Neutrosophic $g^S$ closed sets in neutrosophic topological spaces

A. Atkinswestley$^1$* and S. Chandrasekar$^2$

Abstract
Aim of this present paper is, we introduced and studied new type of generalized closed sets is called Neutrosophic $g^S$ closed sets and Neutrosophic $g^S$ open sets and followed that its properties and application are also discussed.

Keywords
Neutrosophic $g^S$ closed sets, Neutrosophic $g^S$ open sets, Neutrosophic $g^S$ interior, Neutrosophic $g^S$ interior, Neutrosophic topological spaces.

AMS Subject Classification
03E72.

1 Introduction

1.1 Introduction
Neutrosophic sets and system has been developed from intuitionistic fuzzy system with three components $T$ Truth, $F$-Falsehood, $I$-Indeterminacy. This concepts developed by Floretin Smarandache [10]. Neutrosophic sets and system have wide range of applications in the field of decision making, Artificial Intelligence, Information Systems, Computer Science, Applied Mathematics, Mechanics, Medicine, Management Science Electrical & Electronic, etc., New kind of open sets and closed sets are introduced every year in Topological spaces, Neutrosophic Topological spaces introduced by A.A. Salama [22] et al., R. Dhavaseelan [7] introduced Neutrosophic generalized closed sets. Neutrosophic semi closed sets are introduced by P. Ishwarya, [11] et al, V.K. Shanthi [23] et al., introduced Neutrosophic generalized semi closed sets. Aim of this paper is we introduce and study about Neutrosophic $g^S$ closed sets and Neutrosophic $g^S$ open sets in Neutrosophic topological spaces and its properties and Characterization are discussed details.

2 Preliminaries

In this section, we recall needed basic definition and operation of Neutrosophic sets and its fundamental Results.

Definition 2.1 ([10]). Let $X$ be a non-empty fixed set. A Neutrosophic set $A_i^*$ is aoboect having the form

$$A_i^* = \{ x, \mu_{A_i^*}(x), \sigma_{A_i^*}(x), \gamma_{A_i^*}(x) : x \in X \}$$

$\mu_{A_i^*}(x)$-represents the degree of membership function
$\sigma_{A_i^*}(x)$-represents degree indeterminacy and then
$\gamma_{A_i^*}(x)$-represents the degree of non-membership function.

Definition 2.2 ([10]). Neutrosophic set $A_1^*$ is aoboect having the form

$$A_1^* = \{ x, \mu_{A_1^*}(x), \sigma_{A_1^*}(x), \gamma_{A_1^*}(x) : x \in X \}$$

on $X$ and $\forall x \in X$ then complement of $A_1^*$ is,

$$A_1^C = \{ x, \gamma_{A_1^*}(x), 1-\sigma_{A_1^*}(x), \mu_{A_1^*}(x) : x \in X \}$$

Definition 2.3 ([10]). Let $A_1^*$ and $A_2^*$ are two Neutrosophic sets, $\forall x \in X$

$$A_1^* = \{ x, \mu_{A_1^*}(x), \sigma_{A_1^*}(x), \gamma_{A_1^*}(x) : x \in X \}$$
Neutrosophic $g^S$ closed sets in neutrosophic topological spaces — 1787/1791

$A_2^1 = \{ < x, \mu_{A_2^1}(x), \sigma_{A_2^1}(x), \gamma_{A_2^1}(x) > : x \in X \}$.

Then

$A_2^1 \subseteq A_2^2 \iff \mu_{A_2^1}(x) \leq \mu_{A_2^2}(x), \sigma_{A_2^1}(x) \leq \sigma_{A_2^2}(x) \land \gamma_{A_2^1}(x) \geq \gamma_{A_2^2}(x)$.  

**Definition 2.4** ([10]). Let $X$ be a non-empty set, and let $A_1^6$ and $A_2^6$ be two Neutrosophic sets are

$A_1^6 = \{ < x, \mu_{A_1^6}(x), \sigma_{A_1^6}(x), \gamma_{A_1^6}(x) > : x \in X \}$,

$A_2^6 = \{ < x, \mu_{A_2^6}(x), \sigma_{A_2^6}(x), \gamma_{A_2^6}(x) > : x \in X \}$.

