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Darcy-Benard double diffusive Marangoni convection with Soret effect in a composite layer system

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Abstract
The effect of Soret parameter on double diffusive Marangoni convection in a two-layer system, comprising an incompressible two component fluid saturated porous layer over which lies a layer of the same fluid under micro gravity condition is investigated analytically. The upper boundary of the fluid layer is free, the lower boundary of the porous layer is rigid and both the boundaries are insulating to heat and mass. At the interface, the velocity, shear stress, normal stress, heat, heat flux, mass and mass flux are assumed to be continuous. Thermal Marangoni number is obtained by solving ordinary differential equations using method of exact solution. The effect of different physical parameters on double diffusive Marangoni convection are also investigated in detail.

Keywords
Double Diffusive Marangoni Convection, Soret Effect, Thermal Marangoni Number.

AMS Subject Classification
76-XX,76Rxx,76Sxx.

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Contents

1. Introduction

Convection driven by surface tension gradient is termed as Marangoni convection. As surface tension is a strong function of temperature and solute concentration, a small fluctuation in temperature or solute concentration on the free surface can induce convection. The temperature gradient also induces mass flux which is known as the thermal diffusion process or Soret effect.

Bennacer R., Mahidjiba A., Vasseur P., Beji H and Duval R. [3] have studied numerically and analytically natural convection with Soret effect in a binary fluid saturating a shallow horizontal porous layer using the Darcy model and the density variation is also taken into account by the Boussinesq approximation.

M.S.Malashetty, S.N.Gaikwad and Mahantesh Swamy [10] have analysed the double diffusive convection in a two component couple stress liquid layer with Soret effect using both linear and non-linear stability analyses. They have found that the effects of couple stress are quite large and the positive Soret number enhances the stability while the negative Soret number enhances the instability.

S. Saravanan and T. Sivakumar [15] have analysed the onset of Marangoni convection in a non-reactive binary fluid layer in the presence of throughflow and Soret effect with free top surface. The linear stability analysis is followed and an exact solution is obtained for the corresponding eigenvalue problem by assuming that stationary convection is exhibited at the neutral state. They have found that the contribution from the Soret effect is seen only when the throughflow is weak, and for a wider range of upward throughflow when the bottom boundary is conducting. The instability gets advanced/delayed when the Soret parameter assumes negative/positive values.

A. Khadiri, A. Amahmid, M. Hasnaoui and A. Rtibi [1] have studied numerically the Soret effect on double diffusion in a two-dimensional square cavity filled with a saturated Darcy porous medium. P.V.S.N. Murthy and M.F. El-Amin [12] have analyzed Thermo-diffusion effect on free convection heat and mass transfer from a vertical surface embedded in a liquid saturated thermally stratified non - Darcy porous medium using a local non-similar procedure.

M. Bhuvaneswari, S. Sivasankaran and Y. J. Kim [9] have performed a numerical analysis to understand the mixed convection flow, heat and mass transfer with Soret effect in a two-sided lid-driven square cavity. J. P. Pascal and S. J. D. D'Alessio [8] considered the stability of a binary liquid film flowing down a heated incline. They have implemented a theoretical model which captures the Soret effect and the dependence of surface tension on both temperature and solutal concentration.

C. G. Mohan, A. Satheesh [4] have studied a two-dimensional steady state double-diffusive mixed convective flow in a square cavity with Soret effect. Rishi Raj Kairi, Ch. Ram Reddy and Santanu Raut [14] have emphasized the thermo-diffusion and viscous dissipation effects on double diffusive natural convection heat and mass transfer characteristics of non- Newtonian power-law fluid over a vertical cone embedded in a non-Darcy porous medium with variable heat and mass flux conditions.

Chigozie Israel-Cookey, Emeka Amos and Liberty Ebiwareme [5] investigated the effects of Soret and magnetic field on thermosolutal convection in a porous medium with concentration based internal heat source. They have formulated the leading equations assuming that the Boussinesq approximation is valid and the Darcy law is governing the flow. Linear stability analysis was employed to determine the onset of instability. They have also shown the influence of Soret, Hartmann number and internal heat tabular and graphical forms.

R Sumithra and B Komala [13] have investigated the effects of Parabolic and Inverted parabolic Salinity gradients on the onset of Double Diffusive Marangoni Convection in a two-layer system under microgravity condition with free upper boundary and rigid lower boundary using Darcy-Brinkman model, employing the method of Exact solution.

Hussam K. Jawad [7] have investigated in his thesis, natural convection and Soret effect in a multi-layered liquid and porous system. They have shown that the positive sign of Soret parameter indicates that the denser component moves in the direction of colder side where as negative sign indicating the less denser component moving towards colder side of the system under consideration. B Komala and R Sumithra [2] have investigated the effects of Uniform and Non uniform salinity gradients on the onset of Double Diffusive Convection in a composite layer with rigid boundaries. They have solved resulting Eigen value problem by Regular perturbation method and have found that for the stability demanding situations like solar ponds, the parabolic salinity profile is the most conducive where in the onset of double diffusive convection in a composite layer can be delayed. For the heat and mass transfer problems like petroleum and geothermal reservoirs, the inverted parabolic salinity profile is most suitable, where in the onset of double diffusive convection is fast. Marangoni convection plays an important role in material processes associated with unbalanced surface tension. The effect of this type of convection is significant in reduced gravity environment as it dampens buoyancy convection and hydro static pressure. The phenomenon of heat and mass transfer in a system that contain layers of both fluid and porous media is of great importance because of the common occurrence of the system in many environmental, natural, and industrial applications like ground water pollution, migration of minerals and mass transport modeling in living matters, geothermal systems, crude oil production, storage of nuclear waste material, solidification of castings, Extraction of oil from oil sand and deep oil reservoirs, thermal insulation systems and many more.

