\(\mathcal{AC}\) and \(\mathcal{AC}_2\)-Paracompact spaces

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Abstract

The reason for this paper is to present the two new ideas of \(\mathcal{AC}\)-Paracompact spaces and \(\mathcal{AC}_2\)-Paracompact spaces. Additionally we have demonstrated that each \(\mathcal{AC}\)-Paracompactness and \(\mathcal{AC}_2\)-Paracompactness has a topological property. We have likewise presented the \(\mathcal{AC}\)-Normal and its properties.

Keywords

Angelic spaces, \(\mathcal{C}\)-Paracompact, \(\mathcal{C}_2\)-Paracompact, \(\mathcal{C}\)-Normal, \(\mathcal{AC}\)-Paracompact, \(\mathcal{AC}_2\)-Paracompact, \(\mathcal{AC}\)-normal.

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1. Introduction

In 1944, Dieudonné. J [7] presented the paracompact space. The idea of paracompactness [7] is one of the most helpful speculation of compactness. Some notable mathematicians of different occasions have contemplated certain stronger just as more weaker types of paracompactness. \(\mathcal{C}\)-Paracompact and \(\mathcal{C}_2\)-Paracompact were characterized by Arhangel’skii. \(\mathcal{C}\)-Paracompact and \(\mathcal{C}_2\)-Paracompact were concentrated in [17]. Alzahrani S [3] research \(\mathcal{C}\)-normal topological property. Fermlín’s idea of angelic space [6] and a portion of its emanation carry us with he required tools for introducing those outcomes in a mordant idea.

2. Preliminaries

Definition 2.1. [6] A topological space \(T\) is termed as an angelic, if for each relatively countably compact subset \(\mathcal{I}\) of \(T\) the ensuing hold: (a) \(\mathcal{I}\) is relatively compact (b) If \(s \in \mathcal{I}\), then there is a sequence in \(\mathcal{I}\) that converges to \(s\).

Definition 2.2. [14] A topological space \(X\) is paracompact, if each open cover has a locally finite open refinement.

Definition 2.3. [17]

A topological space \(M\) is termed as \(\mathcal{C}\)-paracompact if \(\exists\) a paracompact space \(N\) and a bijective mapping \(p: M \to N\) \(\ni\) the restriction \(p\vert_S: S \to p(S)\) is a homeomorphism for every compact subspace \(S \subseteq M\).

Definition 2.4. [17] A topological space \(M\) is termed as \(\mathcal{C}_2\)-paracompact if \(\exists\) a Hausdorff paracompact space \(N\) and a bijective mapping \(p: M \to N\) \(\ni\) the restriction \(p\vert_S: S \to p(S)\) is a homeomorphism for every compact subspace \(S \subseteq M\).

Definition 2.5. [3] A space \(M\) is termed as \(\mathcal{C}\)-normal if \(\exists\) a normal space \(N\) and function \(p: M \to N\) \(\ni\) the restriction \(p\vert_S: S \to p(S)\) is homeomorphism for each compact subspace \(S \subseteq M\).

Definition 2.6. [10] A space \((M, \tau)\) is termed as submetrizable if \(\exists\) a metric \(d\) on \(M\) \(\ni\) the topology \(\tau_d\) on \(M\) caused by \(d\) is coarser than \(\tau\), i.e. \(\tau_d \subseteq \tau\).

Definition 2.7. [2] A space \((M, \tau)\) is supposed to be epinormal if \(\exists\) a coarser topology \(\tau'\) on \(X\) \(\ni (M, \tau')\) is normal.

Definition 2.8. [4] A topology \(\tau\) on a nonempty set \(M\) is supposed to be minimal Hausdorff if \((M, \tau)\) is Hausdorff and has no Hausdorff topology on \(M\) strictly coarser than \(\tau\).