Then

1. $A_1^6 \cap A_2^6 = \{ < x, \min(\mu_{A_1^6}(x), \mu_{A_2^6}(x)), \max(\sigma_{A_1^6}(x), \sigma_{A_2^6}(x)), \max(\gamma_{A_1^6}(x), \gamma_{A_2^6}(x)) > : x \in X \}$

2. $A_1^6 \cup A_2^6 = \{ < x, \max(\mu_{A_1^6}(x), \mu_{A_2^6}(x)), \min(\sigma_{A_1^6}(x), \sigma_{A_2^6}(x)), \min(\gamma_{A_1^6}(x), \gamma_{A_2^6}(x)) > : x \in X \}$.

**Definition 2.5** ([22]). Let $X$ be a non-empty set and $\tau_N$ be the collection of Neutrosophic subsets of $X$ satisfying the following properties:

1. $0_N, 1_N \in \tau_N$
2. $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$
3. $\cup T \in \tau_N$ for every $\{ T_i \}_{i \in J} \subseteq \tau_N$

Then the space $(X, \tau_N)$ is called a Neutrosophic topological space ($N-T-S$). The element of $\tau_N$ are called $N.O.S.$ (Neutrosophic open set) and their complement is $N.C.S.$ (Neutrosophic closed set).

**Example 2.6.** Let $X = \{ x \}$ and $\forall x \in X$

$A_1 = \left\langle x, \frac{6}{10}, \frac{6}{10}, \frac{6}{10}, \frac{9}{10} \right\rangle$, $A_2 = \left\langle x, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}, \frac{10}{10} \right\rangle$

$A_3 = \left\langle x, \frac{6}{10}, \frac{7}{10}, \frac{10}{10}, \frac{5}{10} \right\rangle$, $A_4 = \left\langle x, \frac{5}{10}, \frac{6}{10}, \frac{9}{10}, \frac{10}{10} \right\rangle$

Then the collection $\tau_N = \{ 0_N, A_1, A_2, A_3, A_4, 1_N \}$ is called a $N-T$-$S$ on $X$.

**Definition 2.7** ([2,7,11,12,23,24]). Let $(X, \tau_N)$ be a $N-T-S$ and

$A_1^6 = \{ < x, \mu_{A_1^6}(x), \sigma_{A_1^6}(x), \gamma_{A_1^6}(x) > : x \in X \}$

be a Neutrosophic set in $X$. Then $A_1^6$ is said to be

1. Neutrosophic $\alpha$-closed set ($N.\alpha CS$) if $N.cl(N.int(N.cl(A_1^6))) \subseteq A_1^6$,

2. Neutrosophic semi closed set ($N.SCS$) if $N.int(N.cl(A_1^6)) \subseteq A_1^6$,

3. Neutrosophic $\beta$-closed set ($N.\beta CS$) if $N.int(N.cl(N.int(A_1^6))) \subseteq A_1^6$,

4. Neutrosophic generalized closed set ($N.GCS$) if $N.cl(A_1^6) \subseteq H$ whenever $A_1^6 \subseteq H$ and $H$ is a $N.O.S.$

5. Neutrosophic $\alpha$ generalized closed set ($N.\alpha GCS$) if $Neu\, \alpha(cl(A_1^6)) \subseteq H$ whenever $A_1^6 \subseteq H$ and $H$ is a $N.O.S.$

6. Neutrosophic generalized semi closed set ($N.GSCS$) if $N.Scl(A_1^6) \subseteq H$ whenever $A_1^6 \subseteq H$ and $H$ is a $N.O.S.$

7. Neutrosophic generalized $\beta$ closed set ($N.G\beta$) if $N.\beta(cl(A_1^6)) \subseteq H$ whenever $A_1^6 \subseteq H$ and $H$ is a $N.O.S.$

### 3. Neutrosophic $g^S$ closed sets

**Definition 3.1.** A Neutrosophic set $A_1^6$ of a Neutrosophic topological space $(X, \tau_N)$ is called Neutrosophic $g^S$ semi closed set (briefly $N.\cdot g^S SC$) if $N.Scl(A_1^6) \subseteq U$ whenever $A_1^6 \subseteq U$ and $U$ is $N.\cdot g^S$-open set in $(X, \tau_N)$.