The above applications motivated to investigate the double diffusive Marangoni convection in composite layer with oret effect.

2. Formulation of the Problem

We consider a horizontal two component fluid saturated, isotropic, densely packed porous layer of thickness d_m underlying a two component fluid layer of thickness d, under micro gravity condition. The lower surface of the porous layer is rigid and the upper surface of the fluid layer is free with the surface tension effects depending on both temperature and concentration. Both the boundaries are kept at different constant temperatures and concentrations. Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z - axis vertically upwards. The basic steady state is assumed to be the quiescent and the governing equations are continuity, momentum, energy and concentration equations including Soret effects with Boussinesq approximation.
For fluid layer,

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla P + \mu \nabla^2 \vec{q} \tag{2.2}$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{2.3}$$

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = \kappa_c \nabla^2 C + \kappa_T \nabla^2 T \tag{2.4}$$

and for porous layer,

$$\nabla_m \cdot \vec{q}_m = 0 \tag{2.5}$$

$$\frac{\rho_0}{\phi} \left(\frac{\partial \vec{q}_m}{\partial t} \right) = -\nabla_m P_m - \frac{\mu}{K} \vec{q}_m \tag{2.6}$$

$$A\frac{\partial T_m}{\partial t}+(\vec{q}_m\cdot\nabla_m)\,T_m=\kappa_m\nabla_m^2T_m \tag{2.7}$$

$$\phi\frac{\partial C_m}{\partial t}+(\vec{q}_m.\nabla)\,C_m=\kappa_{mc}\nabla^2C_m+\kappa_{mT}\nabla^2T_m \tag{2.8}$$

where $\vec{q}=(u,v,w)$ is the velocity vector, t is the time, μ is the fluid viscosity, $P=p+\frac{\mu_pH^2}{2}$ is the total pressure, ρ_0 is the fluid density, $A=\frac{(\rho_0C_p)_m}{(\rho C_p)_f}$ is the ratio of heat capacities, C_p is the specific heat, K is the permeability of the porous medium, T is the temperature, κ is the thermal diffusivity, κ_c the solutal diffusivity, κ_T the Soret coefficient, C is the concentration or the salinity field, ϕ is the porosity.
The subscripts m refer to the porous medium.
The basic steady state is assumed to the quiescent and we consider the solution of the form,
In the fluid layer,

$$[u,v,w,P,T,C]=[0,0,0,P_b(z),T_b(z),C_b(z)] \tag{2.9}$$

and in the porous layer,

$$[u_m,v_m,w_m,P_m,T_m,C_m]=[0,0,0,P_{mb}(z_m),T_{mb}(z_m),C_{mb}(z_m)] \tag{2.10}$$

where the subscript b denotes the basic state.
The temperature distributions $T_b(z)$ and $T_m(z_m)$ are found to be

$$T_b(z)=T_0+\frac{(T_u-T_0)\,z}{d}\quad in\quad 0\le z\le d \tag{2.11}$$

$$T_m(z_m)=T_0+\frac{(T_L-T_0)\,z_m}{d_m}\quad in\quad 0\le z_m\le d_m \tag{2.12}$$

$$T_0=\frac{\kappa_m dT_l-\kappa d_mT_u}{\kappa_m d-\kappa d_m}\quad at\quad z=0. \tag{2.13}$$

The concentration distributions $C_b(z)$ and $C_m(z_m)$ are found to be

$$C_b=C_0+\frac{z(C_u-C_0)}{d}\quad in\quad 0\le z\le d \tag{2.14}$$

$$C_m=C_0+\frac{z_m(C_L-C_0)}{d_m}\quad in\quad 0\le z_m\le d_m \tag{2.15}$$

$$C_0=\frac{\kappa_{mc}dC_l-\kappa_C d_mC_u}{\kappa_{mc}d-\kappa_c d_m}\quad at\quad z=0. \tag{2.16}$$

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form,

$$[\vec{q},P,T,C]=[0,P_b(z),T_b(z),C_b(z)]+\left[\vec{q}',P',\theta,S\right]$$

and

$$[\vec{q}_m,P_m,T_m,C_m]=[0,P_{mb}(z_m),T_{mb}(z_m),C_{mb}(z_m)]+\left[\vec{q}'_m,P'_m,\theta_m,S_m\right]$$

