# On $K$-eccentric and $K$-hyper eccentric indices of Benzenoid $H_{k}$ system 

M. Bhanumathi ${ }^{1}$, R. Rohini ${ }^{2}$ and G. Srividhya $3^{3^{*}}$


#### Abstract

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. Bhanumathi and Easu Julia Rani introduced the first $K$-Eccentric index $B_{1} E(G)$ and the second $K$ - Eccentric index $B_{2} E(G)$ of a graph $G$ as $B_{1} E(G)=$ $\Sigma_{u e}\left[e_{G}(u)+e_{L(G)}(e)\right], B_{2} E(G)=\sum_{u e}\left[e_{G}(u) e_{L(G)}(e)\right]$ They also defined the first $K$-Hyper eccentric index $H B_{1} E(G)$ and the second $K$-Hyper eccentric index $H B_{2} E(G)$ of a graph $G$ as $H B_{1} E(G)=\sum_{u e}\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}, H B_{2} E(G)=$ $\sum_{u e}\left[e_{G}(u) e_{L(G)}(e)\right]^{2}$ where in all the cases $u e$ means that the vertex $u$ and edge $e$ are incident in $G$ and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of $G$. They have defined the multiplicative version of these indices also. In this paper, we calculate the first and second $K$ eccentric, the first and second K-hyper eccentric indices and their multiplicative versions of benzenoid $H_{k}$ system.


## Keywords

$K$-eccentric index, $K$-hyper eccentric index, Multiplicative $K$-eccentric index, Multiplicative $K$-hyper eccentric index, Circo.
${ }^{1}$ Department of Mathematics, Government Arts College for Women, Sivagangai-630562, Tamil Nadu, India.
${ }^{2}$ Department of Mathematics, Government Arts College for Women (Autonomous), Pudukkottai-622001, Tamil Nadu, India.
${ }^{3}$ Department of Mathematics, Government Arts College, Tiruchirappalli-620022, Tamil Nadu, India.
*Corresponding author: ${ }^{1}$ bhanu_ksp@yahoo.com; ${ }^{2}$ rohinianbazhagan7@gmail.com; ${ }^{3}$ vkm292011@hotmail.com Article History: Received 19 September 2020; Accepted 19 November 2020

## Contents

## 1 Introduction

 20972 First and second $K$-Eccentric indices, First and Second $K$-Hyper Eccentric indices of Benzenoid $H_{k}$ system: 2098
3 Multiplicative First and Second $K$-Eccentric indices, Multiplicative First and Second $K$ Hyper Eccentric indices of Benzenoid $H_{k}$ system: 2100
4 Conclusion .2102
References............................................... 2102

## 1. Introduction

A topological index is a real number associated with chemical constitution. It correlates the chemical structure with various physical and chemical properties and biological activity.

All graphs in this paper are simple, finite and undirected. A graph G is a finite nonempty set $V(G)$ together with a prescribed set $E(G)$ of unordered pair of distinct elements of $V$. The cardinality of $V(G)$ and $E(G)$ are represented by $|V(G)|$ and $|E(G)|$, respectively. Let, $d_{G}(v)$ be the degree of a vertex v of $G$ and $N_{G}(v)$ be the neighborhood of a vertex $v$ of
$G$. The distance between the vertices $u$ and $v$ of a connected graph $G$ is represented by $d_{G}(u, v)$. It is defined as the number of edges in a shortest path connects the vertices $u$ and $v$. The eccentricity e ${ }_{G}(v)$ of a vertex $v$ in $G$ is the largest distance between $v$ and any other vertices $u$ of $G$.