**Example 3.2.** Let $X = \{ a_1, a_2 \}$, $\tau_N = \{ 0_N, A_1^6, 1 \}$, is a $N.T.$ on $X$ where $A_1^6 = \langle x, \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \rangle$. Then the Neutrosophic set $A_2^6 = \langle x, \left( \frac{7}{10}, \frac{7}{10}, \frac{7}{10}, \frac{3}{10}, \frac{5}{10}, \frac{4}{10} \right) \rangle$ is a $N.\cdot g^S$-in $X$.

**Theorem 3.3.** Every $N$-closed set is $N.\cdot g^S$.

Proof. Let $A_1^6$ be a $N$-closed set in a $N.\cdot T.\cdot X$. Let $A_1^6 \subseteq U$, where $U$ is $N.\cdot g^S$-open set in $X$. Since $A_1^6$ is closed, we have

$N.cl(A_1^6) = N.Scl(A_1^6) = A_1^6$. Hence $A_1^6$ is $N.\cdot g^S$.

**Example 3.4.** Let $X = \{ a_1, a_2 \}$, $\tau_N = \{ 0_N, A_1^6, 1 \}$, is a $N.T.$ on $X$ where $A_1^6 = \langle x, \left( \frac{6}{10}, \frac{8}{10}, \frac{5}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \rangle$. Then the Neutrosophic set $A_2^6 = \langle x, \left( \frac{7}{10}, \frac{6}{10}, \frac{9}{10}, \frac{3}{10}, \frac{5}{10}, \frac{2}{10} \right) \rangle$ is $N.\cdot g^S$-in $X$ but not $N$-closed.

**Theorem 3.5.** Every $N.SCS$ is $N.\cdot g^S$.

Proof. Let $A_1^6$ be a $N.SCS$ in a $N.\cdot T.\cdot X$. Let $A_1^6 \subseteq U$, where $U$ is $N.\cdot g^S$-open set in $X$. Since $A_1^6$ is $N.SCS$, we have $N.Scl(A_1^6) = A_1^6 \subseteq U$. That is $N.Scl(A_1^6) \subseteq U$. Hence $A_1^6$ is $N.\cdot g^S$.

**Example 3.6.** Let $X = \{ a_1, a_2 \}$, $\tau_N = \{ 0_N, A_1^6, 1 \}$, is a $N.T.$ on $X$ where $A_1^6 = \langle x, \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \rangle$. Then the Neutrosophic set $A_2^6 = \langle x, \left( \frac{7}{10}, \frac{6}{10}, \frac{9}{10}, \frac{3}{10}, \frac{5}{10}, \frac{4}{10} \right) \rangle$ is a $N.\cdot g^S$-in $X$ but not $N.SCS$.

**Theorem 3.7.** Every $N.\cdot \alpha CS$ is $N.\cdot g^S$.

Proof. Let $A_1^6$ be a $N.\cdot \alpha CS$ in a $N.\cdot T.\cdot X$. Let $A_1^6 \subseteq U$, where $U$ is $N.\cdot g^S$-open set in $X$. Since $A_1^6$ is $N.\cdot \alpha CS$, we have $N.Scl(A_1^6) = A_1^6 \subseteq U$. That is $N.Scl(A_1^6) \subseteq U$. Hence $A_1^6$ is $N.\cdot g^S$.
Example 3.8. Let \( X = \{a_1, a_2\} \) and \( \tau_X = \{0_X, N_a, 1\} \), is a N.T on X where \( A_1 = \langle x, (\frac{4}{10}, \frac{3}{10}, \frac{5}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}) \rangle \). Then the Neutrosophic set \( A_2 = \langle x, (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle \) is a N.gSCS in X but not N.\alpha CS.

Theorem 3.9. Every N.gSCS is N.gSCS in N.T X.

Proof. Let \( A_1 \) be a N.gSCS in X. Let \( A_1 \subseteq U \), where U is N. open and so N.\alpha g open set. Since \( A_1 \) is N.gSCS, we have

\[
N.Scl(A_1) \subseteq N.gSCS \subseteq A_1 \subseteq U.
\]

That is N.Scl(A_1) \subseteq U and hence \( A_1 \) is N.gSCS.

