Group action on contra semi open map and almost contra $\beta$-open mapping

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Abstract
The purpose of this paper is to introduce and research the concept of contra semi open map and almost contra $\beta$-open mapping group action.

Keywords
Group acting on open set, open map, contra semi open map and almost contra $\beta$ open mapping with topological spaces.

AMS Subject Classification
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1. Introduction
In 1963 N. Levine and in 1969 N. Biswas have respectively defined and studied the notions called semi open sets and semi closed sets in topology. In the years 1969 and 1970, N. Biswas had defined and studied the notions of semiopen and semi-closed functions respectively. Contra-continuous functions were introduced by Julian Dontchev as functions having the property that inverse images of open sets are closed. In 1997 C.W. Baker has defined and studied the notions of contra open and contra closed functions as dual generations of contra-continuous functions. In the study of modern mathematics, mapping plays an important role, especially in topology and functional analysis.

2. Preliminaries

Definition 2.1. If $A_G \subseteq X_\tau$. It is said that it is

(i) pre open if $A_G \subseteq \text{int}(\text{cl}(A_G))$

(ii) semi-open if $A_G \subseteq \text{cl}(\text{int}(A_G))$

(iii) regular open if $A_G = \text{int}(\text{cl}(A_G))$

(iv) $\alpha$-open if $A_G \subseteq \text{int}($cl(int($A_G)))$

(v) $\beta$-open if $A_G \subseteq \text{cl}(\text{int}(\text{cl}(A_G)))$.

The complement of a pre open, semi open, $\alpha$-open, semi-pre open ($\beta$-open) set is said to be pre closed, semi closed, $\alpha$-closed, semi-pre closed ($\beta$-closed).

The collection of all closed, pre open, semi open, $\alpha$-open, semi-pre open ($\beta$-open) subset of $X$ will be denote by $C(X_\tau), PO(X_\tau), SO(X_\tau), \alpha(X_\tau), SPO(X_\tau), \beta(X_\tau)$.

Definition 2.2. The Intersection of every one of semi closed sets of $X_\tau$ contain $A_G \subseteq X_\tau$ is called the semi closer of $A_G$ is denote by $\text{Scl}(A_G)$.

Definition 2.3. A function $g_\tau : X_\tau \rightarrow Y_\tau$ is called the pre open, semi open, $\alpha$-open but $g_\tau(U)$ is pre open, semi open, $\alpha$-open in $Y_\tau$ for each open set $U$ in $X_\tau$.

Definition 2.4. A function $g_\tau : X_\tau \rightarrow Y_\tau$ is called the pre closed, semi closed, $\alpha$-closed if $g(R)$ is pre closed, semi closed, $\alpha$-closed in $Y_\tau$ for each closed set $R$ in $X_\tau$.

3. Group action on contra semi open

Definition 3.1. A function $g_\tau : X_\tau \rightarrow Y_\tau$ is called the group action on contra semi open if the image of open subset of $X_\tau$ is group acting on semi closed set in $Y_\tau$. Clearly each contra
semi open function is contra semi pre open. Since each semi closed set is semi pre closed.

In this paper short form of contra semi open, contra semi pre open Almost contra semi pre open, Contra alpha open, semi closed, alpha closed set are CSO, $C^\beta O$, $AC^\beta O$, $CaO$, SC, $\alpha C$.

**Theorem 3.2.** For a function $g: X \to Y$, the following statements are equivalent:

(i) $g_t$ is group acted on CSO.

(ii) For every subset $R$ of $Y$ and for every closed set $G$ of $X_t$ with $g_t^{-1}(R) \subseteq G$ there exists a SO set $T$ of $Y_t$ with $R \subseteq T$ and $g_t^{-1}(T) \subseteq G$.

(iii) For every $y \in Y_t$ and for every closed subset $G$ of $X_t$ with $g_t^{-1}(y) \subseteq G$ there exists a SO subset $T$ of $Y$ with $y \in T$ and $g_t^{-1}(T) \subseteq G$.

**Proof.** (i) $\implies$ (ii) Let $R \subseteq Y_t$ and $G$ be a closed subset of $X_t$ with $g_t^{-1}(R) \subseteq G$.

Since $g$ is group acted on $X_t$, $g_t^{-1}(y) \subseteq G$ is semi closed. If we set $T = Y_t - g_t(X_t - G)$ is easily seen that $R \subseteq T$ and $g_t^{-1}(T) \subseteq G$.

(ii) $\implies$ (iii) Let $y \in Y_t$ and $G$ be a closed subset of $X_t$ with $g_t^{-1}(y) \subseteq G$ since group acted on $g_t$ is $C^\beta O$. It is easily seen that $Y_t - g_t(X_t - G)$ is semi closed.