Definition 2.9. [20] A space \(M\) is termed as mildly normal, \(k\)-normal, if any two disjoint closed domains \(U\) and \(V\) of \(M\) \(\exists\) disjoint open sets \(U'\), \(V'\) of \(M\) \(\ni U \subseteq U'\) and \(V \subseteq V'\).
Theorem 3.3. Let $M$ be any $\mathfrak{A}^\mathcal{E}$-paracompact space. Choose an angelic paracompact space $N$ and a bijective function $p : M \to N \ni p(s) \to S$ is a homeomorphism for every angelic compact subspace $S \subseteq M$. Suppose the Alexandroff duplicate spaces $A(M)$ and $A(N)$ of $M$ and $N$ respectively. By reason of $N$ is an angelic paracompact, next $A(N)$ is also an angelic paracompact. Characterize $q : A(M) \to A(N)$ by $q(a) = A(a)$ if $a \in M$. If $a \in M_0$ consider the unique element $b \in M \ni b_0 = a$, then characterize $q(a) = (p(b))'$. Next $q$ is a bijective mapping. Currently, a subspace $S \subseteq A(M)$ is an angelic compact iff $S \cap M$ is an angelic compact in $M$ and for every open set $U$ in $M$ with $S \cap M \subseteq A(M)$, we state $S \cap M'/U'$ is finite. Take $S \subseteq A(M)$ is any angelic compact subspace. To prove $q(S) : S \to q(S)$ is a homeomorphism. Take $a \in S$ is arbitrary. If $a \in S \cap M'$, let $b \in X$ be the unique element $\exists b' = a$. For the smallest basic open neighborhood $\{(f(b))'\}$ of the point $g(a)$ we state that $\{a\}$ is open in $\mathcal{C}$ and $\{g(a)\} \subseteq \{(f(b))'\}$. If $a \in S \cap M$. Let $W$ be any open set in $N \ni g(a) = f(a) \in W$. Consider $H = (W \cup U'/(f(a)'\}) \cap q(G)$ which is a basic open neighborhood of $p(a)$ in $q(S)$. By reason of $p|_{S \cap M} : S \cap M \to p(S \cap M)$ is a homeomorphism, then $\exists$ an open set $U$ in $M$ with $a \in U$ and $p|_{S \cap M(U/S)} \subseteq W$. Currently, $(U \cup U'/(d')) \cap q(G)$ is in $\mathcal{C}$ and $\exists a \in G$ and $q(G) \subseteq H$. Thus, $q(S)$ is continuous. Currently, to prove that $q|_{S}$ is open. Take $K \cap (K'/K')$, here $k \in K$ and $K$ is open in $M$, be any basic open set in $A(M)$, then $K \cap N \cap S \cup (K'S \cup \{K'/K'\})$ is a basic open set in $\mathcal{C}$. By reason of $M \cap N$ is an angelic compact in $M$, then $q|_{S\cap M(M'\cap C)} = q|_{M(M'\cap M(S))}$ is open in $N \cap p(S \cap M)$ as $p|_{M \cap S} : M \cap S \to p(S \cap M)$ is a homeomorphism. Hence $K \cap N$ is open in $N \cap p(N \cap M)$. Also, $p(K' \cap S)/K'$ is open in $N' \cap q(S)$ be a set of isolated points. Hence $q|_{S}$ is an open function. Hence, $p|_{S}$ is a homeomorphism. 

Theorem 3.5. If $M$ is an $\mathfrak{A}^\mathcal{E}$-paracompact space ($\mathfrak{A}^\mathcal{E}_2$-paracompact space), then its Alexandroff duplicate $A(M)$ is also an $\mathfrak{A}^\mathcal{E}$-paracompact space ($\mathfrak{A}^\mathcal{E}_2$-paracompact space).

Definition 3.1. Let $M$ be an angelic space and $S$ be an angelic compact subspace of $M$. If there is a bijection mapping $p : M \to N$, $N$ is an angelic paracompact space and the restriction $p|_{S} : S \to p(S)$ is a homeomorphism, then $M$ is said to be an $\mathfrak{A}^\mathcal{E}$-paracompact space.

Definition 3.2. Let $M$ be an angelic space and $S$ be an angelic compact subspace of $M$. If there is a bijection function $p : M \to N$, $N$ is a Hausdorff angelic paracompact space and the restriction $p|_{S} : S \to p(S)$ is homeomorphism for every angelic compact subspace $S \subseteq M$. Let $q : O \to M$ be a homeomorphism. Hence, $N$ and $p \circ q : O \to N$ has the topological properties.

Theorem 3.3. Every $\mathfrak{A}^\mathcal{E}$-paracompact space ($\mathfrak{A}^\mathcal{E}_2$-paracompact space) is a topological property.

Proof. Suppose $M$ is an $\mathfrak{A}^\mathcal{E}$-paracompact ($\mathfrak{A}^\mathcal{E}_2$-paracompact) space and $M \ni O$. Let $N$ be an angelic paracompact (Hausdorff angelic paracompact) space and $p : M \to N$ be a bijective mapping $\exists$ the restriction $p|_{S} : S \to p(S)$ is homeomorphism for every angelic compact subspace $S \subseteq M$. Let $q : O \to M$ be a homeomorphism. Hence, $N$ and $p \circ q : O \to N$ has the topological properties.

Theorem 3.4. Every $\mathfrak{A}^\mathcal{E}$-paracompact space ($\mathfrak{A}^\mathcal{E}_2$-paracompact space) has an additive property.

