy rough continuous and \( BFR(C) \) is bipolar fuzzy rough open in \( \tau \). But \( BFR(B) \) is bipolar fuzzy rough open in \( \tau \). Therefore \( g^{-1}(BFR(C)) \) is bipolar fuzzy rough open in \( \tau \). Therefore \( h^{-1}(BFR(C)) = f^{-1}(BFR(C)) \cup g^{-1}(BFR(C)) \) is bipolar fuzzy rough open in \( \tau \), whenever \( BFR(C) \) is bipolar fuzzy rough open in \( \tau \). Therefore \( h : (BFR(A), BFR(B)) \rightarrow BFR(C) \) is bipolar fuzzy rough continuous. \( \square \)

**Example 2.16.** Let \( (U = \{x_1, x_2, x_3\}, BFR(A), BFR(B), \tau) \) and \( (U^* = \{y_1, y_2, y_3\}, BFR(C), \tau) \) be two bipolar fuzzy rough topological spaces.

Consider \( U = \{x_1, x_2, x_3\} \)
\[
A = \{x_1/(-0.3, 0.5), x_2/(-0.15, 0.25), x_3/(-0.25, 0.21)\}
\]

\( BFR(A) = \{x_1/(-0.25, 0.4), x_2/(-0.3, 0.25), x_3/(-0.3, 0.21)\} \)
\( BFR(B) = \{x_1/(-0.5, 0.4), x_2/(-0.4, 0.3), x_3/(-0.5, 0.6)\} \)

\( h(A) = \{y_1/(-0.2, 0.4), y_2/(-0.3, 0.3), y_3/(-0.6, 0.6)\} \)

Now, \( (f^{-1}(BFR(C))(x_1) = BF_R^p(C)f^-(x_1) = BF_R^p(C)(y_1 - 1/2) = -0.7 \)
\( (f^{-1}(BFR(C))(x_2) = BF_R^p(C)f^-(x_2) = BF_R^p(C)(y_2 - 1/2) = -0.7 \)
\( (f^{-1}(BFR(C))(x_3) = BF_R^p(C)f^-(x_3) = BF_R^p(C)(y_3 - 1/2) = -0.65 \)

Similarly, the other values are calculated.
\[
(h^-(x_1) = (\rho^-, \sigma^-)(x_1) = \left(\frac{-1 - 2y_1}{2}, \frac{1 - 2y_1}{2}\right), i = 1, 2, 3.
\]

**References**


