**β-Baire space in fuzzy soft topological spaces**

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**Abstract**

In this paper a property which can be used to measure and category named fuzzy soft β-Baire spaces in fuzzy soft topological spaces is investigated. For this purpose fuzzy soft β-dense, nowhere fuzzy soft β-dense and fuzzy soft β-first category sets are defined and some results about these concepts are also obtained.

**Keywords**

Fuzzy soft β-dense set, fuzzy soft nowhere β-dense sets, fuzzy soft β-Baire space, fuzzy soft β-first category.

**AMS Subject Classification**

54A40, 03E72.

**1. Introduction**

Zadeh[8] introduced the concepts of fuzzy sets. Soft sets theory was introduced by Molodtsov [4]. The notion of fuzzy soft set is investigated and discussed in [5]. Chang[2] introduced and developed the concepts of fuzzy topology. In recent years, B. Tanay and M. B. Kandemir [6] much attention has been used to generalize the basic notions of fuzzy topology in soft setting. Further, the concept of fuzzy soft β-open set is introduced by A. M. Abd El-latif[1]. The notion of baire space in fuzzy topology introduced and discussed G.Thangaraj etal [7] In this paper we define new concepts called fuzzy soft β-dense sets, fuzzy soft nowhere β-dense sets. Also we define fuzzy soft β-first category sets and fuzzy soft β-Baire space. We obtain a characterization and investigated some properties of fuzzy soft β-Baire space.

**2. Preliminaries**

Throughout the present paper, X, Y, P and Q denote the fuzzy soft topological spaces. Let $f_A$ be a fuzzy soft set of X.

The closure (resp. the interior) of $f_A$ is denoted by $(f_A)^\beta$ (resp. $(f_A)_{\beta}$). A fuzzy soft set $f_A$ is defined to be fuzzy soft β-open [1] if $f_A \subseteq ((f_A)^\beta)^\beta$. The complement of a fuzzy soft β-open set is called fuzzy soft β-closed. The intersection of all fuzzy soft β-closed sets containing $f_A$ is called the fuzzy soft β-closure of $f_A$ and is denoted by $f_s(f_A)^\beta$. The fuzzy soft β-interior of $f_A$ is defined by the union of all fuzzy soft β-open sets contained in $f_A$ and is denoted by $f_s(f_A)^\beta$.

**Definition 2.1.** [3] A fuzzy soft set $f_A$ in a fsts $(X, E, \tau)$ is called fuzzy soft dense if there exists no fuzzy soft closed sets $g_B$ in $(X, E, \tau)$ such that $F_A \subseteq G_B \subseteq E$.

**Definition 2.2.** [3] A fuzzy soft set $f_A$ in a fsts $(X, E, \tau)$ is called fuzzy soft nowhere dense if there exists no nonempty fuzzy soft open sets $g_B$ in $(X, E, \tau)$ such that $G_B \subseteq f_scl(F_A)$. That is, $f_sint(f_scl(F_A)) = \emptyset$.

**Definition 2.3.** [3] Let $f_A$ be a fuzzy soft set of X. $f_A$ is defined to be of fuzzy soft first category if it can be represented as a countable union of fuzzy soft nowhere dense sets i.e) $\bigcup_{i=1}^{\infty} f_A = f_A$, where $f_A_i$’s are $f_A$ nowhere dense set X. Otherwise fuzzy soft open $f_A$ in X is said to be fuzzy soft second category. If $f_A$ is fuzzy soft first category in X then $f_A^\beta$ is called fuzzy soft residual set in X.

**Definition 2.4.** [3] A fsts $(X, E, \tau)$ is called fuzzy soft Baire space if $f_sint(\bigcup_{i=1}^{\infty} f_A) = \emptyset$ where $f_A_i$’s are $f_A$ nowhere dense sets in $(X, E, \tau)$.
3. Fuzzy soft $\beta$-dense and Fuzzy soft $\beta$-nowhere dense

In this section, we define fuzzy soft $\beta$-dense and fuzzy soft $\beta$-nowhere dense and discuss with some properties.

**Definition 3.1.** A fuzzy soft set $f_A$ in fuzzy soft topological space $(X, \tau)$ is called fuzzy soft $\beta$-dense(fuzzy soft $\beta$-blank) if there exists no fuzzy soft $\beta$-closed set $f_B$ in $(X, \tau)$ such that $f_A \subseteq f_B \subseteq E$, i.e. $fs(f_A)_\beta = 1_E$.