Proof. Suppose $M_\alpha$ is an $\mathfrak{A}^\mathcal{E}$-paracompact ($\mathfrak{A}^\mathcal{E}_2$-paracompact) space for each $\alpha \in A$. To prove that their sum $\oplus_{\alpha \in A} M_\alpha$ is an $\mathfrak{A}^\mathcal{E}$-paracompact ($\mathfrak{A}^\mathcal{E}_2$-paracompact) space. For each $\alpha \in A$, choose an angelic paracompact (Hausdorff angelic paracompact) space $N_\alpha$ and bijective mapping $p_\alpha : M_\alpha \to N_\alpha \ni p_\alpha(b) : S \to p_\alpha(S)$ is a homeomorphism for every angelic compact subspace $S \subseteq M_\alpha$. By reason of $N_\alpha$ is an angelic paracompact (Hausdorff angelic paracompact) for each $\alpha \in A$, then the sum $\oplus_{\alpha \in A} N_\alpha$ is an angelic paracompact (Hausdorff angelic paracompact) space. Consider the function sum $\oplus_{\alpha \in A} p_\alpha : \oplus_{\alpha \in A} M_\alpha \to \oplus_{\alpha \in A} p_\alpha(S)$ described by $\oplus_{\alpha \in A} p_\alpha(m) = p(m)$ if $m \in M_\beta$, $\beta \in A$. Currently, a subspace $S \subseteq \oplus_{\alpha \in A} M_\alpha$ is an angelic compact iff the set $\Lambda_0 = \{\alpha \in A : S \cap M_\alpha \neq \emptyset\}$ is finite, $S \cap M_\alpha$ is an angelic compact in $M_\alpha$ for every $\alpha \in \Lambda_0$. If $S \subseteq \oplus_{\alpha \in A} M_\alpha$ is an angelic compact, subsequently $\oplus_{\alpha \in A} M_\alpha$ is a homeomorphism as $p_{\alpha / \cap M_\alpha}$ is a homeomorphism for every $\alpha \in \Lambda_0$. 

Theorem 3.5. If $M$ is an $\mathfrak{A}^\mathcal{E}$-paracompact ($\mathfrak{A}^\mathcal{E}_2$-paracompact), then its Alexandroff duplicate $A(M)$ is also an $\mathfrak{A}^\mathcal{E}$-paracompact ($\mathfrak{A}^\mathcal{E}_2$-paracompact).
a homeomorphism. Currently, take \( V \subseteq U \) is some open neighbourhood of \( y \); next \( V \cap p(T) \) is open in the subspace \( p(T) \) including \( n \). Thus, \( p^{-1}(V) \cap T \) is open in the subspace \( T \) containing \( m \). Thus, \( p^{-1}(V) \cap B \cap \{ m_n : n \in N \} \neq \emptyset \), so \((p^{-1}(V) \cap T) \cap A \neq \emptyset \). Hence, \( \emptyset \neq p((p^{-1}(V) \cap T) \cap S) \subseteq p((p^{-1}(V)) \cap S) = V \cap p(S) \). Hence, \( n \in (p(S)) \) and \( p(S) \subseteq (p(S)) \). Thus, \( p \) is continuous.

**Corollary 3.8.** If \( M \) is an \( \mathcal{S}_\mathcal{C} \)-paracompact (\( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact) first countable space and \( p : M \to Y \) is a witness of the \( \mathcal{S}_\mathcal{C} \)-paracompact (\( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact) of \( M \), then \( p \) is continuous.

**Corollary 3.9.** If \( M \) is an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact Frechet space, then \( X \) is Hausdorff.

**Proof.** Since \( N \) is a \( T_2 \) angelic paracompact space and \( p : M \to N \) be a bijective mapping \( \exists \) the restriction \( p|_S : S \to p(S) \) is homeomorphism for every angelic compact subspace \( S \subseteq M \). Through Theorem 3.5, \( p \) is continuous. Take \( A, B \) be some disjoint angelic compact space; then \( f(A), f(B) \) are disjoint angelic compact subspaces of \( N \). By reason of \( N \) is \( T_2 \), then \( f(A) \) and \( f(B) \) are disjoint closed subspaces of \( N \). By reason of \( N \) is \( T_2 \) angelic paracompact, \( N \) is normal and thus \( \exists \) open subsets \( G \) and \( H \) of \( Y \) \( \ni f(A) \subseteq G, f(B) \subseteq H \), and \( \exists \) open sets \( U \) and \( V \subseteq U \). By the continuity of \( p \), \( U = p^{-1}(G) \) and \( V = p^{-1}(H) \). Thus, for every disjoint angelic compact subspaces \( A, B, \exists \) open sets \( U \) and \( V \), \( S \subseteq U \). Finally, \( \exists \) open sets \( U \) and \( V \) with the discrete topology. Thus the identity mapping from \( M \) onto \( N \). Hence, \( \exists \) a bijection mapping \( p : M \to N, N \) is an angelic Hausdorff paracompact space and the restriction \( p|_S : S \to p(S) \) is a homeomorphism, then \( M \) is said to be an \( \mathcal{S}_\mathcal{C} \)-paracompact.

**Theorem 3.10.** If \( M \) is a \( T_1 \) space \( \exists \) the only angelic compact subsets are the finite subsets, then \( M \) is an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact space.