Then by (iii) there exists a semi subset $T$ of $Y$ with $y \in T$ such that $T = Y_t - g_t(X_t - G)$ and hence $g_t^{-1}(T)$ is semi closed and consequently $g_t^{-1}(T)$ group acted on contra semi open function.

**Definition 3.3.** A function $g_t: X_t \to Y_t$ is called the group acted on $C^\beta O$ (group acted on $C^\beta O$, $CaO$) if the image of each open subset of $X_t$ is semi open.

**Theorem 3.4.** If a function $g_t: X_t \to Y_t$ is group acted on CSO, $X_t$ is regular then $g_t$ is group acted on semi open function.

**Proof.** Let $Y_t$ be a random point of $Y_t$ and $T$ be a open set of $X_t$ contain $g_t^{-1}(y)$. Since $X_t$ is regular in this exists an open set $S$ in $X_t$ containing $g_t^{-1}(y)$ such that $cl(S) \subseteq T$. Since $g_t$ is group acted on $C^\beta O$, there exists $E \subseteq SO(X_t, X_t)$ such that $g_t(E) \subseteq cl(S)$. Then $g_t(E) \subseteq T$ This is show that $g_t$ is group acted on SO function.

**Theorem 3.5.** Let a function $g_t: X_t \to Y_t$ is group acted on contra semi open and $h_t: Y_t \to Z_t$ is the semi closed function then $h_t \circ g_t: X_t \to Z_t$ is group acted on contra semi open.

**Proof.** Let $T \subseteq X_t$ be an open acted on open set. Since $g_t$ is group acted on contra semi open function, $g_t(T)$ is SC set in $Y_t$ again $h_t$ is pre SC semi closed function. So $h_t(g_t(T)) = h_t \circ g_t(T)$ is semi closed set in $Z_t$. Therefore $h_t \circ g_t$ is group acted on CSO function.

4. Group action on $AC^\beta O$ mapping

**Definition 4.1.** A function $g_t: X_t \to Y_t$ is called the group acting on $AC^\beta O$ if the image of each group acted on $T$ regular open set in $X_t$ is $\beta$-closed in $Y_t$.

**Example 4.2.** Let

$X_t = Y_t = \{1, 2, 3\}$,

$\tau_3 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$

$\tau_2 = \{\emptyset, \{1\}, \{2\}, Y\}$

Let $g_t: X_t \to Y_t$ be defined $g_t(1) = 2, g_t(2) = 3, g_t(3) = 1$. Then $g_t$ is group acted on $AC^\beta O, C^\beta O$.

**Example 4.3.** Let

$X_t = Y_t = \{1, 2, 3\}$,

$\tau_3 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$

$\tau_2 = \{\emptyset, \{1\}, \{2\}, Y\}$

Let $g_t: X_t \to Y_t$ be defined $g_t(1) = 1, g_t(2) = 3, g_t(3) = 2$. Then $g_t$ is group acted on not $AC^\beta O, C^\beta O$.

**Theorem 4.4.** Let $G$ is a topological group then group acted on all $C^\beta O$ map is $AC^\beta O$ but conversely is not true.

**Proof.** Let $E_G \subseteq X_t$ be regular open implies that $g_t(E_G)$ is group acted on $\beta$ in $Y$. Since $g_t: X_t \to Y_t$ is $C^\beta O$. Therefore $g_t$ is group acted on $C^\beta O$.

**Theorem 4.5.** If $g_t: X_t \to Y_t$ is group acted on $AC^\beta O$ and $E_G \subseteq X_t$ is open $g_t(E_G)$ is $\tau_B$ closed in $Y_t$.

**Proof.** Let $E_G \subseteq X_t$ and $g_t: X_t \to Y_t$ exist group acted on $AC^\beta O$, implies that $\beta(g_t(E_G)) \subseteq g_t(E_G))$. Implies that

$\beta(g_t(E_G)) \subseteq g_t(E_G)$.

Since, $g_t(E_G) \subseteq g_t(E_G)$ like $E_G$ is open. However $g_t(E_G) \subseteq \beta\left(g_t(E_G)\right)$, we have $g_t(E_G) \subseteq \beta\left(g_t(E_G)\right)$. Therefore $g_t(E_G)$ is $\tau_B$ closed in $Y_t$.

**Theorem 4.6.** Let $G$ is a topology Group if $g$ is open $(r$-open) and $g_1$ is group acted on $C^\beta O$, then $g_1 \circ g_2$ is group acted on $AC^\beta O$.

**Proof.** Let $R_G \subseteq X_t$ be $r$-open implies $g_2(R_G)$ is open $(r$-open) in $Y_t$ implies that $g_1(g_2(R_G)) = g_1 \circ g_2(R_G)$ is $\beta$ closed in $G$. Hence $g_1 \circ g_2$ is group acted on $AC^\beta O$.