The governing equations are linearized and then the pressure term is eliminated from momentum equation of fluid and porous layers by taking curl twice and only the vertical component is retained. The variables are then non-dimensionalised using $\frac{d^2}{\kappa}$, $\frac{\kappa}{d}$, T_0-T_u and C_0-C_u as the units of time velocity, temperature, and the concentration in the fluid layer and $\frac{d_m^2}{\kappa_m}$, $\frac{\kappa_m}{d_m}$, T_l-T_0 and C_l-C_0 as the corresponding characteristic quantities in the porous layer. The separate length scales are chosen for the two layers (Chen and Chen [6]), D.A Nield [11]), so that each layer is of unit depth with $(x,\,y,\,z)=d(x',\,y',\,z')$ and $(x_m,\,y_m,\,z_m)=d_m(x'_m,\,y'_m,\,z'_m-1)$. The detailed flow fields are obtained for all the depth ratios $\hat{d}=\frac{d_m}{d}$ in both the fluid and porous layers.
The dimensionless equations are rendered to normal mode expansion (following Venkatachalappa M *et al* [16]). It is known that the principle of exchange of instabilities holds for Double Diffusive convection in both fluid and porous layers separately for certain choice of parameters. Therefore, we assume that the principle of exchange of instabilities holds even for the composite layers (following Nield [11]). Denoting the differential operator $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial z_m}$ by D and D_m respectively, an Eigen value problem consisting of the following ordinary differential equations are obtained,
In $0\le z\le 1$

$$\left(D^2-a^2\right)^2W(z)=0 \tag{2.17}$$

$$\left(D^2-a^2\right)\Theta(z)+W(z)=0 \tag{2.18}$$

$$\left(D^2-a^2\right)S(z)+\left(\frac{1}{\tau_c}-S_{r1}\right)W(z)=0 \tag{2.19}$$

In $-1\le z_m\le 0$

$$\left(D_m^2-a_m^2\right)^2W_m(z_m)=0 \tag{2.20}$$

$$\left(D_m^2-a_m^2\right)\Theta_m(z_m)+W_m(z_m)=0 \tag{2.21}$$

$$\left(D_m^2-a_m^2\right)S_m(z_m)-\left(S_{r2}+\frac{1}{\tau_{mc}}\right)W_m(z_m)=0 \tag{2.22}$$

where $\beta^2=\frac{K}{d_m^2}=Da$ is the Darcy number, $\hat{\mu}=\frac{\nu_m}{\nu}$ is the viscosity ratio, $\tau_c=\frac{\kappa_c}{\kappa}$, $\tau_T=\frac{\kappa_T}{\kappa}$ are the diffusivity ratio in fluid layer, $\tau_{mc}=\frac{\kappa_{mc}}{\kappa_m}$, $\tau_{mT}=\frac{\kappa_{mT}}{\kappa_m}$ are the temperature and concentration diffusivity ratio in the porous layer, a and a_m are the non-dimensional horizontal wave numbers in fluid and porous regions respectively, Θ and Θ_m are the temperature in fluid and porous layers respectively, S and S_m are the concentration in fluid and porous layers respectively, $S_{r1}=\frac{\tau_T(T_0-T_u)}{\tau_c(C_0-C_u)}$, $S_{r2}=\frac{\tau_{mT}(T_0-T_u)}{\tau_{mc}(C_0-C_u)}$ are the Soret parameters in fluid and porous regions respectively.
These ordinary differential equations are solved exactly by using the following boundary conditions.

3. Boundary conditions

At the upper boundary,

$$W(1)=0, \quad D^2W(1)+Ma^2\Theta(1)+M_s a^2 S(1)=0,$$
$$D\Theta(1)=0, \quad DS(1)=0,$$

At the interface,

$$\hat{T}\hat{d}^3\beta^2\left(D^3W(0)-3a^2DW(0)\right)=$$
$$-D_mW_m(0)+\hat{\mu}\beta^2D_m^3W_m(0)-\hat{\mu}\beta^2 3a_m^2 D_mW_m(0),$$
$$\hat{T}W(0)=W_m(0), \quad \hat{T}\hat{d}DW(0)=D_mW_m(0),$$
$$\Theta(0)=\hat{T}\Theta_m(0), \quad D\Theta(0)=D_m\Theta_m(0),$$
$$\hat{T}\hat{d}^2\left(D^2+a^2\right)W(0)=\hat{\mu}\left(D_m^2+a_m^2\right)W_m(0),$$
$$S(0)=\hat{S}S_m(0), \quad DS(0)=D_mS_m(0),$$

At the lower boundary,

$$W_m(-1)=0, \quad D_mW_m(-1)=0,$$
$$D_m\Theta_m(-1)=0, \quad D_mS_m(-1)=0.$$

where $M=-\frac{\partial\sigma_t}{\partial T}\frac{(T_0-T_u)d}{\nu\kappa}$, $M_s=-\frac{\partial\sigma_t}{\partial S}\frac{(C_0-C_u)d}{\nu\kappa}$ are the thermal and solute Marangoni numbers.
$\hat{T}=\frac{(T_L-T_0)}{(T_0-T_U)}$, $\hat{S}=\frac{(C_L-C_0)}{(C_0-C_U)}$, $\hat{d}=\frac{d_m}{d}$,
$\hat{\kappa}=\frac{\kappa_m}{\kappa}=\frac{\hat{d}}{\hat{T}}$ and $\hat{\kappa}_s=\frac{\kappa_{sm}}{\kappa_s}=\frac{\hat{d}}{\hat{S}}$.