**Proof.** Suppose \( M \) is a \( T_1 \) space \( \exists \) the only angelic compact subspaces of \( M \) are the finite subsets of \( M \). Since \( T_1 \), finally some angelic compact subspace of \( M \) is discrete. Then, take \( N = M \) and let \( Y \) with the discrete topology. Thus the identity mapping from \( M \) onto \( N \). Hence, \( \exists \) bijection mapping \( p : M \to N, N \) is an angelic Hausdorff paracompact space and the restriction \( p|_S : S \to p(S) \) is a homeomorphism, then \( M \) is said to be an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact.

**Theorem 3.11.** Let \( M \) be the Hausdorff locally angelic compact space. Then \( M \) is an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-Paracompact space.

**Proof.** Since \( M \) is any Hausdorff locally angelic compact topological space. Then \( \exists \) a \( T_2 \) angelic compact space \( N \) and hence \( N \) is \( T_2 \) angelic paracompact, and a bijective function \( p : M \to N \) \( \ni p \) is continuous. By reason of \( p \) is continuous, Next for some angelic compact subspace \( S \subseteq M \), we have \( p|_S : S \to p(S) \) is a homeomorphism because 1 to 1, onto, and continuity are acquired from \( p \), and \( p|_S \) is closed as \( S \) is an angelic compact and \( p(S) \) is Hausdorff.

**Example 3.12.** A Tychonoff \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact space is not locally compact.

**Proof.** Consider the quotient space \( \mathbb{R}/\mathbb{N} \). We can describe it as follows: Let \( i = \sqrt{-1} \). Let \( N = \mathbb{R}/\mathbb{N} \cup i \). Define \( p : \mathbb{R} \to Y \) as follows:

\[
p(x) = \begin{cases} x; & \text{if } x \in \mathbb{R}/\mathbb{N} \\ i; & \text{if } x \in \mathbb{N}
\end{cases}
\]

Now consider on \( \mathbb{R} \) the usual topology \( U \). Define on \( N \) the topology \( \tau = \{ \mathbb{W} \subseteq Y : p^{-1}(\mathbb{W}) \in U \} \). Then \( p : (U, \tau) \to (N, \pi) \) is a closed quotient mapping. We can describe the open neighborhoods of each element in as follows: The open neighborhoods of \( i \in N \) are \( (U/\mathbb{N}) \cup \{ i \}, \) here \( U \) is an open set in \( (N, U) \ni \mathbb{N} \subseteq U \). The open neighborhoods of any \( y \in \mathbb{R}/\mathbb{N} \) are \( (y - \varepsilon, y + \varepsilon) \mathbb{N} \) where \( \varepsilon \) is a positive real number. It is well known that \( (N, \pi) \to (X, \tau') \) is an angelic compact and \( T_2 \)天使ic. Therefore, \( (N, \pi) \to (X, \tau') \) is an angelic compactness.

**Definition 3.13.** A topological space \( (M, \tau) \) is termed as lower angelic compact if \( \exists \) a coarser topology \( \tau' \) on \( M \) \( \ni (X, \tau') \) is \( T_2 \) angelic compact.

**Theorem 3.14.** If \( (M, \tau) \) lower angelic compact space, Then \( M \) is an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact.

**Proof.** Suppose \( \tau' \) is a \( T_2 \) angelic compact topology on \( M \) \( \ni \tau' \subseteq \tau \). Next \( (M, \tau') \) is \( T_2 \) angelic paracompact and the identity mapping \( i_{id} : (M, \tau') \to (M, \tau') \) is a continuous function. If \( S \) is some angelic compact subspace of \( (M, \tau) \), next the restriction of the identity mapping on \( S \) onto \( i_{id}(S) \) is a homeomorphism as \( S \) is an angelic compact, \( i_{id}(S) \) is Hausdorff being a subspace of the \( T_2 \) space \( (M, \tau') \) and every continuous 1-1 function of an angelic compact space onto a Hausdorff space is a homeomorphism. Hence, \( M \) is an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact.

**Theorem 3.15.** If \( (M, \tau) \) is an \( \mathcal{S}_\mathcal{C} \)-\( \mathcal{C}_2 \)-paracompact countably angelic compact Frechet, then \( (M, \tau) \) is lower angelic compact.