To take an account on contributions of pairs of incident elements, Kulli [5] introduced the first and second $K$ Banhatti indices. In [4], Bhanumathi and Easu Julia Rani introduced the first $K$-Eccentric index $B_{1} E(G)$ and the second $K$ - Eccentric index $B_{2} E(G)$ of a graph $G$ as

$$
B_{1} E(G)=\sum_{u e}\left[e_{G}(u)+e_{L(G)}(e)\right], B_{2} E(G)=\sum_{u e}\left[e_{G}(u) e_{L(G)}(e)\right]
$$

and also defined the first $K$-Hyper eccentric index $H B_{1} E(G)$ and the second $K$-Hyper eccentric index $H B_{2} E(G)$ of a graph $G$ as $H B_{1} E(G)=\sum_{u e}\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}, H B_{2} E(G)=\sum_{u e}$ $\left[e_{G}(u) e_{L(G)}(e)\right]^{2}$ where in all the cases ue means that the vertex $u$ and edge $e$ are incident in $G$ and $e_{L(G)}(e)$ is the eccentricity of e in the line graph $L(G)$ of G [4].

Table 1

| Edge set | No. of edges <br> $e=u v$ | Eccentricity of end vertices <br> $(e(u), e(v))$ | Eccentricity of $e$ in <br> $L(G) e_{L(G)}(e)$ |
| :---: | :---: | :---: | :---: |
| $E_{1}$ | 6 | $(2 k+1,2 k+1)$ | $2 k+1$ |
| $E_{2}$ | 6 | $(2 k+1,2 k+2)$ | $2 k+1$ |
| $E_{3}$ | 12 | $(2 k+2,2 k+3)$ | $2 k+2$ |
| $E_{4}$ | 6 | $(2 k+3,2 k+3)$ | $2 k+3$ |
| $E_{5}$ | 12 | $(2 k+3,2 k+4)$ | $2 k+3$ |
| $E_{6}$ | 24 | $(2 k+4,2 k+5)$ | $2 k+4$ |
| $E_{7}$ | 6 | $(2 k+5,2 k+5)$ | $2 k+5$ |
| $E_{8}$ | 18 | $(2 k+5,2 k+6)$ | $2 k+5$ |
| $E_{9}$ | 36 | $(2 k+6,2 k+7)$ | $2 k+6$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $E_{3(k-2)-2}$ | 6 | $(2 k+2(k-2)-1,2 k+2(k-2)-1)$ | $2 k+2(k-2)-1$ |
| $E_{3(k-2)-1}$ | $6(k-2)$ | $(2 k+2(k-2)-1,2 k+2(k-2))$ | $2 k+2(k-2)-1$ |
| $E_{3(k-2)}$ | $12(k-2)$ | $(2 k+2(k-2), 2 k+2(k-1)-1)$ | $2 k+2(k-2)$ |
| $E_{3(k-1)-2}$ | 6 | $(2 k+2(k-1)-1,2 k+2(k-1)-1)$ | $2 k+2(k-1)-1$ |
| $E_{3(k-1)-1}$ | $6(k-1)$ | $(2 k+2(k-1)-1,2 k+2(k-1))$ | $2 k+2(k-1)-1$ |
| $E_{3(k-1)}$ | $12(k-1)$ | $(2 k+2(k-1), 2 k+2(k-1)+1)$ | $2 k+2(k-1)$ |
| $E_{3(k-1)+1}$ | 6 | $(2 k+2(k-1)+1,2 k+2(k-1)+1)$ | $2 k+2(k-1)+1$ |

## 2. First and second $K$-Eccentric indices, First and Second $K$-Hyper Eccentric indices of Benzenoid $H_{k}$ system:

The circumcoronene homologous series of benzenoid also belongs to the family of molecular graphs that has several copy of benzene $C_{6}$ on its circumference. The terms of this series are represented as, $H_{1}$-benzene, $H_{2}$-coronene, $H_{3}$ circumcoronene and $H_{4}$ circumcircumcoronene etc. A benzeniod system is a connected geometric figure. It is obtained by arranging congruent regular hexagons in a plane. Consequently two hexagons are either disjoint or have a common edge.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The eccentricities of $u, v \in V(G)$ are denoted by $e(u), e(v)$ respectively and for $e=u v \in E(G)$, denote the eccentricities of the end vertices of the edge e by $(e(u), e(v))$.

