Spherical fuzzy graph
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**Abstract**
Classification of degrees and their properties are deliberated in the spherical fuzzy graph. Also the spherical fuzzy graph are interrogated using the order, size, completeness and their regularity.

**Keywords**
Effective degree, neighborhood degree, spherical complete fuzzy graph, regular spherical fuzzy graph.

**AMS Subject Classification**
03E72, 03F55.

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## 1. Introduction

In numerous reasonable circumstances such as operation management, networking and economical interpretation, the graph-theoretical portrayals of the data have been discovered to be more powerful and advantageous to manage the data embedded among various articles, characteristics, choices. The cerebration of fuzzy graphs put forward by Kaufmann [2] grounded by the fuzzy relation [3]. The conceptual idea of fuzzy vertex and fuzzy edge progressed by Rosenfeld [4]. An intuitionistic fuzzy graph evolved from fuzzy graph, studied by Parvathi et. al., [5]. Pythagorean fuzzy graphs generalized out of intuitionistic fuzzy graph [6] and the spherical fuzzy graph acquainted by Akram et. al., [1]. In this paper, spherical fuzzy subgraph, complete spherical fuzzy graph, minimum and maximum degrees of spherical fuzzy graphs are demonstrated. The effective degree, neighborhood degree, closed neighborhood degree and their minimum and maximum neighborhood degrees are also defined in spherical fuzzy graph. The spherical regular fuzzy graph, the order and their size of spherical fuzzy graph are elucidated with their properties.

## 2. Preliminaries

**Definition 2.1.** \(^1\) Let \(\mathbb{V}\) be the non-empty set possess spherical fuzzy graph \(\mathbb{SFG}\) is \(\mathbb{G} = (\mathbb{P}, \mathbb{Q})\), here \(\mathbb{P}\) and \(\mathbb{Q}\) are spherical fuzzy set and spherical fuzzy relation on \(\mathbb{V}\) in such

\[
\alpha_{\mathbb{G}}(p,q) \leq \min\{\alpha_p(p), \alpha_q(q)\}; \\
\gamma_{\mathbb{G}}(p,q) \leq \min\{\gamma_p(p), \gamma_q(q)\}; \\
\beta_{\mathbb{G}}(p,q) \leq \max\{\beta_p(p), \beta_q(q)\}.
\]

and \(0 \leq \alpha_{\mathbb{G}}(p,q) + \gamma_{\mathbb{G}}(p,q) + \beta_{\mathbb{G}}(p,q) \leq 1 \ \forall p, q \in \mathbb{V}\). The spherical fuzzy vertex set \(\mathbb{P}\) is of \(\mathbb{G}\) and the spherical edge set \(\mathbb{Q}\) is of \(\mathbb{G}\). The insubstantial edge is \(\mathbb{Q}(p,q) = 0 \ \forall (p,q) \in \mathbb{V} \times \mathbb{V} - \mathbb{E}\). The spherical fuzzy digraph has no symmetry relation.

**Example 2.2.** \(^1\) Let the vertex set \(\mathbb{V} = \{p, q, r, s\}\) and the edge set \(\mathbb{E} = \{pq, qr, rs, ps\}\) in \(\mathbb{G}^* = (\mathbb{V}, \mathbb{E})\). Take the spherical fuzzy set \(\mathbb{P} = (\alpha_p, \gamma_p, \beta_p)\) in \(\mathbb{V}\) and the spherical fuzzy edge set in \(\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}\) defined by

\[
(\alpha_p, \gamma_p, \beta_p) = (0.6, 0.5, 0.3), \\
(\alpha_q, \gamma_q, \beta_q) = (0.7, 0.3, 0.6), \\
(\alpha_r, \gamma_r, \beta_r) = (0.3, 0.8, 0.4), \\
(\alpha_s, \gamma_s, \beta_s) = (0.6, 0.4, 0.5)
\]
A spherical fuzzy graph is complete if
\[ d_\delta(p) = (d_{\alpha}(p), d_{\gamma}(p), d_{\beta}(p)) \]
is elucidated as
\[ (SFG, \delta) = (SFG, \alpha, \gamma, \beta) \]
defined by
\[ \Delta(G) = \min \{d_\delta(p)\} \]
where,
\[ \Delta_\alpha(G) = \max \{d(\alpha)(p)\} \]
\[ \Delta_\gamma(G) = \max \{d(\gamma)(p)\} \]
\[ \Delta_\beta(G) = \max \{d(\beta)(p)\} \].

Then, it is a spherical fuzzy graph.