**Proof.** Consider a \( T_2 \) angelic paracompact space \( (N, \tau) \) and a bijection function \( p : (M, \tau) \to (N, \tau) \) \( \ni \) the restriction \( p|_S : S \to p(S) \) is homeomorphism for every angelic compact subspace \( S \subseteq M \). By reason of \( M \) is Frechet, then \( p \) is continuous. Hence, \( (M, \tau) \) is countably angelic compact. By reason of \( \tau \) is also an angelic paracompact, then \( (M, \tau) \) is \( T_2 \) angelic compact. Characterize a topology \( \tau' \) on \( M \) as follows: \( \tau' = \{ p^{-1}(U) : U \subseteq \tau' \} \). Then \( \tau' \) is coarser than \( \tau \) and \( p : (M, \tau') \to (N, \tau) \) is a bijection continuous function. Let \( W \subseteq \tau' \) be arbitrary; then \( W = p^{-1}(U) \) for some \( U \subseteq \tau \). Thus, \( p(W) = p(p^{-1}(U)) = U \). Hence, \( p \) is open and \( p \) is homeomorphism. Thus, \( (M, \tau') \) is \( T_2 \) angelic compact. Therefore, \( (M, \tau) \) is lower angelic compact.
4. **$\mathcal{C}$-Normal and its Properties**

**Definition 4.1.** A space $M$ is termed as an $\mathcal{C}$-normal if $\exists$ a normal space $N$ and a bijective $p : M \rightarrow N$ so that the restriction $p|_S : S \rightarrow p(S)$ is homeomorphism for each angelic compact subspace $S \subseteq M$.

**Definition 4.2.** A space $M$ is termed as Angelic countably normal if there exists a normal space $N$ and a bijective $p : M \rightarrow N$ so that the restriction $p|_S : S \rightarrow p(S)$ is a homeomorphism for each angelic countable subspace $S \subseteq M$.

**Example 4.3.** An $\mathcal{C}$-normal space is not an $\mathcal{C}$-paracompact.

**Proof.** Suppose $\mathbb{R}$ with $L = \{\emptyset, \mathbb{R}\} \cup \{(\infty, x) : x \in \mathbb{R}\}$. In this space $(\mathbb{R}, L)$, any two nonempty closed sets are intersect; thus, $(\mathbb{R}, L)$ is normal and thus $\mathcal{C}$-normal. $(\mathbb{R}, L)$ is not Hausdorff as any two nonempty open sets must intersect. A subset $S \subseteq \mathbb{R}$ is an angelic compact if it has a maximum element. Suppose that $(\mathbb{R}, L)$ is $\mathcal{C}$-paracompact. Take $N$ is Hausdorff paracompact space and $p : R \rightarrow Y$ be a bijection $p|_S : S \rightarrow p(S)$ is homeomorphism for every angelic compact subspace $S \subseteq R$. Let $S = (\infty, 0]$; then $S$ is an angelic compact in $(\mathbb{R}, L)$ and $S$ as a subspace is not Hausdorff because any two nonempty open sets in $S$ must intersect. However, $S$ will be homeomorphic to $p(S)$ and $p(S)$ is Hausdorff, being a subspace of a Hausdorff space, and this is a contradiction. Thus, $(\mathbb{R}, L)$ cannot be $\mathcal{C}$-paracompact. $\blacksquare$

**Example 4.4.** An infinite $\mathcal{C}$-normal space is not an $\mathcal{C}$-paracompact space.

**Proof.** Let $M = [0, \infty)$. Define $\tau = \{\emptyset, M\} \cup \{(0, x) : x \in \mathbb{R}, 0 < x\}$. Consider $(\mathbb{R}, L)$ is just the angelic subspace of $(\mathbb{R}, L)$. (i.e., $\tau = L_M = L_{(0, \infty)}$. Now consider $(M, \tau_0)$, where $\tau_0$ is the particular topology point. We have that $\tau$ is coarser than $\tau_0$ because any nonempty open set in $\tau$ must contain $0$. Thus, $(M, \tau_0)$ cannot be an angelic paracompact. Observe that $(M, \tau)$ is normal because there are no two nonempty closed disjoint subsets. Thus, $(M, \tau)$ is an $\mathcal{C}$-normal. Now, a subset $S$ of $M$ is an angelic compact iff $S$ has maximal element. If $S$ has maximal element, then any open cover for $S$ will be covered by one member of the open cover, the one that contains the maximal element. If $S$ has no maximal element, then $S$ cannot be finite. If $S$ is unbounded above, then $\{0, n : n \in \mathbb{N}\}$ would be an open cover for $S$ has no finite subcover. If $S$ is bounded above, let $y = sup S$ and pick an increasing sequence $(c_n) \subseteq S \ni c_n \rightarrow y$, where the convergence is taken in the usual metric topology on $M$. Then $\{0, c_n : n \in \mathbb{N}\}$ is an open cover for $S$ that has no finite subcover. Thus, $S$ would not be an angelic compact. $(M, \tau)$ is Frechet. That is because $M$ is first countable. If $x \in M$, then $B(x) = \{0, x + 1/n : n \in \mathbb{N}\}$ is a countable local base for $x$.