**Remark 3.2.** Every fuzzy soft dense set is fuzzy soft $\beta$-dense set but the converse is not true in general.

**Example 3.3.** Let $X = \{a, b\}, E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq B = \{e_1, e_3\}$ and $C = \{e_2, e_3\}$ and let fuzzy soft sets $f_A = \{(f(e_1) = (a_0, 0), f(e_3) = (a_0, 0))\}$ and $f_B$ = \{(f(e_1) = (a_0, 0), f(e_2) = (a_0, 0), f(e_3) = (a_0, 0))\} be a fuzzy soft open sets. Then $fs(f_A)_\beta = 1_E$. Therefore it is a fuzzy soft dense set in $(X, \tau)$ but it is not a fuzzy soft dense set.

**Definition 3.4.** A fuzzy soft set $f_A$ in fuzzy soft topological space $(X, \tau)$ is called fuzzy soft $\beta$-nowhere dense if there exists no fuzzy soft $\beta$-open set in $(X, \tau)$ such that $f_B \subseteq f_A \subseteq E$, i.e. $fs(f_A)_\beta = 0_E$.

**Remark 3.5.** Every fuzzy soft dense set is a fuzzy soft $\beta$-nowhere dense set. The converse is not true in general as shown in example.

**Example 3.6.** Let $X = \{a, b, c\}, E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq B = \{e_1, e_3\}$ and $C = \{e_2, e_3\}$ and let fuzzy soft sets $f_A = \{(f(e_1) = (a_0, 0), f(e_2) = (a_0, 0), f(e_3) = (a_0, 0))\}$ and $f_B$ = \{(f(e_1) = (a_0, 0), f(e_2) = (a_0, 0), f(e_3) = (a_0, 0))\}$ be a fuzzy soft open sets. Then $fs(f_A)_\beta = 1_E$. Therefore it is a fuzzy soft dense set in $(X, \tau)$ but it is not a fuzzy soft nowhere dense set.

**Lemma 3.7.** A fuzzy soft set $f_A$ in $(X, \tau)$ is fuzzy soft $\beta$-nowhere dense set if and only if $fs(f_A)_\beta$ has no interior points.

**Proof:** Let $f_B$ be a fuzzy soft $\beta$-nowhere dense set. Then there exists a fuzzy soft $\beta$-open set and fuzzy soft dense set $f_B$ such that $f_B \subseteq 1_E - f_A$. Since $f_B$ is fuzzy soft $\beta$-open set $fs(f_B)_\beta = fs(1_E - f_A)_\beta = 1_E - fs(f_A)_\beta$. Then $fs(f_B)_\beta = 1_E - fs(f_A)_\beta$. Hence $fs(f_A)_\beta = 0_E$.
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Proof: (i). Now $f_A \subseteq f_B$ which implies $f_S(f_A)_\beta \subseteq f_S(f_B)_\beta$ then $f_S(f_S(f_A)_\beta) \subseteq f_S(f_S(f_B)_\beta)$. Since $f_{\text{fs}}$ is $\beta$-nowhere dense in $(X, E, \tau)$, then $f_S(f_S(f_A)_\beta) \subseteq 0$. Hence $f_A$ is $\text{fs}f_\beta$-nowhere dense set in $(X, E, \tau)$. (ii) and (iii) proof is similar to that of (i).

Definition 3.14. Let $f_A$ be a fuzzy soft set of $X$. $f_A$ is defined to be of fuzzy soft $\beta$-first category $(\text{fs}f_\beta$-first category) if it can be represented as a countable union of $\text{fs}f_\beta$-nowhere dense sets. i.e $f_S(\cup_{i=0}^{\infty} f_A)_\beta = f_A$ where $f_A$'s are $\text{fs}f_\beta$-nowhere dense set $X$. Otherwise fuzzy soft $\beta$-open $f_A$ in $X$ is said to be fuzzy soft $\beta$-second category. If $f_A$ is fuzzy soft $\beta$-first category in $X$ then $f_A^*$ is called fuzzy soft $\beta$-residual set in $X$.

Remark 3.15. Every fuzzy soft first category set is of $\text{fs}f_\beta$-first category. But the converse is not true in general as seen in example.

Example 3.16. Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$ $B = \{e_1, e_2\}$, $C = \{e_2\}$ and $D = \{e_2, e_3\}$ and let fuzzy soft sets $f_A = \{f(e_1) = \{a_0.9, b_0.5, c_0.1\}, f(e_2) = \{\{a_0.9, b_0.6, c_0.3\}, f(e_3) = \{a_0.9, b_0.6, c_0.3\}\}$. Consider the fuzzy soft topology $\tau = \{0E, 1E, f_A, f_B, f_B \cup f_A \cap f_B, f_B \cap f_A \cup f_B \cup f_B, (f_B \cup f_B)\}$ defined over $(X, \tau)$. Now let us consider $f_E = \{f(e_1) = \{a_0.9, b_0.5, c_0.1\}, f(e_2) = \{a_0.9, b_0.6, c_0.3\}, f(e_3) = \{a_0.9, b_0.6, c_0.3\}\}$. By hypothesis, $f_S(f_E)_\beta = 0E$ which implies that $(f_S(f_E))_\beta = 0E$. Hence $f_S(f_E)_\beta = 0E$.