Let $V$ be the vertex set of $H_{k}$ and $E$ be the edge set in $H_{k}$, then $|V|=6 k^{2}$ and $|E|=9 k^{2}-3 k$ for the structure of $H_{k}$. First, we shall determine the number of edges $e=u v$ with the eccentricity of the end vertices $e(u), e(v)$ and eccentricity of the edge e in $L(G)$. We give these values in the following Table 1.

Theorem 2.1. For any positive integer number $k$, let $H_{k}$ be the general form of circumcoronene series of benzenoid system, then
(i) $\quad B_{1} E\left(H_{k}\right)=6 \sum_{i=1}^{k}[8 k+4(2 i-1)]+6 \sum_{i=1}^{k-1}[8 k+4(2 i-1)$

$$
+1]+12 \sum_{i=1}^{k-1} i[8 k+4(2 i)+1]
$$



Figure 1. The Circumcoronene homologous Series of Benzenoid $H_{k}(k \geq 1)$ with edges
(ii)

$$
\begin{aligned}
& B_{2} E\left(H_{k}\right)=6 \sum_{i=1}^{k}\left[(2(2 k+2 i-1))^{2}\right] \\
& \quad+6 \sum_{i=1}^{k-1} i\left[(2 k+2 i-1)^{2}+(2 k+2 i)(2 k+2 i-1)\right] \\
& \quad+12 \sum_{i=1}^{k-1} i\left[(2 k+2 i)^{2}+(2 k+2 i)(2 k+2 i-1)\right]
\end{aligned}
$$

(iii) $H B_{1} E\left(H_{k}\right)=6 \sum_{i=1}^{k}\left[(2(2 k+2 i-1))^{2}+(2(2 k+2 i-1))^{2}\right]$

$$
\begin{aligned}
& +6 \sum_{i=1}^{k-1} i\left[(2(2 k+2 i-1))^{2}+((2 k+2 i)+(2 k+2 i-1))^{2}\right] \\
& +12 \sum_{i=1}^{k-1} i\left[(2(2 k+2 i))^{2}+((2 k+2 i+1)+(2 k+2 i))^{2}\right]
\end{aligned}
$$

(iv) $H B_{2} E\left(H_{k}\right)=6 \sum_{i=1}^{k}\left[\left((2 k+2 i-1)^{2}\right)^{2}+\left((2 k+2 i-1)^{2}\right)^{2}\right]$

$$
\begin{aligned}
& +6 \sum_{i=1}^{k-1} i\left[\left((2 k+2 i-1)^{2}\right)^{2}+((2 k+2 i)(2 k+2 i-1))^{2}\right] \\
& +12 \sum_{i=1}^{k-1} i\left[\left((2 k+2 i)^{2}\right)^{2}+((2 k+2 i+1)(2 k+2 i))^{2}\right]
\end{aligned}
$$

Proof. Consider the General form of $H_{k}$-Circumcoronene graph.