Example 3.10. Let \( X = \{a_1, a_2\} \), \( \tau_X = \{0_X, N_a, 1\} \), is a N.T on X where \( A_1 = \langle x, (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle \). Then the Neutrosophic set \( A_2 = \langle x, (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle \) is a N.gSCS in X. But not N.\alpha gSCS.

Theorem 3.11. Every N.gSCS is N.\beta gCS in N.T X.

Proof. Let \( A_1 \) be a N.gSCS in X. Let \( A_1 \subseteq U \), where U is N. open set and so N.\alpha g open. Since \( A_1 \) is N.gSCS, we have N.Scl(A_1) \subseteq U. Therefore

\[
N.Scl(A_1) \subseteq N.\beta gCS \subseteq A_1 \subseteq U.
\]

That is N.\beta gCS(A_1) \subseteq U. And hence \( A_1 \) is N.\beta gCS.

Example 3.12. Let \( X = \{a_1, a_2, a_3\} \), \( \tau_X = \{0_X, N_a, 1_X\} \), is a N.T on X where

\[
A_1 = \langle x, (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle.
\]

Then the Neutrosophic set

\[
A_2 = \langle x, (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle
\]

is a N.\beta gCS in X. But not N.gSCS.

Remark 3.13. The following diagram shows the relationships of g^S semi closed sets with some other Neutrosophic sets.

Theorem 3.14. In a N.T X, if a Neutrosophic set \( A_1 \) is both N.\alpha gOS and N.\beta gSCS, then \( A_1 \) is N.\beta gSCS.

Proof. Suppose a Neutrosophic set \( A_1 \) of a N.T X is both N.\alpha gOS and N.\beta gSCS. Now \( A_1 \subseteq A_1 \), where \( A_1 \) is N. open set and so \( A_1 \) is N.\alpha gOS. This implies that N.Scl(A_1) \subseteq A_1, since \( A_1 \) is N.\beta gSCS. Also, we have \( A_1 \subseteq N.Scl(A_1) \), which implies N.Scl(A_1) = A_1. Hence \( A_1 \) is N.\beta gSCS.

Theorem 3.15. If a Neutrosophic set \( A_1 \) is a N.\gamma gSCS in X such that \( A_1 \subseteq A_2 \subseteq N.Scl(A_1) \), then \( A_2 \) is also a N.\gamma gSCS in X.

Proof. Let \( U \) be a N.\alpha gOS in X, such that \( A_2 \subseteq U \), then \( A_1 \subseteq U \). Since \( A_1 \) is N.\gamma gSCS, then by definitions N.Scl(A_1) \subseteq U. Now \( A_2 \subseteq N.Scl(A_1) \),

\[
N.Scl(A_2) \subseteq N.Scl(A_1) = N.Scl(A_1) \subseteq U.
\]

That is N.Scl(A_2) \subseteq U. Hence \( A_2 \) is a N.\gamma gSCS.

Theorem 3.16. A Finite union of N.\gamma gSCS is a N.\gamma gSCS.

Proof. Let \( A_1 \) and \( A_2 \) be N.\gamma gSCS in a N.T X. To prove that \( A_1 \cup A_2 \) is N.\gamma gSCS. Let \( A_1 \cup A_2 \subseteq U \), where U be N.\alpha gOS. Then \( A_1 \subseteq U \) and \( A_2 \subseteq U \), and so N.Scl(A_1) \subseteq U and N.Scl(A_2) \subseteq U since \( A_1 \) and \( A_2 \) are N.\gamma gSCS. This implies that

\[
N.Scl(A_1) \cup N.Scl(A_2) \subseteq U,
\]

and so N.Scl(A_1 \cup A_2) \subseteq U. Hence \( A_1 \cup A_2 \) is N.\gamma gSCS. Thus a finite union of N.\gamma gSCS is a N.\gamma gSCS. We introduce N.\gamma gSOS.

4. Neutrosophic g^S open sets

Definition 4.1. A Neutrosophic set \( A_1 \) of a N.T X is called N.\gamma gS open set (briefly N.\gamma gSOS) if its complement \( A_1^C \) is N.\gamma gSCS.

