Malaya Journal of Matematik, Vol. 8, No. 4, 2113-2115, 2020
https://doi.org/10.26637/MJM0804/0134

Group action on contra semi open map and almost contra β-open mapping

S. Sivakumar[1] and T. Ramanathan[2*]

Abstract
The purpose of this paper is to introduce and research the concept of contra semi open map and almost contra β-open mapping group action.

Keywords
Group acting on open set, open map, contra semi open map and almost contra β open mapping with topological spaces.

AMS Subject Classification
54H11, 55-06, 55R35.

[1,2] *PG and Research Department of Mathematics, Government Arts College, C. Mutlur, Chidambaram, Tamil Nadu, India.*
***Corresponding author**: [1]ssk.2012@yahoo.in; [2]trmaths2018@gmail.com
Article History: Received **18** September **2020**; Accepted **21** November **2020**

Contents

1. Introduction

In 1963 N. Levine and in 1969 N. Biswas have respectively defined and studied the notions called semi open sets and semi closed sets in topology. In the years 1969 and 1970, N. Biswas had defined and studied the notions of semiopen and semi-closed functions respectively. Contra-continuous functions were introduced by Julian Dontchev as functions having the property that inverse images of open sets are closed. In 1997 C.W. Baker has defined and studied the notions of contra open and contra closed functions as dual generations of contra-continuous functions. In the study of modern mathematics, mapping plays an important role, especially in topology and functional analysis.

2. Preliminaries

Definition 2.1. *If $A_G \subseteq X_\tau$. It is said that it is*

(i) pre open if $A_G \subseteq \operatorname{int}(\operatorname{cl}(A_G))$

(ii) semi-open if $A_G \subseteq cl(\operatorname{int}(A_G))$

(iii) regular open if $A_G = \operatorname{int}(\operatorname{cl}(A_G))$

(iv) α -open if $A_G \subseteq \operatorname{int}(\operatorname{cl}(\operatorname{int}(A_G)))$

(v) β-open if $A_G \subseteq cl(\operatorname{int}(\operatorname{cl}(A_G)))$.

The complement of a pre open, semi open, α-open, semi-pre open (β-open) set is said to be pre closed, semi closed, α-closed, semi-pre closed ($\beta-$closed).

The collection of all closed, pre open, semi open, α-open, semi pre open (β-open) subset of X will be denote by $C(X_\tau), \mathrm{PO}(X_\tau), \mathrm{SO}(X_\tau), \alpha(X_\tau), SPO(X), (\beta(X_\tau)$.

Definition 2.2. *The Intersection of every one of semi closed sets of X_τ contain $A_G \subseteq X_\tau$ is called the semi closer of A_G is denote by $Scl(A_G)$.*

Definition 2.3. *A function $g_\tau : X_\tau \to Y_\tau$ is called the pre open, semi open, α-open but $g_\tau(U)$ is pre open, semi open, α-open in Y_τ for each open set U in X_τ.*

Definition 2.4. *A function $g_\tau : X_\tau \to Y_\tau$ is called the pre closed, semi closed, α-closed if $g(R)$ is pre closed, semi closed, α-closed in Y_τ for each closed set R in X_τ.*

3. Group action on contra semi open

Definition 3.1. *A function $g_\tau : X_\tau \to Y_\tau$ is called the group action on contra semi open if the image of open subset of X_τ is group acting on semi closed set in Y_τ. Clearly each contra*

semi open function is contra semi pre open. Since each semi closed set is semi pre closed.
In this paper short form of contra semi open, contra semi pre open Almost contra semi pre open, Contra alpha open, semi closed, alpha closed set are CSO, $C\beta O$, $AC\beta O$, $C\alpha O$, SC, αC.