Now, suppose that $M$ is an $\mathcal{C}$-paracompact. Choose an angelic paracompact space $Y$ and a bijective mapping $p : M \rightarrow N$ so that $p|_S : S \rightarrow p(S)$ is a homeomorphism for each angelic compact subspace $A$ of $M$. By Corollary 3.8, $p$ is continuous. Thus, for some nonempty open subset $U$ of $N$ we have that $p^{-1}(U)$ is open in $M$. By reason of $p$ is a bijective, $M$ is infinite. For each $y \in M$, pick an open neighborhood $U_y$ of $y \ni \{U_y : y \in Y\}$ is an infinite open cover for $M$. By reason of each $U_y$ contains the element $p(0)$, then the open cover $\{U_y : y \in Y\}$ cannot have any locally finite open refinement and thus $Y$ is not paracompact, which is a contradiction. Therefore, $M$ is $\mathcal{C}$-normal but not an $\mathcal{C}$-paracompact. $\blacksquare$

**Lemma 4.5.** If $p : M \rightarrow N$ is a bijective function, $N$ is an $\mathcal{C}$-normal space and any finite subset of $M$ is discrete, then $N$ is $T_1$.

**Proof.** By reason of $p : M \rightarrow N$ is a bijective function $\ni p|_S : S \rightarrow p(S)$ is a homeomorphism for each angelic compact subspace $S \subseteq M$. Assume $M$ has more than one element and take $a, b$ are distinct elements of $N$. Let $c$ and $d$ be the unique elements of $M \ni p(c) = a$ and $p(d) = b$. Then $p|_{\{c, d\}} : \{c, d\} \rightarrow \{a, b\}$ is a homeomorphism and $\{c, d\}$ is a discrete subspace of $M$. Thus, $p(\{c\}) = \{a\}$ and $p(\{d\}) = \{b\}$ are both open in $\{a, b\}$ as a subspace of $N$. Thus, $\exists$ an open neighborhood $U_a \subseteq N$ of $a \ni U_a \cap \{a, b\} = \{a\}$; hence, $b \notin U_a$, and similarly $\exists$ an open neighborhood $U_b \subseteq N$ of $b \ni a \notin U_b$. Thus, $N$ is $T_1$. $\blacksquare$

**Example 4.6.** $\mathbb{R}$ with $\tau_p(\mathbb{R}, \tau_p)$ is not an $\mathcal{C}$-normal space.

**Proof.** $\mathbb{R}$ with $\tau_p$, here the particular point is $p \in \mathbb{R}$, is not $\mathcal{C}$-normal. By reason of $\tau_p = \{\emptyset\} \cup \{U \subseteq \mathbb{R} : p \in U\}$. We known that $(\mathbb{R}, \tau_p)$ is neither $T_1$ nor normal space and if $A \subseteq \mathbb{R}$, then $\{x, p : x \in A\}$ is an open cover for $S$, thus a subset $S$ of $R$ is an angelic compact iff it is finite. To show $(\mathbb{R}, \tau_p)$ is not $\mathcal{C}$-normal, suppose that $(\mathbb{R}, \tau_p)$ is $\mathcal{C}$-normal. Take $N$ is a normal space, $p : M \rightarrow N$ be a function $\ni$ the restriction $p|_S : S \rightarrow p(S)$ is a homeomorphism for every angelic compact subspace $S \subseteq (\mathbb{R}, \tau_p)$. Consider the ensuing two cases for the space $N$.

**Case (i):** $M$ is $T_1$. Take $S = \{a, b\}$, where $a \neq b$; then $S$ is an angelic compact subspace of $(\mathbb{R}, \tau_p)$. By assumption $p|_S : S \rightarrow p(S) = \{p(a), p(b)\}$ is a homeomorphism. By reason of $p(S)$ is a discrete subspace of $M$ and $M$ is $T_1$, then $p(S)$ is a discrete subspace of $M$. Hence, $p|_S$ is not continuous which is contradiction as $p|_S$ is a homeomorphism.

**Case (ii):** $M$ is not $T_1$. To prove the topology on $M$ is the particular point topology with $p(b)$ as its particular point. Assume that $N$ is not the particular point topology then $\exists$ a non-empty open set $U \subseteq \mathbb{R} \ni p(b) \notin U$. Choose $y \in U$ and take $x \in \mathbb{R}$ is the unique real number $p(x) = y$. Suppose $\{a, b\}$ and $\notin \mathbb{B}$ because $p(x) = y \in U$, $p(b) \notin U$, and $p$ is 1-1. Take $p|_{\{a,b\}} : \{a, b\} \rightarrow \{y, p(b)\}$. Currently, $\{y\}$ is open in the subspace $\{y, p(b)\}$ of $N$ as $\{y\} = Y \cap \{y, p(b)\}$, but $p^{-1}(\{y\}) = \{x\}$ and $\{x\}$ is not open in the subspace $\{a, b\}$ of $(\mathbb{R}, \tau_p)$, which means $p|_{\{a,b\}}$ is not continuous. Any particular point space consisting of more than one point cannot be normal, so which contradiction as $N$ is normal. Hence, $(\mathbb{R}, \tau_p)$ is not an $\mathcal{C}$-normal. $\blacksquare$
Theorem 4.7. Every angelic compact non-normal space is not an $\mathcal{A}c'$-normal.