4. Exact Solution

The solutions of the Equations (2.17) and (2.20) are independent of $\Theta(z), S(z), \Theta_m(z_m), S_m(z_m)$. The expressions for W and W_m are obtained as,

$$W(z)=\Delta_8\lambda_{37}$$

and

$$W_m(z_m)=\Delta_8\left[\lambda_{38}+\lambda_{39}\right]$$

where

$$\Delta_1=\lambda_{33}\Delta_8, \quad \Delta_2=\lambda_{34}\Delta_8, \quad \Delta_3=\lambda_{36}\Delta_8,$$
$$\Delta_4=\lambda_{32}\Delta_8, \quad \Delta_5=\lambda_{35}\Delta_8, \quad \Delta_6=\lambda_{31}\Delta_8,$$
$$\Delta_7=\lambda_{30}\Delta_8, \quad \lambda_1=\frac{1-6a_m^2\hat{\mu}\beta^2}{2\hat{T}\hat{d}^3\beta^2a^3},$$
$$\lambda_2=\frac{a_m-4a_m^3\hat{\mu}\beta^2}{2\hat{T}\hat{d}^3\beta^2a^3}, \quad \lambda_3=\frac{1-\hat{T}\hat{d}a\lambda_1}{\hat{T}\hat{d}},$$
$$\lambda_4=\frac{a_m-\hat{T}\hat{d}a\lambda_2}{\hat{T}\hat{d}}, \quad \lambda_5=a_m sinha_m+cosha_m,$$
$$\lambda_6=sinha_m+a_m cosha_m, \quad \lambda_7=\lambda_3 cosha+\lambda_1 sinha,$$
$$\lambda_8=\lambda_4 cosha+\lambda_2 sinha, \quad \lambda_9=\hat{T}\hat{\mu}a^2,$$
$$\lambda_{10}=\hat{T}\hat{d}a, \quad \lambda_{12}=\hat{\mu}a_m\Delta_8, \quad \lambda_{13}=\frac{\lambda_{12}}{\lambda_9-\lambda_{11}},$$
$$\lambda_{14}=\frac{\lambda_{10}}{\lambda_9-\lambda_{11}}, \quad \lambda_{15}=\Delta_8\lambda_{13}cosha,$$
$$\lambda_{16}=sinha-\lambda_{14}cosha, \quad \lambda_{17}=\lambda_{14}\hat{T}cosha_m,$$
$$\lambda_{18}=sinha_m+\lambda_{13}\hat{T}cosha_m, \quad \lambda_{19}=-\lambda_{14}\hat{T}a_m sinha_m,$$
$$\lambda_{20}=a_m cosha_m, \quad \lambda_{21}=\lambda_6+\lambda_{13}\hat{T}a_m sinha_m,$$
$$\lambda_{22}=cosha_m-\frac{\lambda_7\lambda_{17}}{\lambda_{15}}, \quad \lambda_{23}=sinha_m-\frac{\lambda_8\lambda_{17}}{\lambda_{15}},$$
$$\lambda_{24}=\lambda_{17}-\frac{\lambda_{17}\lambda_{16}}{\lambda_{15}}, \quad \lambda_{25}=\lambda_5-\frac{\lambda_7\lambda_{19}}{\lambda_{15}},$$
$$\lambda_{26}=\lambda_{20}-\frac{\lambda_8\lambda_{19}}{\lambda_{15}}, \quad \lambda_{27}=\lambda_{21}-\frac{\lambda_{16}\lambda_{19}}{\lambda_{15}},$$
$$\lambda_{28}=\lambda_{26}-\frac{\lambda_{23}\lambda_{25}}{\lambda_{22}}, \quad \lambda_{29}=\lambda_{27}-\frac{\lambda_{24}\lambda_{25}}{\lambda_{22}},$$
$$\lambda_{30}=\frac{-\lambda_{29}}{\lambda_{28}}, \quad \lambda_{31}=-\frac{\lambda_{23}\lambda_{30}+\lambda_{24}}{\lambda_{22}},$$
$$\lambda_{32}=\frac{\lambda_7\lambda_{31}+\lambda_8\lambda_{30}+\lambda_{16}}{-\lambda_{15}}, \quad \lambda_{33}=\lambda_{13}-\lambda_{14}\lambda_{32},$$
$$\lambda_{34}=\lambda_3\lambda_{31}+\lambda_4\lambda_{30}, \quad \lambda_{35}=\hat{T}\lambda_{33},$$
$$\lambda_{36}=\lambda_1\lambda_{31}+\lambda_2\lambda_{30}$$
$$\lambda_{37}=(\lambda_{33}+\lambda_{34}z)cosh(az)+(\lambda_{36}+\lambda_{32}z)sinh(az),$$
$$\lambda_{38}=\lambda_{35}cosh(a_mz_m)+\lambda_{31}sinh(a_mz_m),$$
$$\lambda_{39}=\lambda_{30}cosh(\sigma z_m)+sinh(\sigma z_m).$$

The Temperature distributions $\Theta(z)$ and $\Theta_m(z_m)$ are obtained from the Equations (2.18) and (2.21) by substituting expressions for $W(z)$ and $W_m(z_m)$ and are as below.