4. Fuzzy soft $\beta$-Baire space

In this section, we define fuzzy soft $\beta$-Baire space and we discuss with some characterization of this space.

Definition 4.1. A fuzzy soft topological space $(X, E, \tau)$ is called fuzzy soft $\beta$-Baire space $(\text{fs}f_\beta$-Baire space) if $f_S(\cup_{i=0}^{\infty} f_A)_\beta = 0E$ where $f_A$'s are $\text{fs}f_\beta$-nowhere dense sets in $(X, E, \tau)$.

Example 4.2. Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$ $B = \{e_1, e_3\}$, $C = \{e_2\}$ and $D = \{e_2, e_3\}$ and let fuzzy soft sets $f_A = \{f(e_1) = \{a_0.9, b_0.5, c_0.1\}, f(e_2) = \{a_0.9, b_0.6, c_0.3\}, f(e_3) = \{a_0.9, b_0.6, c_0.3\}\}$. Consider the fuzzy soft topology $\tau = \{0E, 1E, f_A, f_B, f_B \cup f_A \cap f_B, f_B \cap f_A \cup f_B \cup f_B, (f_B \cup f_B)\}$ defined over $(X, \tau)$. Now let us consider $f_E = \{f(e_1) = \{a_0.9, b_0.5, c_0.1\}, f(e_2) = \{a_0.9, b_0.6, c_0.3\}, f(e_3) = \{a_0.9, b_0.6, c_0.3\}\}$. Then $f_E$ is a $\text{fs}f_\beta$-first category set in $(X, E, \tau)$ and $f_E$ is $\text{fs}f_\beta$-residual set in $(X, E, \tau)$ but it is not a fuzzy soft category set.
Remark 4.8. Every $f\beta$-$\text{Baire}$ space is a $f\beta$-$\text{second category space}$ but the converse not necessarily true. From example 3.16, the $f\beta$-$\text{nowhere dense sets}$ $fs((fA \cup fB \cup fC)^c \cup fD)^c) \not\subseteq 1_E$ which is a $f\beta$-$\text{second category space}$ but it is not $f\beta$-$\text{Baire}$ space.

Proposition 4.9. If $fs((\bigcap_{i=1}^{n} fA_i)^c) = 1_E$ where $fA_i$’s are $f\beta$-$\text{dense}$ and $f\beta$-$\text{open sets}$ in $fs((X, E, \tau)$ iff $(X, E, \tau)$ is $f\beta$-$\text{Baire}$ space.

Proof: Let $fA_i$’s are $f\beta$-$\text{dense}$ sets in $(X, E, \tau$). Then $fs((\bigcap_{i=1}^{n} fA_i)^c) = 1_E$ Which implies $(fs((\bigcap_{i=1}^{n} fA_i)^c)^c = 0_E$. That is $fs((\bigcap_{i=1}^{n} fA_i)^c) = 0_E$. Hence $fs((\bigcap_{i=1}^{n} fA_i)^c)^c = 0_E$. Consequently $fs((\bigcap_{i=1}^{n} fA_i)^c) = 0_E$, where $(fs((\bigcap_{i=1}^{n} fA_i)^c)^c = 0_E$ and $fA_i$’s are $f\beta$-$\text{closed sets}$ in $(X, E, \tau$). By Proposition 4.3, $(X, E, \tau)$ is $f\beta$-$\text{Baire}$ space.

Conversely, Let $fA_i$’s are $f\beta$-$\text{dense}$ sets and $f\beta$-$\text{open sets}$ in $(X, E, \tau$). By proportion 3.11, $fA_i$’s are $f\beta$-$\text{nowhere}$ dense sets in $X$. Then $fA = \bigcup_{i=1}^{n} fA_i^c$ is a $f\beta$-$\text{first category}$ set in $(X, E, \tau$). Now $fs((\bigcap_{i=1}^{n} fA_i)^c)^c = fs((\bigcap_{i=1}^{n} fA_i)^c) = (fs((\bigcap_{i=1}^{n} fA_i)^c)^c)^c$. Since $(X, E, \tau)$ is $f\beta$-$\text{Baire}$ space, by theorem 4.4, we get $fs((\bigcap_{i=1}^{n} fA_i)^c)^c = 0_E$. Then $(fs((\bigcap_{i=1}^{n} fA_i)^c)^c = 0_E$. This implies that $(fs((\bigcap_{i=1}^{n} fA_i)^c)^c = 1_E$.