$$
\begin{aligned}
& \text { (i) } \quad B_{1} E\left(H_{k}\right)=\sum_{u e}\left[e_{H_{k}}(u)+e_{L\left(H_{k}\right)}(e)\right] \\
& =\sum_{e=u v \in E(G)}\left[e_{H_{k}}(u)+e_{L\left(H_{k}\right)}(e)+e_{H_{k}}(v)+e_{L\left(H_{k}\right)}(e)\right] \\
& =\sum_{u v \in E_{1}(G)}\left[e_{G}(u)+e_{L(G)}(e)+e_{G}(v)+e_{L(G)}(e)\right]+\ldots \\
& +\sum_{u v \in E_{3(k-1)+1}(G)}\left[e_{G}(u)+e_{L(G)}(e)+e_{G}(v)+e_{L(G)}(e)\right] \\
& =6 \sum_{i=1}^{k}[8 k+4(2 i-1)]+6 \sum_{i=1}^{k-1}[8 k+4(2 i-1)+1] \\
& +12 \sum_{i=1}^{k-1} i[8 k+4(2 i)+1] \\
& \text { (ii) } \quad B_{2} E\left(H_{k}\right)=\sum_{u e}\left[e_{H_{k}}(u) \times e_{L\left(H_{k}\right)}(e)\right] \\
& =\sum_{u v \in E_{1}(G)}\left[e_{G}(u) e_{L(G)}(e)+e_{G}(v) e_{L(G)}(e)\right]+\ldots \\
& +\sum_{u v \in E_{3(k-1)+1}}\left[e_{G}(u) e_{L(G)}(e)+e_{G}(v) e_{L(G)}(e)\right] \\
& =6 \sum_{i=1}^{k}\left[(2(2 k+2 i-1))^{2}\right] \\
& +6 \sum_{i=1}^{k-1} i\left[(2 k+2 i-1)^{2}+(2 k+2 i)(2 k+2 i-1)\right] \\
& +12 \sum_{i=1}^{k-1} i\left[(2 k+2 i)^{2}+(2 k+2 i)(2 k+2 i-1)\right] \\
& \text { (iii) } \quad H B_{1} E\left(H_{k}\right)=\sum_{u e}\left[e_{H_{k}}(u)+e_{L\left(H_{k}\right)}(e)\right]^{2} \\
& =\sum_{u v \in E_{1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}+\left[e_{G}(v)+(e)\right]\right]^{2}+\ldots \\
& +\sum_{u v \in E_{3(k-1)+1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}+\left[e_{G}(v)+e_{L(G)}(e)\right]\right]^{2} \\
& =6 \sum_{i=1}^{k}\left[\left(2(2 k+2 i-1)^{2}\right)+(2(2 k+2 i-1))^{2}\right] \\
& +6 \sum_{i=1}^{k-1} i\left[(2(2 k+2 i-1))^{2}+((2 k+2 i)+(2 k+2 i-1))^{2}\right] \\
& +12 \sum_{i=1}^{k-1} i\left[(2(2 k+2 i))^{2}+((2 k+2 i+1)+(2 k+2 i))^{2}\right]
\end{aligned}
$$

(iv) $\quad H B_{2} E\left(H_{k}\right)=\sum_{u e}\left[e_{H_{k}}(u) \times e_{L\left(H_{k}\right)}(e)\right]^{2}$

$$
\begin{aligned}
= & \sum_{e=u v \in E_{1}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}+\left[e_{G}(v) \times e_{L(G)}(e)\right]\right]^{2}+\ldots \\
+ & \sum_{e=u v \in E_{3(k-1)+1}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}+\left[e_{G}(v) \times e_{L(G)}(e)\right]\right]^{2} \\
= & 6 \sum_{i=1}^{k}\left[\left((2 k+2 i-1)^{2}\right)^{2}+\left((2 k+2 i-1)^{2}\right)^{2}\right] \\
& +6 \sum_{i=1}^{k-1} i\left[\left((2 k+2 i-1)^{2}\right)^{2}+((2 k+2 i)(2 k+2 i-1))^{2}\right] \\
& +12 \sum_{i=1}^{k-1} i\left[\left((2 k+2 i)^{2}\right)^{2}+((2 k+2 i+1)(2 k+2 i))^{2}\right]
\end{aligned}
$$

For example, let us evaluate the indices for $H_{4}$. Consider the $H_{4}$-Circumcircumcoronene graph.