$$\Theta(z)=\Delta_8\left[\alpha_{14}cosh(az)+\alpha_{12}sinh(az)-F(z)\right]$$
$$\Theta_m(z_m)=\Delta_8\left[\alpha_{15}cosh(a_mz_m)+\alpha_{13}sinh(a_mz_m)-F(z_m)\right]$$

where

$$F(z)=sinh(az)\left(\frac{z\lambda_{33}}{2a}+\frac{z^2\lambda_{34}}{4a}-\frac{z\lambda_{32}}{4a^2}\right)+$$
$$cosh(az)\left(\frac{z\lambda_{36}}{2a}-\frac{z\lambda_{34}}{4a^2}+\frac{z^2\lambda_{32}}{4a}\right),$$
$$F(z_m)=sinh(a_mz_m)\left(\frac{z_m\lambda_{35}}{2a_m}+\frac{z_m^2\lambda_{31}}{4a_m}-\frac{z_m}{4a_m^2}\right)+$$
$$cosh(a_mz_m)\left(\frac{z_m\lambda_{30}}{2a_m}-\frac{z_m\lambda_{31}}{4a_m^2}+\frac{z_m^2}{4a_m}\right),$$
$$\alpha_1=\frac{\lambda_{36}}{2a}-\frac{\lambda_{34}}{4a^2}, \quad \alpha_2=\alpha_{21}+\alpha_{22},$$
$$\alpha_3=\frac{\lambda_{30}}{2a_m}-\frac{\lambda_{31}}{4a_m^2}, \quad \alpha_4=\alpha_{41}+\alpha_{42}$$
$$\alpha_{21}=sinha\left(\frac{\lambda_{33}}{2a}+\frac{\lambda_{34}}{2a}-\frac{\lambda_{34}}{4a}+\frac{\lambda_{36}}{2}+\frac{\lambda_{32}}{4}-\frac{\lambda_{32}}{4a^2}\right),$$
$$\alpha_{22}=cosha\left(\frac{\lambda_{33}}{2}+\frac{\lambda_{34}}{4}-\frac{\lambda_{34}}{4a^2}+\frac{\lambda_{36}}{2a}+\frac{\lambda_{32}}{2a}-\frac{\lambda_{32}}{4a}\right),$$

$$\alpha_{41} = sinha_m\left(\frac{\lambda_{35}}{2a_m}+\frac{\lambda_{31}}{4a_m}-\frac{1}{4a_m^2}+\frac{\lambda_{30}}{2}+\frac{1}{4}\right),$$
$$\alpha_{42} = cosha_m\left(\frac{\lambda_{35}}{2}+\frac{\lambda_{31}}{4}+\frac{1}{4a_m}+\frac{\lambda_{30}}{2a_m}-\frac{\lambda_{31}}{4a_m^2}\right),$$
$$\alpha_5=\alpha_3+\alpha_1-\frac{\alpha_2}{e^a},\quad \alpha_6=ae^{-2a}-a,$$
$$\alpha_7=\alpha_5-\frac{\alpha_4}{e^{a_m}},\quad \alpha_8=a_m\left(1-e^{-2a_m}\right),$$
$$\alpha_9=1+e^{-2a},\quad \alpha_{10}=\hat{T}\left(1+e^{-2a_m}\right),$$
$$\alpha_{11}=\frac{\alpha_2}{ae^a}+\frac{\hat{T}\alpha_4}{a_me^{a_m}},\quad \alpha_{12}=\frac{\alpha_7\alpha_{10}+\alpha_8\alpha_{11}}{\alpha_6\alpha_{10}+\alpha_8\alpha_9},$$
$$\alpha_{13}=\frac{\alpha_7-\alpha_6\alpha_{12}}{\alpha_8},\quad \alpha_{14}=\alpha_{12}e^{-2a}+\frac{\alpha_2}{ae^a},$$
$$\alpha_{15}=\frac{\alpha_{13}a_me^{-a_m}-\alpha_4}{a_me^{a_m}}$$

The Concentration distributions $S(z)$ and $S_m(z_m)$ are obtained from the Equations (2.19) and (2.22) by substituting expressions for $W(z)$ and $W_m(z_m)$ and are as below.

$$S(z)=\Delta_8\left[\beta_{17}cosh(az)+\beta_{14}sinh(az)-\beta_{21}\right]$$
$$S_m(z_m)=\Delta_8\left[\beta_{16}cosh(a_mz_m)+\beta_{15}sinh(a_mz_m)-\beta_{11}\right]$$