Definition 3.2. A spherical fuzzy graph is complete if
\[ \alpha(p, q) = \min \{\alpha(p), \alpha(q)\}, \]
\[ \gamma(p, q) = \min \{\gamma(p), \gamma(q)\}, \]
\[ \beta(p, q) = \max \{\beta(p), \beta(q)\}. \]

Example 3.3. Let the vertex set \( \mathcal{V} = \{p, q, r\} \) and the edge set \( \mathcal{E} = \{pq, qr, pr\} \) in \( G^* = (\mathcal{V}, \mathcal{E}) \). Take the spherical fuzzy set \( P = (\alpha_p, \gamma_p, \beta_p) \) in \( \mathcal{V} \) and the spherical fuzzy edge set in \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \) defined by
\[ (\alpha(p, q), \gamma(p, q), \beta(p, q)) = (0.8, 0.5, 0.6), \]
\[ (\alpha(p, q), \gamma(p, q), \beta(p, q)) = (0.2, 0.3, 0.7), \]
\[ (\alpha(p, q), \gamma(p, q), \beta(p, q)) = (0.5, 0.6, 0.8) \]
and
\[ (\alpha(p, q), \gamma(p, q), \beta(p, q)) = (0.2, 0.3, 0.7), \]
\[ (\alpha(p, q), \gamma(p, q), \beta(p, q)) = (0.2, 0.3, 0.8), \]
\[ (\alpha(p, q), \gamma(p, q), \beta(p, q)) = (0.5, 0.5, 0.8) \].

Then, it is a complete SFG.

Definition 3.4. The minimum degree of SFG, \( G = (P, Q) \) is designated \( \delta(G) = (\delta_\alpha(G), \delta_\gamma(G), \delta_\beta(G)) \) where,
\[ \delta_\alpha(G) = \min \{d_\alpha(p)\} \]
\[ \delta_\gamma(G) = \min \{d_\gamma(p)\} \]
\[ \delta_\beta(G) = \min \{d_\beta(p)\} \].
Definition 3.12. The effective degree of a vertex $p$ of SFG, $G = (P, Q)$ is elucidated by $d_{\delta}(p) = (d_{\delta_\alpha}(p), d_{\delta_\gamma}(p), d_{\delta_\beta}(p))$ where, $d_{\delta_\alpha}(p) = \min\{d_{\delta_\alpha}(p) | p \in P\}$, $d_{\delta_\gamma}(p) = \min\{d_{\delta_\gamma}(p) | p \in P\}$, $d_{\delta_\beta}(p) = \min\{d_{\delta_\beta}(p) | p \in P\}$.

Definition 3.13. The minimum effective degree of $G = (P, Q)$ in a SFG is elucidated by $\delta_{\delta}(G) = (\delta_{\delta_\alpha}(G), \delta_{\delta_\gamma}(G), \delta_{\delta_\beta}(G))$ where, $\delta_{\delta_\alpha}(G) = \max\{d_{\delta_\alpha}(p) | p \in P\}$, $\delta_{\delta_\gamma}(G) = \max\{d_{\delta_\gamma}(p) | p \in P\}$, $\delta_{\delta_\beta}(G) = \max\{d_{\delta_\beta}(p) | p \in P\}$.

Definition 3.14. The maximum effective degree of $G = (P, Q)$ in a SFG is elucidated by $\Delta_{\delta}(G) = (\Delta_{\delta_\alpha}(G), \Delta_{\delta_\gamma}(G), \Delta_{\delta_\beta}(G))$ where, $\Delta_{\delta_\alpha}(G) = \max\{d_{\delta_\alpha}(p) | p \in P\}$, $\Delta_{\delta_\gamma}(G) = \max\{d_{\delta_\gamma}(p) | p \in P\}$, $\Delta_{\delta_\beta}(G) = \max\{d_{\delta_\beta}(p) | p \in P\}$.

Example 3.15. In Example (3.6),

\[
\begin{align*}
d_{\delta}(p) &= (0, 0, 0) \\
d_{\delta}(q) &= (0, 0, 0) \\
d_{\delta}(r) &= (0.6, 0.7, 0.4) \\
d_{\delta}(s) &= (0.6, 0.7, 0.4) \\
\delta_{\delta}(G) &= (0, 0, 0) \\
\Delta_{\delta}(G) &= (0.6, 0.7, 0.4)
\end{align*}
\]