Figure 2

Let $V$ be the vertex set and $E$ be the edge set in $H_{4}=$ Circumcircumcoronene, then $|V|=96$ and $|E|=132$. Also, the number of edges with eccentricities of end vertices $e=$ $u v \in E(G)$ and $e \in L(G)$ are given as follows:

Table 2

| Edge <br> set | No. of <br> edges | Eccentricity <br> of end vertices <br> $(e(u), e(v))$ | Eccentricity <br> of $e$ in <br> $L(G) e_{L(G)}(e)$ |
| :---: | :---: | :---: | :---: |
| $E_{1}$ | 6 | $(9,9)$ | 9 |
| $E_{2}$ | 6 | $(9,10)$ | 9 |
| $E_{3}$ | 12 | $(10,11)$ | 10 |
| $E_{4}$ | 6 | $(11,11)$ | 11 |
| $E_{5}$ | 12 | $(11,12)$ | 11 |
| $E_{6}$ | 24 | $(12,13)$ | 12 |
| $E_{7}$ | 6 | $(13,13)$ | 13 |
| $E_{8}$ | 18 | $(13,14)$ | 13 |
| $E_{9}$ | 36 | $(14,15)$ | 14 |
| $E_{10}$ | 6 | $(15,15)$ | 15 |

(i) $\quad B_{1} E\left(H_{4}\right)=\sum_{u e}\left[e_{H_{4}}(u)+e_{L\left(H_{4}\right)}(e)\right]$

$$
\begin{aligned}
= & \sum_{u v \in E_{1}(G)}\left[e_{G}(u)+e_{L(G)}(e)+e_{G}(v)+e_{L(G)}(e)\right]+\ldots \\
& +\sum_{u v \in E_{10}(G)}\left[e_{G}(u)+e_{L(G)}(e)+e_{G}(v)+e_{L(G)}(e)\right] \\
= & 6588
\end{aligned}
$$

(ii) $\quad B_{2} E\left(H_{4}\right)=\sum_{u e}\left[e_{H_{4}}(u) \times e_{L\left(H_{4}\right)}(e)\right]$

$$
\begin{aligned}
&= \sum_{e=u v \in E_{1}(G)}\left[e_{G}(u) e_{L(G)}(e)+e_{G}(v) e_{L(G)}(e)\right]+\ldots \\
&+\sum_{e=u v \in E_{10}(G)}\left[e_{G}(u) e_{L(G)}(e)+e_{G}(v) e_{L(G)}(e)\right] \\
&=41868
\end{aligned}
$$

(iii) $H B_{1} E\left(H_{4}\right)=\Sigma_{u e}\left[e_{H_{4}}(u)+e_{L\left(H_{4}\right)}(e)\right]^{2}$

$$
\begin{aligned}
& =\sum_{e=u v \in E_{1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}+\left[e_{G}(v)+e_{L(G)}(e)\right]\right]^{2}+\ldots \\
& \quad+\sum_{e=u v \in E_{10}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}+\left[e_{G}(v)+e_{L(G)}(e)\right]\right]^{2} \\
& =167580
\end{aligned}
$$

(iv) $H B_{2} E\left(H_{4}\right)=\sum_{u e}\left[e_{H_{4}}(u) \times e_{L\left(H_{4}\right)}(e)\right]^{2}$

$$
\begin{aligned}
& =\sum_{e=u v \in E_{1}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}+\left[e_{G}(v) \times e_{L(G)}(e)\right]\right]^{2}+\ldots \\
& \quad+\sum_{e=u v \in E_{10}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}+\left[e_{G}(v) \times e_{L(G)}(e)\right]\right]^{2} \\
& =7105236
\end{aligned}
$$

Thus we have $B_{1} E\left(H_{4}\right)=6588, B_{2} E\left(H_{4}\right)=41868, H B_{1} E\left(H_{4}\right)$ $=167580$ and $H B_{2} E\left(H_{4}\right)=7105236$.

Corollary 2.2. $H_{1}$ be the first terms of this Circumcoronene series of Benzene $H_{k}$. Then
(i) $B_{1} E\left(H_{1}\right)=72$
(ii) $B_{2} E\left(H_{1}\right)=108$
(iii) $H B_{1} E\left(H_{1}\right)=432$
(iv) $H B_{2} E\left(H_{1}\right)=972$.