where,

$$\beta_{11}=F(z_m)\left(\frac{1}{\tau_{mc}}-S_{r2}\right)-S_{r2}\beta_{12},$$
$$\beta_{12}=\left(\alpha_{15}cosh[a_mz_m]+\alpha_{13}sinh[a_mz_m]\right),$$
$$\beta_1=S_{r1}\left(\alpha_{14}+\alpha_{12}\right),$$
$$\beta_2=\left(\frac{P_{36}}{2a}-\frac{P_{34}}{4a^2}-\right)\left(\frac{1}{\tau_c}-S_{r1}\right)+\beta_{22},$$
$$\beta_{22}=S_{r1}a\left(\alpha_{14}-\alpha_{12}\right),$$
$$\beta_{21}=F(z)\left(\frac{1}{\tau_c}-S_{r1}\right)-\beta_{22},$$
$$\beta_{22}=S_{r1}(\alpha_{14}cosh[az]+\alpha_{12}sinh[az]),$$
$$\beta_3=\frac{df(1)}{dz}\left(\frac{1}{\tau_c}-S_{r1}\right)+S_{r1}a\left(\alpha_{14}-\alpha_{12}\right),$$
$$\beta_4=-S_{r2}\left(\alpha_{15}+\alpha_{13}\right),$$
$$\beta_5=\left(\frac{P_{30}}{2a_m}-\frac{P_{31}}{4a_m^2}-\right)\left(\frac{1}{\tau_{mc}}-S_{r2}\right)+\beta_{51}$$
$$\beta_{51}=S_{r2}a_m\left(\alpha_{15}-\alpha_{13}\right),$$
$$\beta_6=\frac{df_m(-1)}{dz}\left(\frac{1}{\tau_{mc}}-S_{r2}\right)+\beta_{61},$$
$$\beta_{61}=S_{r2}a_m\left(\alpha_{15}sinh[a_m]-\alpha_{13}cosh[a_m]\right),$$
$$\beta_7=\frac{\beta_3}{ae^a}-\beta_1-\hat{S}\beta_4,\quad \beta_8=1+e^{-2a},$$
$$\beta_9=\hat{S}\left(1+e^{2a_m}\right),\quad \beta_{10}=\beta_7+\frac{\beta_6\hat{S}}{a_me^{-a_m}},$$
$$\beta_{11}=a\left(e^{-2a-1}\right),\quad \beta_{12}=a_m\left(e^{2a_m}-1\right),$$
$$\beta_{13}=\beta_3e^{-a}-\beta_2+\frac{\beta_6}{e^{-a_m}-\beta_5},$$
$$\beta_{14}=\frac{\beta_{10}\beta_{12}-\beta_9\beta_{13}}{\beta_9\beta_{11}-\beta_8\beta_{12}},\quad \beta_{15}=\frac{\beta_{14}\beta_8+\beta_{10}}{\beta_9},$$
$$\beta_{16}=\frac{\beta_{15}a_me^{-a_m}-\beta_6}{a_me^{a_m}},\quad \beta_{17}=\frac{\beta_{14}ae^{-a}+\beta_3}{ae^a}$$

The Thermal Marangoni number is obtained by

$$D^2W(1)+Ma^2\Theta(1)+M_sa^2S(1)=0$$

as

$$M=\frac{-\left(a^2M_sM_1+M_2\right)}{a^2M_3} \tag{4.1}$$

where,

$$M_1=\beta_{17}e^a+\beta_{14}e^{-a}-F(z)\left(\frac{1}{\tau_c}-S_{r1}\right)-M_{11},$$
$$M_{11}=S_{r1}\left(\alpha_{14}cosh[az]+\alpha_{12}sinh[az]\right),$$
$$M_2=sinha\left(2aP_{34}+a^2P_{36}+a^2P_{32}\right)$$
$$+cosha\left(a^2P_{33}+a^2P_{34}+2aP_{32}\right),$$
$$M_3=\alpha_{14}e^a+\alpha_{12}e^{-a}-sinha\left(\frac{P_{33}}{2a}+\frac{P_{34}}{4a}-\frac{P_{32}}{4a^2}\right)-$$
$$cosha\left(\frac{P_{32}}{4a}+\frac{P_{36}}{2a}-\frac{P_{34}}{4a^2}\right).$$

5. Graphical Interpretations

The effects of horizontal wave number a on the thermal Marangoni number M is shown in **Figure 1** for fixed values of $Da=0.005$, $\hat{\mu}=1$, $M_s=10$, $S_{r1}=0.75$, $S_{r2}=0.25$, $\hat{S}=1$, $\hat{T}=1$. The curves are diverging, indicating that for larger values of depth ratios $\hat{d}$, the effect of horizontal wave number a is more. Also, by increase in the value of the horizontal wave number a, the value of the thermal Marangoni number M increases for a fixed depth ratio $\hat{d}$, thus the Darcy-Benard double diffusive Marangoni convection is retarded and hence the system is stabilized.

The effects of solute Marangoni number M_s on the thermal Marangoni number M is shown in **Figure 2** for fixed values of $a=2$, $\hat{\mu}=1$, $Da=0.005$, $S_{r1}=0.75$, $S_{r2}=0.25$, $\hat{S}=1$, $\hat{T}=1$. The increase in the value of the solute Marangoni number M_s, the value of the thermal Marangoni number M increases, thus the Darcy-Benard double diffusive Marangoni convection is retarded making the system stable.