Definition 3.16. The neighborhood of any vertex $p$ in $G = (P, Q)$ of a SFG is elucidated as $N(p) = (N_{\alpha}(p), N_{\gamma}(p), N_{\beta}(p))$ where,

\[
\begin{align*}
N_{\alpha}(p) &= \{ q \in P : \alpha_{\delta}(p, q) = \alpha_{\delta}(p) \land \alpha_{\delta}(p) \} \\
N_{\gamma}(p) &= \{ q \in P : \gamma_{\delta}(p, q) = \gamma_{\delta}(p) \land \gamma_{\delta}(p) \} \\
N_{\beta}(p) &= \{ q \in P : \beta_{\delta}(p, q) = \beta_{\delta}(p) \land \beta_{\delta}(p) \}
\end{align*}
\]

and $N[p] = N(p) \cup p$ is called the closed neighborhood of $p$.

Definition 3.17. The neighborhood degree of a vertex in $G = (P, Q)$ of a SFG is elucidated as

\[
\delta_{s}(p) = (\delta_{s_\alpha}(p), \delta_{s_\gamma}(p), \delta_{s_\beta}(p))
\]

where,

\[
\begin{align*}
\delta_{s_\alpha}(p) &= \Sigma_{p \in s_{\alpha}}(p) \alpha_{\delta}(p) \\
\delta_{s_\gamma}(p) &= \Sigma_{p \in s_{\gamma}}(p) \gamma_{\delta}(p) \\
\delta_{s_\beta}(p) &= \Sigma_{p \in s_{\beta}}(p) \beta_{\delta}(p)
\end{align*}
\]

Definition 3.18. The minimum neighborhood degree of $G = (P, Q)$ in a SFG is elucidated as

\[
\delta_{r}(G) = (\delta_{s_\alpha}(G), \delta_{s_\gamma}(G), \delta_{s_\beta}(G))
\]

where,

\[
\begin{align*}
\delta_{s_\alpha}(G) &= \bigwedge\{\delta_{s_\alpha}(p) | p \in P\} \\
\delta_{s_\gamma}(G) &= \bigwedge\{\delta_{s_\gamma}(p) | p \in P\} \\
\delta_{s_\beta}(G) &= \bigwedge\{\delta_{s_\beta}(p) | p \in P\}
\end{align*}
\]

Definition 3.19. The maximum neighborhood degree of $G = (P, Q)$ in a SFG is elucidated as

\[
\Delta_{s}(G) = (\Delta_{s_\alpha}(G), \Delta_{s_\gamma}(G), \Delta_{s_\beta}(G))
\]

where,

\[
\begin{align*}
\Delta_{s_\alpha}(G) &= \bigvee\{\delta_{s_\alpha}(p) | p \in P\} \\
\Delta_{s_\gamma}(G) &= \bigvee\{\delta_{s_\gamma}(p) | p \in P\} \\
\Delta_{s_\beta}(G) &= \bigvee\{\delta_{s_\beta}(p) | p \in P\}
\end{align*}
\]

Example 3.20. In Example (3.3),

\[
\begin{align*}
d_{s}(p) &= (0.7, 0.9, 1.5) \\
d_{s}(q) &= (1.3, 1.1, 1.4) \\
d_{s}(r) &= (1.0, 0.8, 1.3) \\
\delta_{s}(G) &= (0.7, 0.8, 1.3) \\
\Delta_{s}(G) &= (1.3, 1.1, 1.5)
\end{align*}
\]

Definition 3.21. The closed neighborhood degree of a vertex $p$ of $G = (P, Q)$ in a SFG is elucidated as

\[
d_{s}[p] = (d_{s_\alpha}(p), d_{s_\gamma}(p), d_{s_\beta}(p))
\]

where,

\[
\begin{align*}
d_{s_\alpha}[a] &= \Sigma_{q \in s_{\alpha}}(p) \alpha_{\delta}(p) \land \alpha_{\delta}(p) \\
d_{s_\gamma}[a] &= \Sigma_{q \in s_{\gamma}}(p) \gamma_{\delta}(p) \land \gamma_{\delta}(p) \\
d_{s_\beta}[a] &= \Sigma_{q \in s_{\beta}}(p) \beta_{\delta}(p) \land \beta_{\delta}(p)
\end{align*}
\]

Definition 3.22. The minimum closed neighborhood degree of $G = (P, Q)$ in a SFG is elucidated as

\[
\delta_{s}[G] = (\delta_{s_\alpha}[G], \delta_{s_\gamma}[G], \delta_{s_\beta}[G])
\]

where,

\[
\begin{align*}
\delta_{s_\alpha}[G] &= \bigwedge\{d_{s_\alpha}(p) | p \in P\} \\
\delta_{s_\gamma}[G] &= \bigwedge\{d_{s_\gamma}(p) | p \in P\} \\
\delta_{s_\beta}[G] &= \bigwedge\{d_{s_\beta}(p) | p \in P\}
\end{align*}
\]