Proof. Consider $M$ an angelic compact non-normal space. Assume $M$ is an $\mathcal{A}c'$-normal, then $\exists$ a normal space $N$ and a bijective mapping $p: M \to N \ni$ the restriction $p|_S: S \to p(S)$ is homeomorphism for every angelic compact subspace $S \subseteq M$. Therefore, $N$ is normal and $M$ is not an angelic compact non-normal space. Hence $M$ cannot be an $\mathcal{A}c'$-normal.

Theorem 4.8. Let $M$ be an $\mathcal{A}c'$-normal space. If every countable subspace of $M$ is included in an angelic compact subspace, then $M$ is an angelic countably normal.

Proof. Take $M$ is any $\mathcal{A}c'$-normal space $\ni$ $S$ is any countable subspace of $M$, then $\exists$ an angelic compact subspace $E \ni S \subseteq E$. Take $N$ is a normal space and $p: M \to N \ni$ a bijective mapping $\ni p|_S: S \to p(S)$ is homeomorphism for every angelic compact subspace $S$ of $M$. Presently, take $S$ is a countable subspace of $M$. Choose an angelic compact subspace $E$ of $\ni S \subseteq E$, next $p|_E: E \to p(E)$ is homeomorphism, hence $p|_S: S \to p(S)$ is homeomorphism as $\ni p|_E|_S = p|_S$.

Theorem 4.9. Let $M$ be an $\mathcal{A}c'$-normal. If $M$ is a Frechet Lindelöf space $\ni$ any finite subspace of $M$ is discrete, then $M$ is an $\mathcal{A}c'$-paracompact.

Proof. Consider $M$ is an $\mathcal{A}c'$-normal, then $\exists$ a normal space $N$ and a bijection function $p: M \to N \ni$ the restriction $p|_S: S \to p(S)$ is homeomorphism for every angelic compact subspace $S \subseteq M$. By Lemma 4.1, $Y$ is $T_1$ and hence $T_5$. By reason of $M$ is Frechet, then $p$ is continuous. By reason of $M$ is Lindelöf and $p$ is continuous and onto, then $N$ is Lindelöf. By reason of any $T_1$ Lindelöf space is an angelic paracompact, then $N$ is $T_2$ angelic paracompact. Therefore, $M$ is an $\mathcal{A}c'$-paracompact space.

Theorem 4.10. If $(M, \tau)$ is Lindelöf epinormal space then $(M, (M, \tau')$ is an $\mathcal{A}c'$-2-paracompact.

Proof. Suppose $(M, \tau)$ is some Lindelöf epinormal space. Choose a coarser topology $\tau'$ on $M \ni (M, \tau')$ is $T_4$. By reason of $(M, \tau)$ is Lindelöf and $\tau'$ is coarser than $\tau$ we have $(M, \tau')$ is $T_3$ and Lindelöf, and hence Hausdorff paracompact. Therefore, $(M, \tau')$ is an $\mathcal{A}c'$-2-paracompact as the identity function $id: (M, \tau) \to (M, \tau')$. Hence $(M, \tau)$ is an $\mathcal{A}c'$-paracompact.

Theorem 4.11. Let $(M, \tau)$ be an $\mathcal{A}c'$-2-paracompact Frechet space. Then $(M, \tau)$ is an epinormal.

Proof. Take $(M, \tau)$ is some $\mathcal{A}c'$-2-Paracompact Frechet space and $(M, \tau)$ is normal. Suppose that $(M, \tau)$ is not normal. Let $(N, \tau')$ be a $T_2$ angelic paracompact space and $p: (M, \tau) \to (N, \tau')$ be a bijective mapping $\ni$ the restriction $p|_S: S \to p(S)$ is homeomorphism for every angelic compact subspace $S \subseteq M$. By reason of $M$ is Frechet, $p$ is continuous; Theorem 3.5, define $\tau* = \{p^{-1}(U) : U \in \tau'\}$. It is clear $\tau*$ is a topology on $M$ coarser than $\tau \ni p: (M, \tau') \to (N, \tau')$ is continuous. If $W \in \tau*$, then $W = p^{-1}(U)$ here $U \in \tau'$. Thus, $p(W) = p(p^{-1}(U)) = U$, gives $p$ is open and homeomorphism. hence, $(M, \tau*)$ is $T_4$, hence, $(M, \tau)$ is an epinormal.

Corollary 4.12. Let $(M, \tau)$ be an $\mathcal{A}c'$-2-paracompact Frechet space, then $(M, \tau)$ is completely Hausdorff.

Example 4.13. Any $\mathcal{A}c'$-2-paracompact Frechet space is an epinormal.