The effects of Darcy number Da on the thermal Marangoni number M is shown in **Figure 3** for fixed values of $a=2$, $\hat{\mu}=1$, $M_s=10$, $S_{r1}=0.75$, $S_{r2}=0.25$, $\hat{S}=1$, $\hat{T}=1$. From the figure it is evident that the curves for different values Da are diverging and for smaller values of depth ratios $\hat{d}$, the effect of Darcy number Da is same. Also by increase

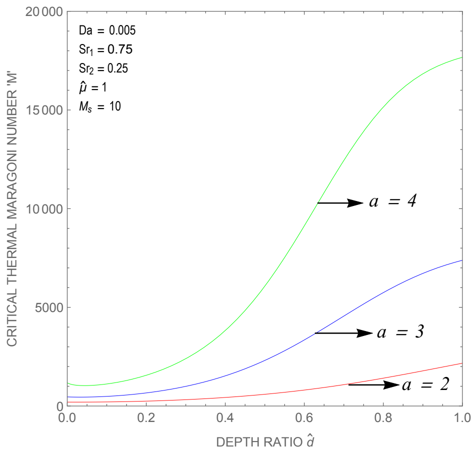

Figure 1. The Effect of the horizontal wave number a on thermal Marangoni number M

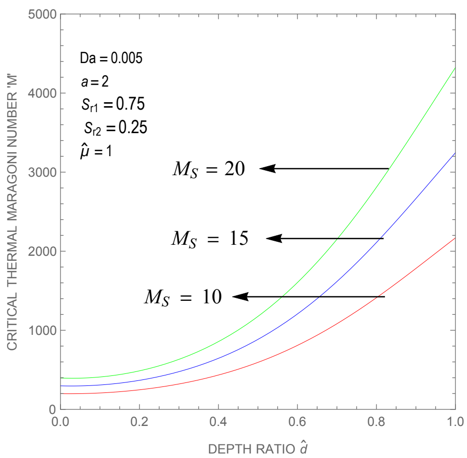

Figure 2. The Effect of solute Marangoni number M_s on thermal Marangoni number M

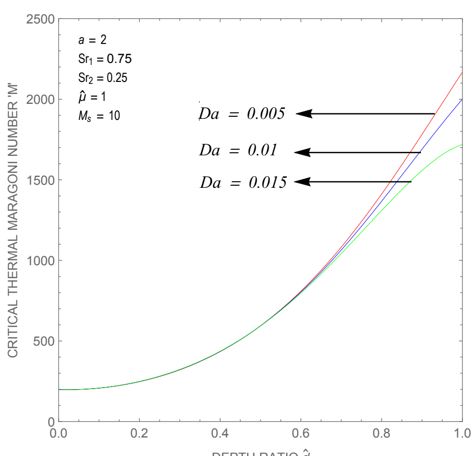

Figure 3. The Effect of Darcy number Da on thermal Marangoni number M

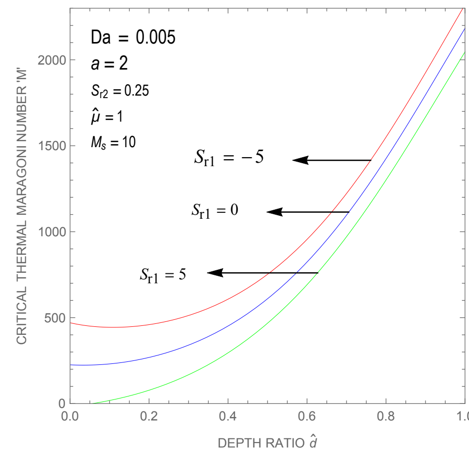

Figure 4. The Effect of Soret parameter of fluid layer S_{r1} on thermal Marangoni number M

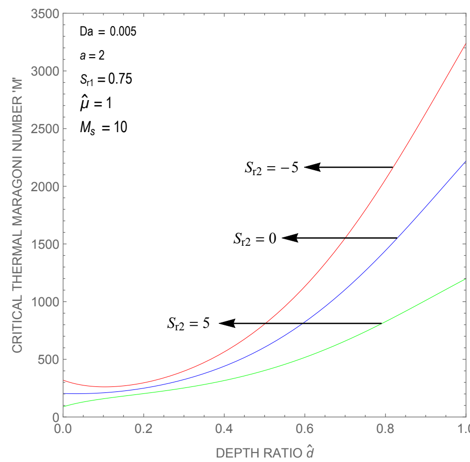

Figure 5. The Effect of Soret parameter of porous layer S_{r2} on critical thermal Marangoni number M

in the value of the Dary number Da, the value of the thermal Marangoni number M decreases, thus the Darcy-Benard double diffusive Marangoni convection is accelerated making the system unstable.

The effects of the soret parameter $S_{r1} = \dfrac{\tau_T(T_0 - T_u)}{\tau_c(C_0 - C_u)}$ in fluid layer, on the thermal Marangoni number M is shown in **Figure 4** for fixed values of $a = 2$, $\hat{\mu} = 1$, $M_s = 10$, $Da = 0.005$, $S_{r2} = 0.25$, $\hat{S} = 1$, $\hat{T} = 1$. By increase in the value of the Soret parameter S_{r1}, the value of the thermal Marangoni number M decreases, thus the Darcy-Benard double diffusive Marangoni convection is accelerated making the system unstable.

The effects of the soret parameter $S_{r2} = \dfrac{\tau_{mT}(T_0 - T_u)}{\tau_{mc}(C_0 - C_u)}$ in porous layer, on the thermal Marangoni number M is shown in **Figure 5** for fixed values of $a = 2$, $\hat{\mu} = 1$, $M_s = 10$, $Da = 0.005$, $S_{r1} = 0.75$, $\hat{S} = 1$, $\hat{T} = 1$. By increase in the value of the Soret parameter S_{r2}, the value of the thermal Marangoni number M decreases, thus the Darcy-

Benard double diffusive Marangoni convection is accelerated making the system unstable.