**Theorem 4.7.** If $g_2$ is group acted on $ACO$ or $[\text{group acted on } CrO]$, $g_1$ is group acted on $\beta$ closed then $g_1 \circ g_2$ is group acted on almost contra $\beta$ open.
Theorem 4.11. If \( g_1 : X \rightarrow Y \) and \( g_2 : Y \rightarrow Z \) are group actions on \( X \) and \( Y \) respectively, then the composition \( g_2 \circ g_1 : X \rightarrow Z \) is a group action on \( X \) by \( Z \).

Proof. Let \( E_G \subseteq X \) be a group action on \( r \)-open in \( X \Rightarrow g_2(E_G) \) is closed \([r-\text{closed}]\) in \( Y \Rightarrow g_1(g_2(E_G)) = g_1 \circ g_2(E_G) \),

is group acted on \( \beta \)-closed in \( G \). Hence \( g_1 \circ g_2 \) is a group acted on almost contra \( \beta \)-open.

Theorem 4.8. If \( g_1 : X \rightarrow Y \) be group acted on almost contra \( \beta \)-open then \( \beta(g_1(E_G)) \subseteq g_1(E_G) \).

Proof. Let \( E_G \subseteq X \) be group acted on almost contra \( \beta \)-open then \( g_1(E_G) \) is \( \beta \)-closed in \( Y \) and \( g_1(E_G) = g_2(E_G) \). This implies

\[
\beta(g_1(E_G)) \subseteq \beta(g_2(E_G)).
\]

Since \( g_1(E_G) \) is group acted on \( \beta \)-open in \( Y \)

\[
\beta(g_1(E_G)) = g_1(E_G).
\]

Therefore \( \beta(g_1(E_G)) = g_1(E_G) \) for every subset \( E_G \) of \( X \).

Example 4.9. Let

\[
X = Y = \{1, 2, 3\},
\]

\[
\tau_1 = \{\emptyset, \{1\}, \{1, 2\}, X\},
\]

\[
\tau_2 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, Y\}.
\]

Let \( g_1 : X \rightarrow Y \) be the identity map then \( \beta(g_1(E_G)) = g_1(E_G) \), for every subset \( E_G \) of \( X \). However \( g_1 \) is group acted on not \( \text{CSO} \), since \( g_1(\{1, 2\}) = \{1, 2\} \) is not \( \beta \)-closed.

Theorem 4.10. Let \( X, Y, Z \) be the topological group and the group acting on each \( \beta \)-closed set is \( r \)-open in \( Y \) then the group acting on \( \text{AC} \) is composed of two \( \text{AC} \) maps.

Proof. Let \( g_1 : X \rightarrow Y \) and \( h_1 : Y \rightarrow Z \) be groups acting on a \( \text{AC} \) map. Let \( E_G \) be any \( r \)-open set in \( X \Rightarrow g_1(E_G) \) is \( \beta \)-closed in \( Y \Rightarrow g_2(E_G) \) is group acting on \( r \)-open in \( Y \). It means that \( h_1(g_1(E_G)) = h_1 \circ g_1(E_G) \) is \( \beta \)-closed. Therefore \( h_1 \circ g_1 \) is group acting on \( \text{AC} \).

Let \( g_1 : X \rightarrow Y \), \( h_1 : Y \rightarrow Z \) be group acting on \( \text{AC} \) map. Let \( E_G \) be any \( r \)-open set in \( X \Rightarrow g_1(E_G) \) is \( r \)-closed in \( Y \Rightarrow g_1(E_G) \) is \( \beta \)-closed in \( Y \Rightarrow g_1(E_G) \) is group acting on \( r \)-open in \( Y \). It implies that \( h_1(g_1(E_G)) \) in \( Z \) is \( r \)-closed and in \( Z \) \( h_1 \circ g_1(E_G) \) is \( \beta \)-closed. Therefore \( h_1 \circ g_1 \) is group acting on \( \text{AC} \).

Theorem 4.11. If \( g_1 : X \rightarrow Y \) is group acted on \( \text{AC} \) and \( E_G \) is an open set of \( X \) then \( g_{E_G} : (X, \tau_1(E_G)) \rightarrow (Y, \tau_2) \) is group acted on \( \text{AC} \).

Proof. Let \( R \) be an open group acted on \( r \)-open set in \( E_G \). Then \( R = E_G \cap T_G \) for some open set \( T_G \) of \( X \) and so \( R \) is open in \( X \Rightarrow g_1(R) \) is group acted on \( \beta \) closed in \( Y \). But \( g_1(R) = g_{E_G}(R) \). Therefore \( g_{E_G} \) is group acted on \( \text{AC} \).

5. Conclusion

We introduced different functions related to \( \text{CSO} \) map and \( \text{AC} \) map in this paper and analysed their properties, concepts and results available in the literature. Moreover, we introduce the concepts of group acted to the topological spaces’ \( \text{CSO} \) map, \( \text{AC} \) map. Further analysis of many topics in topological space group behaviour.

References


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