Proof. Suppose that two countably infinite sets are termed as almost disjoint if their intersection is finite. Consider a subfamily of $\{\omega_0\}^{\omega_0} = \{A \subseteq \omega_0 : A \text{ is infinite}\}$ a mad family on $\omega_0$ if it is a maximal (with respect to inclusion) pairwise almost disjoint subfamily. Take $A$ is a pairwise almost disjoint subfamily of $\{\omega_0\}^{\omega_0}$. The Mrowka space $\mathcal{P}(A)$ is describe by $\omega_0 \cup A$, every point of $\omega_0$ is isolated, and a basic open neighborhood of $W \in A$ has $\{W\} \cup (W/F)$, with $F \in [\omega_0] < \omega_0 \ni B \subseteq \omega_0 : B \text{ is finite}$. Since $\exists$ an almost disjoint family $A \ni \{\omega_0\}^{\omega_0} \ni |A| > \omega_0$ and the Mrowka space $\mathcal{P}(A)$ is a Tychonoff, separable, first countable, and locally angelic compact space that is neither countably angelic compact, angelic paracompact, nor normal. $A$ is a mad family iff $\mathcal{P}(A)$ is pseudocompact. The Mrowka space $\mathcal{P}(A)$ is an $\mathcal{A}c'$-2-paracompact, being $T_2$ locally angelic compact. $\mathcal{P}(A)$ is also Frechet, being first countable. Hence Mrowka space is an epinormal.

Remark 4.14. Any minimal Hausdorff $\mathcal{A}c'$-2-paracompact Frechet space is an angelic compact.

Theorem 4.15. Let $M$ be a minimal Hausdorff second countable space. The ensuing are equivalent.

(i) $M$ is an $\mathcal{A}c'$-2-paracompact.
(ii) $M$ is locally angelic compact.
(iii) $M$ is an angelic compact
(iv) $M$ is an epinormal.
(v) $M$ is metrizable.
(vi) $M$ is lower compact.
(vii) $M$ is minimal $T_3$.

Proof. (i) $\Rightarrow$ (ii) By reason of any second countable space is first countable and any first countable space is Frechet, then Theorem 4.5, gives that $M$ is $T_2$ angelic compact and hence locally angelic compact. (ii) $\Rightarrow$ (iii) By reason of any $T_2$ locally angelic compact space is Tychonoff, by the minimality, $M$ is an angelic compact. (iii) $\Rightarrow$ (iv) Any $T_2$ angelic compact space is $T_4$. (iv) $\Rightarrow$ (v) Any epinormal space is $T_{21/2}$. By minimilaty, $M$ is angelic compact and hence $T_3$. By reason of any $T_3$ second countable space is metrizable, the result follows. (v) $\Rightarrow$ (vi) By minimality, $M$ is $T_{21/2}$ angelic compact...
and hence lower angelic compact. \((vi) \Rightarrow (vii)\) Again, by minimality, \(M\) is \(T_2\) angelic compact and hence \(T_\alpha\). By reason of any minimal \(T_\alpha\) space is an angelic compact. \((viii) \Rightarrow (i)\) By reason of any minimal \(T\alpha_2\) space is angelic compact, \(M\) will be \(T_2\) angelic paracompact and hence \(\mathcal{A}C_2\)-paracompact.

**Example 4.16.** A minimal Hausdorff second countable \(\mathcal{A}C\)-paracompact space is Cannot be \(\mathcal{A}C_2\)-paracompact.

**Proof.** Let \(M = \{a, b, c_i, a_i, b_i : i, j \in \mathbb{N}\}\) here all these elements are distinct. Characterize the ensuing neighborhood system on \(M\):

For each \(i, j \in \mathbb{N}\), \(a_i, b_i\) is isolated and \(b_i\) is isolated.

For each \(i \in \mathbb{N}\), \(B(c_i) = \{c_i, a_i, b_i : j \geq n : n \in \mathbb{N}\}\).

\(\mathcal{B}(a) = \{V^n(a) = \{a, a_i : i \geq n\} : n \in \mathbb{N}\}\).

\(\mathcal{B}(a) = \{V^n(b) = \{b, b_i : i \geq n\} : n \in \mathbb{N}\}\).

Denote the unique topology on \(M\) caused by the above neighborhood system by \(\tau\). Next \(\tau\) is minimal Hausdorff and \((M, \tau)\) is cannot compact. By reason of \(M\) is countable and each local base is countable, then the neighborhood system is a countable base for \((M, \tau)\), so it is second countable but not \(\mathcal{A}C_2\)-paracompact because it is not \(T_{2\frac{1}{2}}\) as the closure of any open neighborhood of a must intersect the closure of any open neighborhood of \(b\).

\[\blacksquare\]

**5. Conclusion**

Our primary outcomes incorporates the two new ideas of \(\mathcal{A}C\)-Paracompact spaces and \(\mathcal{A}C_2\)-Paracompact spaces. Likewise demonstrated that, each \(\mathcal{A}C\)-Paracompactness and \(\mathcal{A}C_2\)-Paracompactness has a topological property. We likewise explored the \(\mathcal{A}C\)-normal and its properties.

**References**


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