Definition 4.10. A surjective function $\varphi : (X, E, \tau) \rightarrow (X, E, \sigma)$ is defined to be
(i) $f\beta$-$\text{slightly continuous}$ if $fs(\varphi^{-1}(fA))^c) \neq 0_E$ whenever $fs(fA)^c) = 0_E$ for a fs set $fA$ of $\sigma$.
(ii) $f\beta$-$\text{slightly open}$ if $fs(\varphi^{-1}(fB))^c) \neq 0_E$ whenever $fs(fB)^c) = 0_E$ for a fs set $fB$ of $\tau$.

Theorem 4.11. Let $\varphi : (P, E_1, \tau) \rightarrow (Q, E_2, \sigma)$ be a surjective function. The following statements hold:
(i) If $\varphi$ is $f\beta$-$\text{slightly continuous}$ and $fA$ is $f\beta$-$\text{dense}$ in $P$, then $f\beta(\varphi(fA))$ in $Q$.
(ii) If $\varphi$ is $f\beta$-$\text{slightly open}$ and $fB$ is $f\beta$-$\text{dense}$ in $Q$, then $\varphi^{-1}(fB)$ is $f\beta$-$\text{dense}$ in $P$.

Proof: (i). Let $\varphi$ be a $f\beta$-$\text{slightly continuous}$ function and $fA$ be a $f\beta$-$\text{dense}$ set in $P$. Suppose that $\varphi(fA)$ is not $f\beta$-$\text{dense}$. Then $1_{E_1} \neq fs(\varphi(fA))^c) \neq 0_{E_1}$ and $0_{E_1} \neq 1_{E_2} = fs(\varphi(fA))^c$.

Let $fc = 1_{E_2} - fs(\varphi(fA))^c$. Then $fc$ is a $f\beta$-$\text{open set}$.

Since $\varphi$ is $f\beta$-$\text{slightly continuous}$, $fs(\varphi^{-1}(fc))^c) = 0_{E_2}$.

Also $fs(\varphi^{-1}(fc))^c \subseteq fs(fA) \subseteq \varphi^{-1}(fc) \cap \varphi^{-1}(\varphi(fA)) = \varphi^{-1}(fc) \cap \varphi^{-1}(\varphi(fA)) = 0_{E_2}$. This is a contradiction since $fA$ is $f\beta$-$\text{dense}$. Hence $\varphi(fA)$ is $f\beta$-$\text{dense}$.

(ii). Let $fB$ be $f\beta$-$\text{slightly open}$ and $fB$ be $f\beta$-$\text{dense}$ in $Q$. Suppose that $\varphi^{-1}(fB)$ is not $f\beta$-$\text{dense}$ in $P$. Then there exists a nonempty $f\beta$-$\text{open set}$ $fD$ of $P$ such that $fD \cap \varphi^{-1}(fB) = 0_{E_1}$. Since $f\varphi$ is $f\beta$-$\text{slightly open}$ open $fs(\varphi(fD))^c) \neq 0_{E_2}$. Moreover, we have $fs(\varphi(fD))^c \subseteq fD \subseteq \varphi(fD) \subseteq 0_{E_2}$. This is a contradiction since $fB$ is $f\beta$-$\text{dense}$. Hence $\varphi^{-1}(fB)$ is $f\beta$-$\text{dense}$.

Theorem 4.12. Let $\varphi : P \rightarrow Q$ be a $f\beta$-$\text{slightly continuous}$ and $f\beta$-$\text{slightly open surjection}$. If $P$ is a $f\beta$-$\text{Baire space}$, then $Q$ is a $f\beta$-$\text{Baire space}$.

Proof: Let $P$ be a $f\beta$-$\text{Baire space}$ and $fB_i \subseteq Q$ be a $f\beta$-$\text{dense}$ set for each $i \in I$ where $I$ is the set of natural numbers. Since $\varphi$ is $f\beta$-$\text{slightly open}$ $\varphi^{-1}(fB_i)$ is $f\beta$-$\text{dense}$ in $P$. Since $P$ is a $f\beta$-$\text{Baire space}$, $\cap_{i \in I} \varphi^{-1}(fB_i)$ is $f\beta$-$\text{dense}$ in $P$.

By theorem 4.11, $\varphi$ is $f\beta$-$\text{slightly continuity}$, $\varphi(\cap_{i \in I} \varphi^{-1}(fB_i)) = \cap_{i \in I} fB_i$ is $f\beta$-$\text{dense}$ in $Q$. This shows that $Q$ is a $f\beta$-$\text{Baire space}$.

5. Conclusion

Thus in this paper the concepts of fuzzy soft $\beta$-$\text{dense}$ and fuzzy soft $\beta$-$\text{nowhere}$ dense were introduced. Also the concepts of fuzzy soft $\beta$-$\text{Baire}$ space were being introduced and discussed. Some characterizations of these spaces and some basic interesting properties of such fuzzy baires space were obtained.

References


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