Theorem 4.2. A Neutrosophic set \( A_1 \) of a N.T X is N.\gamma gSOS iff \( \mathcal{F} \subseteq N.Sint(A_1^C) \), whenever \( \mathcal{F} \) is N.\alpha gCS and \( \mathcal{F} \subseteq A_1^C \).

Proof. Suppose \( A_1 \) is N.\gamma gSOS. Then \( A_1^C \) is N.\gamma gSCS. Let \( \mathcal{F} \) be N.\alpha gCS in X and \( \mathcal{F} \subseteq A_1 \). Then \( A_1 \subseteq \mathcal{F} \subseteq \mathcal{F} \subseteq A_1 \), \( \mathcal{F} \) is N.\alpha gOS. Since \( A_1 \) is N.\gamma gSCS, we have N.Scl(A_1^C) \subseteq \mathcal{F}, which implies \( \mathcal{F} \subseteq N.Sint(A_1) \). As N.Scl(A_1^C) = (N.Sint(A_1))^C.

Conversely, assume that \( \mathcal{F} \subseteq N.Sint(A_1) \), whenever \( \mathcal{F} \subseteq A_1^C \) and \( \mathcal{F} \) is N.\alpha gCS in a N.T X. Let \( A_1^C \subseteq G \), where \( G \) is N.\alpha gOS in X. Then \( G \subseteq A_1 \), where \( G \) is N.\alpha gCS, which implies that \( G \subseteq N.Sint(A_1) \), implies that \( (N.Sint(A_1)) \subseteq G \). That is N.Scl(A_1^C) \subseteq G. Hence \( A_1^C \) is N.\gamma gSCS and so \( A_1 \) is N.\gamma gSOS.

Theorem 4.3. Every N. open set is a N.\gamma gSOS.

Proof. Let \( A_1 \) be a N. open set in a N.T X. Then \( A_1^C \) is N. closed set. And so \( A_1^C \) is N.\gamma gSCS. Hence \( A_1^C \) is N.\gamma gSOS.

Example 4.4. Let \( X = \{a_1, a_2\} \), \( \tau_X = \{0_X, N_a, 1_N\} \), is a N.T on X where

\[
A_1 = \langle x, (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}) \rangle.
\]
Then the Neutrosophic set
\[ A_1^* = \left\{ x, \left( \frac{0}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{1}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\}, \]
the Neutrosophic set \( A_2^* \) is \( N.g\#SOS \) but not a \( N \) open set in \( X \).

**Theorem 4.5.** Every \( N.S \) open set is a \( N.g\#SOS \).

**Proof.** Let \( A_1^* \) be a \( N.S \) open set in a \( N.T.X \). Then \( A_1^* \) is \( N.SCS \). And so \( A_1^* \) is \( N.g\#SCS \). Hence \( A_1^* \) is \( N.g\#SOS \).

**Example 4.6.** Let \( X = \{ a_1, a_2, a_3 \}, \tau_N = \{ 0_N, A_1^*, 1_N \} \), is a \( N.T. \) on \( X \) where
\[ A_1^* = \left\{ x, \left( \frac{3}{10}, \frac{5}{10}, \frac{6}{10} \right), \left( \frac{5}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{8}{10}, \frac{5}{10}, \frac{2}{10} \right) \right\}. \]
Then the Neutrosophic set
\[ A_2^* = \left\{ x, \left( \frac{0}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{1}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\} \]
\( N.g\#SOS \). But not a \( N.SOS \) in \( X \).

**Theorem 4.7.** Every \( N.aOS \) is a \( N.g\#SOS \).

**Proof.** Let \( A_1^* \) be a \( N.aOS \) in a \( N.T.X \). Then \( A_1^* \) is \( N.aCS \). And so \( A_1^* \) is \( N.g\#SCS \). Hence \( A_1^* \) is \( N.g\#SOS \).

**Example 4.8.** Let \( X = \{ a_1, a_2, a_3 \}, \tau_N = \{ 0_N, A_1^*, 1_N \} \), is a \( N.T. \) on \( X \) where
\[ A_1^* = \left\{ x, \left( \frac{4}{10}, \frac{5}{10}, \frac{6}{10} \right), \left( \frac{5}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{7}{10}, \frac{5}{10}, \frac{3}{10} \right) \right\}. \]
Then the Neutrosophic set
\[ A_2^* = \left\{ x, \left( \frac{0}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{1}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right) \right\} \]
is \( N.g\#SOS \) but not a \( N.aOS \) in \( X \).