6. Consequences

- The Darcy-Benard double diffusive Marangoni convection can be accelerated by increasing the physical parameters Darcy number Da, the Soret parameters S_{r1} of fluid layer and the Soret parameter S_{r2} of porous layer making the system unstable.
- The Darcy-Benard double diffusive Marangoni convection can be retarded by increasing the physical parameters wave number a and solute Marangoni number M_s making the system stable.
- The effects of horizontal wave number a, the Darcy number Da, solute Marangoni number M_s and the Soret parameters S_{r1} of fluid layer are dominant for larger values of depth ratio $\hat{d}$ i.e., for porous layer dominant composite layer system.
- The effects of the Soret parameter S_{r2} of porous layer is prominent for smaller values of depth ratio $\hat{d}$ i.e., for fluid layer dominant composite layer system.

References

[1] A. Khadiri, A. Amahmid, M. Hasnaoui and A. Rtibi, Soret Effect on Double-Diffusive Convection in a Square Porous Cavity Heated and Salted from Below, *An International Journal of Computation and Methodology*, 57(11)(2010), 848-868.

[2] B. Komala and R. Sumithra, Effects of Uniform and Non Uniform Salinity Gradients on the Onset of Double Diffusive Convection in a Composite Layer: An Analytical Study, *ARPN Journal of Engineering and Applied Sciences*, 14(4)(2019), 856-868.

[3] Bennacer, R., Mahidjiba, A., Vasseur, P., Beji, H. and Duval, R., The Soret effect on convection in a horizontal porous domain under cross temperature and concentration gradients, *International Journal of Numerical Methods for Heat and Fluid Flow*, 13(2)(2003), 199-215.

[4] C. G. Mohan, A. Satheesh , Computational Investigation of Double-Diffusive Mixed Convective Flow in an Enclosed Square Cavity with Soret Effect, *Frontiers in Heat and Mass Transfer*, 8(36)(2017), 1–10.

[5] Chigozie Israel-Cookey, Emeka Amos and Liberty Ebiwareme, Soret and Magnetic Field Effects on Thermosolutal Convection in a Porous Medium with Concentration Based Internal Heat Source, *American Journal of Fluid Dynamics*, 8(2)(2018), 41-46.

[6] F. Chen, C.F. Chen and A.J. Pearlstein, Convective instability in superposed fluid and anisotropic porous layers, *Physics of Fluids*, 3(4)(1991), 556-565.

[7] Hussam K. Jawad, Natural Convection And Soret Effect In A Multi-Layered Liquid And Porous System, 2012.

[8] J. P. Pascal and S. J. D. D'Alessio, Thermosolutal Marangoni effects on the inclined flow of a binary liquid with variable density I, *Linear Stability Analysis*, 1(8)(2016), 083603(20).

[9] M. Bhuvaneswari, S. Sivasankaran and Y. J. Kim, Numerical Study on Double Diffusive Mixed Convection with a Soret Effect in a Two-Sided Lid-Driven Cavity, *Numerical Heat Transfer, Part A: Applications*, 59(7)(2011), 543-560.

[10] M.S.Malashetty, S.N.Gaikwad and MahanteshSwamy, An analytical study of linear and non-linear double diffusive convection with Soret effect in couple stress liquids, *International Journal of Thermal Sciences*, 45(9)(2006), 897-907.

[11] Nield D.A, Onset of Convection in a Fluid Layer overlying a layer of porous layer, *J. Fluid Mech.*, 1977, 513-522.

[12] P.V.S.N. Murthy and M.F. El-Amin, Thermo-Diffusion Effect on Free Convection Heat and Mass Transfer in a Thermally Linearly Stratified Non-Darcy Porous Media, *The Open Transport Phenomena Journal*, 3(2011), 49-55.

[13] R. Sumithra and B. Komala, Effects of Parabolic and Inverted Parabolic Salinity Gradients on Double Diffusive Marangoni Convection in a Composite Layer: An Exact Study, *International Journal of Application or Innovation in Engineering and Management*, 3(10)(2014), 258-273.

[14] Rishi Raj Kairi, Ch. Ram Reddy and Santanu Raut, Influence of viscous dissipation and thermo-diffusion on double diffusive convection over a vertical cone in a non-Darcy porous medium saturated by a non-Newtonian fluid with variable heat and mass fluxes, *Nonlinear Engineering*, 7(1)(2018), 65–72.

[15] S. Saravanan, T. Sivakumar, Exact solution of Marangoni convection in a binary fluid with throughflow and Soret effect, *Applied Mathematical Modelling*, 33(2009), 3674–3681.

[16] Venkatachalappa M, Prasad Shivakumara I S. and Sumithra R, Hydrothermal growth due to double diffusive convection in composite materials, *Proceedings of 14th National Heat and Mass Transfer Conference and 3rd ISHMT ASME Joint Heat and Mass Transfer Conference*, December 1997, 29-31.

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