Corollary 2.3. $\mathrm{H}_{2}$ be the second terms of this Circumcoronene series of Benzene $H_{k}$. Then
(i) $B_{1} E\left(H_{2}\right)=714$
(ii) $B_{2} E\left(H_{2}\right)=2154$
(iii) $H B_{1} E\left(H_{2}\right)=8634$
(iv) $H B_{2} E\left(H_{2}\right)=82182$

Corollary 2.4. $\mathrm{H}_{3}$ be the third terms of this Circumcoronene series of Benzene $H_{k}$. Then
(i) $B_{1} E\left(H_{3}\right)=2646$
(ii) $B_{2} E\left(H_{3}\right)=12366$
(iii) $H B_{1} E\left(H_{3}\right)=49770$
(iv) $H B_{2} E\left(H_{3}\right)=1134150$

## 3. Multiplicative First and Second $K$-Eccentric indices, Multiplicative First and Second $K$ Hyper Eccentric indices of Benzenoid $H_{k}$ system:

Theorem 3.1. For any positive integer number $k$, let $H_{k}$ be the general form of circumcoronene series of benzenoid system, then
(i) $B \Pi_{1} E\left(H_{k}\right)=6 \prod_{i=1}^{k}\left[4(2 k+2 i-1)^{2}\right]$

$$
\begin{aligned}
& \times 6 \prod_{i=1}^{k-1} i[2(2 k+2 i-1)(4 k+4 i-1)] \\
& \times 12 \prod_{i=1}^{k-1} i[2(2 k+2 i)(4 k+4 i+1)]
\end{aligned}
$$

(ii) $B \Pi_{2} E\left(H_{k}\right)=6 \prod_{i=1}^{k}\left[(2 k+2 i-1)^{4}\right]$

$$
\begin{aligned}
& \times 6 \prod_{i=1}^{k-1} i\left[(2 k+2 i-1)^{3}(2 k+2 i)\right] \\
& \times 12 \prod_{i=1}^{k-1} i\left[(2 k+2 i)^{3}(2 k+2 i+1)\right]
\end{aligned}
$$

(iii) $\quad H B \Pi_{1}\left(H_{k}\right)=6 \prod_{i=1}^{k}\left[16(2 k+2 i-1)^{4}\right]$

$$
\begin{aligned}
& \times 6 \prod_{i=1}^{k-1} i\left[(4(2 k+2 i-1))^{2}\right] \\
& \times\left[((2 k+2 i)+(2 k+2 i-1))^{2}\right] \\
& \times 12 \prod_{i=1}^{k-1} i\left[4(2 k+2 i)^{2}\right]+\left[((2 k+2 i+1)+(2 k+2 i))^{2}\right]
\end{aligned}
$$

(iv) $\quad H B \Pi_{2}\left(H_{k}\right)=6 \prod_{i=1}^{k}\left[(2 k+2 i-1)^{8}\right]$

$$
\begin{aligned}
& \times 6 \prod_{i=1}^{k-1} i\left[(2 k+2 i-1)^{6}\right] \times[(2 k+2 i)] \\
& \times 12 \prod_{i=1}^{k-1} i\left[(2 k+2 i)^{6}\right][(2 k+2 i+1)]
\end{aligned}
$$

Proof. Consider the General form of Hk - Circumcoronene
graph, Using Table 1, we obtain the following:

$$
\begin{aligned}
& \text { (i) } B \Pi_{1} E\left(H_{k}\right)=\prod_{u e}\left[e_{H_{k}}(u)+e_{L\left(H_{k}\right)}(e)\right] \\
& =\prod_{u v \in E_{1} G}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]\left[e_{G}(v)+e_{L(G)}(e)\right]\right] \times \ldots \\
& \times \prod_{u v \in E_{3(k-1)+1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]\left[e_{G}(v)+e_{L(G)}(e)\right]\right] \\
& =6 \prod_{i=1}^{k}\left[4(2 k+2 i-1)^{2}\right] \\
& \times 6 \prod_{i=1}^{k-1} i[2(2 k+2 i-1)(4 k+4 i-1)] \\
& \times 12 \prod_{i=1}^{k-1} i[2(2 k+2 i)(4 k+4 i+1)] \\
& \text { (ii) } B \prod_{2}\left(H_{k}\right)=\prod_{\text {ue }}\left[e_{H_{k}}(u) \times e_{L\left(H_{k}\right)}(e)\right] \\
& =\prod_{e=u v \in E_{1}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]\left[e_{G}(v) \times e_{L(G)}(e)\right]\right] \times \ldots \\
& \times \prod_{e=u v \in E_{3(k-1)+1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]\left[e_{G}(v)+e_{L(G)}(e)\right]\right] \\
& =6 \prod_{i=1}^{k}(2 k+2 i-1)^{4} \times 6 \prod_{i=1}^{k-1} i\left[(2 k+2 i-1)^{3}(2 k+2 i)\right] \\
& \times 12 \prod_{i=1}^{k-1} i\left[(2 k+2 i)^{3}(2 k+2 i+1)\right] \\
& \text { (iii) } \quad H B \Pi_{1}\left(H_{k}\right)=\Pi_{u e}\left[e_{H_{k}}(u)+e_{L\left(H_{k}\right)}(e)\right]^{2} \\
& =\prod_{e=u v \in E_{1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}\left[e_{G}(v)+e_{L(G)}(e)\right]^{2}\right] \times \ldots \\
& \times \prod_{e=u v \in E_{3(k-1)+1}(G)}\left[\left[e_{G}(u)+e_{L(G)}(e)\right]^{2}\left[e_{G}(v)+e_{L(G)}(e)\right]^{2}\right] \\
& =6 \prod_{i=1}^{k}\left[16(2 k+2 i-1)^{4}\right] \times 6 \prod_{i=1}^{k-1} i\left[(4(2 k+2 i-1))^{2}\right] \\
& +\left[((2 k+2 i)+(2 k+2 i-1))^{2}\right] \times 12 \prod_{i=1}^{k-1} i\left[4(2 k+2 i)^{2}\right] \\
& +\left[((2 k+2 i+1)+(2 k+2 i))^{2}\right] \\
& \text { (iv) } H B \Pi_{1}\left(H_{k}\right)=\prod_{u e}\left[e_{H_{k}}(u) \times e_{L\left(H_{k}\right)}(e)\right]^{2} \\
& =\prod_{e=u v \in E_{1}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}\left[e_{G}(v) x+e_{L(G)}(e)\right]^{2}\right] \times \ldots \\
& \times \prod_{e=u v \in E_{3(k-1)+1}(G)}\left[\left[e_{G}(u) \times e_{L(G)}(e)\right]^{2}\left[e_{G}(v) \times e_{L(G)}(e)\right]^{2}\right] \\
& =6 \prod_{i=1}^{k}\left[(2 k+2 i-1)^{8}\right] \times 6 \prod_{i=1}^{k-1} i\left[(2 k+2 i-1)^{6}\right] \\
& \times[(2 k+2 i)] \times 12 \prod_{i=1}^{k-1} i\left[(2 k+2 i)^{6}\right][((2 k+2 i+1)]
\end{aligned}
$$

Using MATLAB programme, we have calculated these indices for $H_{1}, H_{2}$ and $H_{3}$. Those values are given below corollaries.

Corollary 3.2. $H_{1}$ be the first terms of this Circumcoronene series of Benzene $H_{k}$. Then
(i) $B \Pi_{1} E\left(H_{1}\right)=2176782336$
(ii) $B \Pi_{2} E\left(H_{1}\right)=2.824295365 \times 10^{11}$
(iii) $H B \Pi_{1} E\left(H_{1}\right)=4.738381338 \times 10^{18}$
(iv) $H B \Pi_{2} E\left(H_{1}\right)=7.976644308 \times 10^{22}$