**Theorem 4.9.** In a \( N.T.X \), every \( N.g\#SOS \) is \( N.g\#SOS \).

**Proof.** Let \( A_1^* \) be \( N.g\#SOS \) in a \( N.T.X \). Then \( A_1^* \) is \( N.g\#SCS \) in \( X \). And so \( A_1^* \) is \( N.g\#SCS \). That is \( A_1^* \) is \( N.g\#SOS \) in \( X \).

**Example 4.10.** Let \( X = \{ a_1, a_2 \}, \tau_N = \{ 0_N, A_1^*, 1_N \} \), is a \( N.T. \) on \( X \) where
\[ A_1^* = \left\{ x, \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \right\}. \]
Then the Neutrosophic set
\[ A_2^* = \left\{ x, \left( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right), \left( \frac{4}{10}, \frac{5}{10}, \frac{6}{10} \right) \right\} \]
is a \( N.gSOS \) in \( X \). But not \( N.g\#SOS \).

**Theorem 4.11.** In a \( N.T.X \), every \( N.g\#SOS \) is \( N.g\#SOS \).

**Proof.** Let \( A_1^* \) be \( N.g\#SOS \) in a \( N.T.X \). Then \( A_1^* \) is \( N.g\#SCS \) in \( X \). And so \( A_1^* \) is \( N.g\#SCS \). That is \( A_1^* \) is \( N.g\#SOS \).

**Example 4.12.** Let \( X = \{ a_1, a_2, a_3 \}, \tau_N = \{ 0_N, A_1^*, 1_N \} \), is a \( N.T. \) on \( X \) where
\[ A_1^* = \left\{ x, \left( \frac{2}{10}, \frac{5}{10}, \frac{7}{10} \right), \left( \frac{5}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \right\}. \]
Then the Neutrosophic set
\[ A_2^* = \left\{ x, \left( \frac{5}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{6}{10} \right) \right\} \]
is \( N.g\#SOS \) but not \( N.g\#SOS \).

**Theorem 4.13.** If \( N.Sint(A_1^*) \subseteq A_2 \subseteq A_1^* \) and if \( A_1^* \) is \( N.g\#SOS \), then \( A_1^* \) is \( N.g\#SOS \) in a \( N.T.X \).

**Proof.** We have \( N.Sint(A_1^*) \subseteq A_2 \subseteq A_1^* \). Then \( A_1^* \subseteq A_2 \subseteq N.Scl(A_1^*) \) and since \( (A_1^*) \) is \( N.g\#SCS \) and then, we have \( A_2 \) is \( N.g\#SCS \) in \( X \). And hence \( A_2 \) is \( N.g\#SOS \) in a \( N.T.X \).

**Theorem 4.14.** A Neutrosophic set \( A_1^* \) is \( N.g\#SCS \) and \( N.Scl(A_1^*) \cap (N.Scl(A_1^*))^c = \emptyset \), then \( N.Scl(A_1^*) \cap (A_1^*)^c \) is \( N.g\#SOS \).

**Proof.** Let \( A_1^* \) be \( N.g\#SCS \) in a \( N.T.X \). Let \( \mathcal{F} \subseteq N.Scl(A_1^*) \cap (A_1^*)^c \). Then \( \mathcal{F} \) is zero and so \( \mathcal{F} \subseteq N.Sint(N.Scl(A_1^*) \cap (A_1^*)^c) \). Then \( N.Scl(A_1^*) \cap (A_1^*)^c \) is \( N.g\#SOS \) in \( N.T.X \).

**Theorem 4.15.** Finite intersection of \( N.g\#SOS \) is \( N.g\#SOS \).

**Proof.** Let \( A_1^* \) and \( A_2^* \) be \( N.g\#SOS \) sets in a \( N.T.X \). To prove that \( A_1^* \cap A_2^* \) is \( N.g\#SOS \). Let \( \mathcal{F} \subseteq A_1^* \cap A_2^* \) where \( \mathcal{F} \) be \( N.gCS \). Then \( \mathcal{F} \subseteq A_1^* \subseteq A_1 \). Then \( \mathcal{F} \subseteq N.Sint(A_1^*) \). That is \( \mathcal{F} \subseteq N.Sint(A_1^*) \cap N.Scl(A_2^*) \). Hence \( A_1^* \cap A_2^* \) is \( N.g\#SOS \) Thus finite intersection of \( N.g\#SOS \) is \( N.g\#SOS \).