Definition 3.23. The maximum closed neighborhood degree of $G = (P, Q)$ in a SFG is elucidated as

\[
\Delta_{s}[G] = (\Delta_{s_\alpha}(G), \Delta_{s_\gamma}(G), \Delta_{s_\beta}(G))
\]

where,

\[
\begin{align*}
\Delta_{s_\alpha}[G] &= \bigvee\{d_{s_\alpha}(p) | p \in P\} \\
\Delta_{s_\gamma}[G] &= \bigvee\{d_{s_\gamma}(p) | p \in P\} \\
\Delta_{s_\beta}[G] &= \bigvee\{d_{s_\beta}(p) | p \in P\}
\end{align*}
\]
4. Regular Spherical Fuzzy Graph

**Definition 4.1.** Regular of a SFG, $G = (P, Q)$ is defined by all the vertices have the same closed neighborhood degree. To be specifically, $\delta_r(\alpha)[G] = \Delta_r(\alpha)[G]; \delta_r(\gamma)[G] = \Delta_r(\gamma)[G]; \delta_r(\beta)[G] = \Delta_r(\beta)[G].$

**Example 4.2.** In Example (3.3),

\[
\begin{align*}
\delta_r[p] &= (1.5, 1.4, 2.1), \\
\delta_r[q] &= (1.5, 1.4, 2.1), \\
\delta_r[r] &= (1.5, 1.4, 2.1)
\end{align*}
\]

which implies

\[
\begin{align*}
\delta_r[G] &= (1.5, 1.4, 2.1), \\
\Delta_r[G] &= (1.5, 1.4, 2.1).
\end{align*}
\]

**Proposition 4.3.** Every spherical complete fuzzy graph in $G$ is spherical regular fuzzy graph.

*Proof.* Let SFG, $G = (P, Q)$ has completeness implies closed neighborhood degree are elucidated in Definition (3.2) and in Definition (3.21) respectively. This leads to every vertex of a minimum and maximum closed neighborhood degree are equal in $G$. To be specifically, $\delta_r(\alpha)[G] = \Delta_r(\alpha)[G]; \delta_r(\gamma)[G] = \Delta_r(\gamma)[G]; \delta_r(\beta)[G] = \Delta_r(\beta)[G].$ This tends to $G$ is spherical regular fuzzy graph. Hence the proof. $\square$

5. Elucidation of Order and Size in SFG

**Definition 5.1.** The order of SFG, $G$ is elucidated by $\Theta(G) = (\Theta_\alpha(G), \Theta_\gamma(G), \Theta_\beta(G))$ where

\[
\begin{align*}
\Theta_\alpha(G) &= \Sigma_{p \in P} \alpha_p(p), \\
\Theta_\gamma(G) &= \Sigma_{p \in P} \gamma_p(p), \\
\Theta_\beta(G) &= \Sigma_{p \in P} \beta_p(p).
\end{align*}
\]

**Definition 5.2.** The size of SFG, $G$ is elucidated by $\Omega(G) = (\Omega_\alpha(G), \Omega_\gamma(G), \Omega_\beta(G))$ where

\[
\begin{align*}
\Omega_\alpha(G) &= \Sigma_{p \neq q} \Omega_\alpha(p, q), \\
\Omega_\gamma(G) &= \Sigma_{p \neq q} \Omega_\gamma(p, q), \\
\Omega_\beta(G) &= \Sigma_{p \neq q} \Omega_\beta(p, q).
\end{align*}
\]

**Example 5.3.** In Example (3.3), $\Theta(G) = (1.5, 1.4, 2.1); \Omega(G) = (0.9, 1.4, 2.3)$

**Proposition 5.4.** Every vertex of a closed neighborhood degree of a complete SFG has same order in $G$. To be specifically, $\Theta_\alpha(G) = (d_N\alpha[p]: p \in P); \Theta_\gamma(G) = (d_N\gamma[p]: p \in P); \Theta_\beta(G) = (d_N\beta[p]: p \in P).$

6. Conclusion

In the present study, grasped some contemporary concepts of spherical fuzzy graphs which is the addendum of intuitionistic, Pythagorean and picture fuzzy graphs. Considerably more work should be possible to explore the structure of spherical fuzzy graphs. It is much convenient in the application field such as pattern recognition, decision making and network analysis.

**References**


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