Corollary 3.3. $\mathrm{H}_{2}$ be the second terms of this Circumcoronene series of Benzene $H_{k}$. Then
(i) $B \Pi_{1} E\left(H_{2}\right)=2.086352657 \times 10^{64}$
(ii) $B \Pi_{2} E\left(H_{2}\right)=2.901497086 \times 10^{92}$
(iii) $H B \Pi_{1} E\left(H_{2}\right)=3.1023 e+040$
(iv) $H B \Pi_{2} E\left(H_{2}\right)=8.4187 e+184$

Corollary 3.4. $\mathrm{H}_{3}$ be the third terms of this Circumcoronene series of Benzene $H_{k}$. Then
(i) $B \Pi_{1} E\left(H_{3}\right)=1.3789 e^{+093}$
(ii) $B \Pi_{2} E\left(H_{3}\right)=7.3558 e^{+128} \times 1.0328 e^{+234}$
(iii) $H B \Pi_{1} E\left(H_{3}\right)=1.9013 e^{+186}$
(iv) $H B \prod_{2} E\left(H_{3}\right)=5.4107 e^{+257} \times 5.7517 e^{+151}$ $\times 6.7910 e^{+251} \times 2.7308 e^{+064}$

## 4. Conclusion

In chemical graph theory a topological index of a molecular graph characterizes its topology. Here, we have computed the first, second $K$-eccentric indices, $K$-hyper eccentric indices and multiplicative first, second $K$-eccentric and $K$-hyper eccentric indices of benzenoid $H_{k}$ system.

## References

${ }^{[1]}$ M. Bhanumathi, K. Easu Julia Rani, S. Balachandran, The edge version of inverse sum index of Connected graph, International Journal of Mathematical Archive, 7(1)(2016), 8-12.
${ }^{[2]}$ M. Bhanumathi, K. Easu Julia Rani, On $K$-eccentric indices and K hyper-eccentric indices of graphs, Aryabhatta Journal of Mathematics and Informatics, 9(1)(2017), 509520.
[3] M. Bhanumathi, K. Easu Julia Rani, On Some Multiplicative Topological Indices, International Journal on Recent Trends in Life Science and Mathematics, 4(2017), 09-18.
${ }^{[4]}$ M. Bhanumathi. K. Easu Julia Rani, Harmonic Eccentric Index of Hexagonal Chain, International Journal of Elixir Appl. Math. Appl. Math, 104C(2017), 45871-45880.
${ }^{[5]}$ V. R. Kulli, On $K$ Banhatti Indices of Graphs, Journal of Computer and Mathematical Sciences, 7(4)(2016), 213218.
${ }^{[6]}$ V. R. Kulli, Second Multiplicativee K Banhatti Index and Coindex of Graphs, Journal of Computer and Mathematical Sciences, 7(5)(2016), 254-258.
${ }^{[7]}$ V. R. Kulli , Multiplicative K hyper-Banhatti indices and coindices of graphs, International Journal of Mathematical Archive, (2016), 1-6.
${ }^{[8]}$ V. R. Kulli, On $K$ hyper-Banhatti indices and coindices of graphs Research gate, (2016), 1-6.
${ }^{[9]}$ V. R. Kulli, First Multiplicative $K$ Banhatti Index and Coindex of Graphs, Annals of Pure and Applied Mathematics, 11(2)(2016), 79-82
${ }^{[10]}$ M. Bhanumathi, R. Rohini, G. Srividhya, On $K$-Eccentric and K-Hyper Eccentric indices of Benzenoid Hk system, National Conference on Recent Trends in Pure and Applied Mathematics, Government Arts College for Women, Sivagangai, Tamilnadu on 13th Feb. 2019.
${ }^{[11]}$ V. R. Kulli, Multiplicative hyper-zagreb indices and coindices of graphs: computing these indices of some nanostructures, International Research Journal of Pure Algebra, 6(7)(2016), 342-347.

ISSN(P):2319-3786
Malaya Journal of Matematik
ISSN(O):2321-5666