**Definition 4.16.** For any Neutrosophic set \( a \) in any \( N.T.X \), \( N.g\#Scl(A_1^*) = \cap U : U \) is \( N.g\#SCS \) and \( A_1^* \subseteq U \).

**Theorem 4.17.** In a \( N.T.X \), a Neutrosophic set \( A_1^* \) is \( N.g\#SOS \) closed iff \( A_1^* = N.g\#Scl(A_1^*) \).
Theorem 4.23. In a Neutrosophic g*S space, let A be a Neutrosophic g*S set. Then A is a Neutrosophic g*S set if and only if A is a Neutrosophic g*S set.

Proof. Let A = N g*SSc(A_i). Then A = N g*SSc(A_i). Hence A = N g*SSc(A_i).

Conversely, suppose that A_i = N g*SSc(A_i), that is A_i = A_i \cap \{ F : F is a Neutrosophic g*S set and A_i \subseteq F \}. This implies that A_i = A_i \cap \{ F : F is a Neutrosophic g*S set and A_i \subseteq F \}. Hence A_i = N g*SSc(A_i).

Theorem 4.24. In a Neutrosophic g*S space, let A be a Neutrosophic g*S set. Then A is a Neutrosophic g*S set if and only if A is a Neutrosophic g*S set.

Proof. Let X be a Neutrosophic g*S space. Let F be a Neutrosophic g*S set in X. Then F is a Neutrosophic g*S set in X. Since X is a Neutrosophic g*S space, F is a Neutrosophic g*S space.

Definition 4.25. In a Neutrosophic g*S space, let A be a Neutrosophic g*S set. Then A is a Neutrosophic g*S set if and only if A is a Neutrosophic g*S set.

Theorem 4.25. Every Neutrosophic g*S space is a Neutrosophic g*S space.

Proof. Suppose X is a Neutrosophic g*S space. Let F be a Neutrosophic g*S set in X. Then F is a Neutrosophic g*S set in X. Since X is a Neutrosophic g*S space, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.

Conversely, assume that every Neutrosophic g*S set in X is a Neutrosophic g*S set. Let F be a Neutrosophic g*S set in X, then F is a Neutrosophic g*S set in X. By hypothesis, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.

Theorem 4.26. Every Neutrosophic g*S space is a Neutrosophic g*S space.

Proof. Suppose X is a Neutrosophic g*S space. Let F be a Neutrosophic g*S set in X. Then F is a Neutrosophic g*S set in X. Since X is a Neutrosophic g*S space, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.

Conversely, assume that every Neutrosophic g*S set in X is a Neutrosophic g*S set. Let F be a Neutrosophic g*S set in X, then F is a Neutrosophic g*S set in X. By hypothesis, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.

Theorem 4.27. Every Neutrosophic g*S space is a Neutrosophic g*S space.

Proof. Suppose X is a Neutrosophic g*S space. Let F be a Neutrosophic g*S set in X. Then F is a Neutrosophic g*S set in X. Since X is a Neutrosophic g*S space, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.

Conversely, assume that every Neutrosophic g*S set in X is a Neutrosophic g*S set. Let F be a Neutrosophic g*S set in X, then F is a Neutrosophic g*S set in X. By hypothesis, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.

Theorem 4.28. Every Neutrosophic g*S space is a Neutrosophic g*S space.

Proof. Suppose X is a Neutrosophic g*S space. Let F be a Neutrosophic g*S set in X. Then F is a Neutrosophic g*S set in X. Since X is a Neutrosophic g*S space, F is a Neutrosophic g*S set in X. Therefore F is a Neutrosophic g*S set in X.
Neutrosophic g$S$ closed sets in neutrosophic topological spaces — 1791


T. Rajesh kannan, S. Chandrasekar, Neutrosophic Pre-α, Semi-α & Pre-β Irresolute Functions(Communicated).

A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic topological spaces , Journal computer Sci. Engineering,