Theorem 3.2. *For a function of $g_\tau : X_\tau \to Y_\tau$, the following statements are equivalent:*

(i) *g_τ is group acted on CSO.*

(ii) *For every subset R of Y_τ and for every closed set G of X_τ with $g_\tau^{-1}(R) \subseteq G$ there exists a SO set $\mathscr{T}$ of Y_τ with $R \subseteq \mathscr{T}$ and $g_\tau^{-1}(\mathscr{T}) \subseteq G$.*

(iii) *For every $y \in Y_\tau$ and for each every closed subset G of X_τ with $g_\tau^{-1}(\mathscr{Y}) \subseteq G$ there exists a SO subset $\mathscr{T}$ of $\mathscr{Y}$ with $\mathscr{Y} \in \mathscr{T}$ and $g^{-1}(\mathscr{T}) \subseteq G$.*

Proof. $(i) \Rightarrow (ii)$
Let $R \subseteq Y_\tau$ and G be a closed subset of X_τ with $g_\tau^{-1}(R) \subseteq G$. Since g is group acted on CSO $g_\tau(X_\tau - G)$ is semi closed. If we set $\mathscr{T} = Y_\tau - g_\tau(X_\tau - G)$ is can be easily seen that $R \subseteq \mathscr{T}$ and $g^{-1}(\mathscr{T}) \subseteq G$.
$(ii) \Rightarrow (iii)$
Let $y \in Y_\tau$ and G be a closed subset of X_τ with $g^{-1}(\mathscr{Y}) \subseteq G$ since group acted on g_τ is $CSO g_\tau(X_\tau - G)$ is semi closed. If we get $\mathscr{T} = Y_\tau - g_\tau(X_\tau - G)$ it can be easily seen that $\mathscr{Y} \subset \mathscr{T}$ and ${g_\tau}^{-1}(\mathscr{T}) \subset G$.
$(ii) \Rightarrow (iii)$
Let E be group acted an open subset of X_τ Let $\mathscr{Y} \in Y_\tau - g_\tau(E)$ and $G = X_\tau - E$. Then by (iii) there exists a semi subset $\mathscr{T}$ of Y_τ with $Y_\tau \varepsilon \mathscr{T}, g_\tau^{-1}(\mathscr{T}) \subset G$ then $Y_\tau \varepsilon \mathscr{T} \subset Y_\tau - g_\tau(E)$ and hence $g_\tau(E)$ is semi closed and therefore g_τ is group action on contra semi open function. □

Definition 3.3. *A function $g_\tau : X_\tau \to Y_\tau$ is called the group acted on CSO (group acted on CSO, $C\alpha O$) if the image of each open subset of X_τ is group acted on closed $(SC, \alpha - C)$ subset of Y_τ.*

Theorem 3.4. *If a function $g_\tau : X_\tau \to Y_\tau$ is group acted on CSO, X_τ is regular then g_τ is group acted on semi open function.*

Proof. Let Y_τ be a random point of Y_τ and T be a open set of X_τ contain $g_\tau^{-1}(\mathscr{Y})$. Since X_τ is regular to hand exists an open set S in X_τ containing $g_\tau^{-1}(\mathscr{Y})$ such that $\mathrm{cl}(S)\varepsilon T$. Since g_τ is group acted on CSO, there exists $E\varepsilon SO(X_\tau, X)$, such that $g_\tau(E) \subseteq cl(S)$. Then $g_\tau(E) \subset T$ This is show that g_τ is group acted on SO function. □

Theorem 3.5. *Let a function $g_\tau : X_\tau \to Y_\tau$ is group acted on contra semi open and $h_\tau : Y_\tau \to Z_\tau$ is the semi closed function then $h_\tau \circ g_\tau : X_\tau \to Z_\tau$ is group acted on contra semi open.*

Proof. Let $T \subset X_\tau$ be an group acted on open set. Since g_τ is group acted on contra semi open function. $g_\tau(T)$ is SC set in Y_τ again h_τ is pre SC semi closed function. So $h_\tau(g_\tau(T)) = h_\tau \circ g_\tau(T)$ is semi closed set in Z_τ. Therefore $h_\tau \circ g_\tau$ is group acted on CSO function. □

4. Group action on $AC\beta O$ mapping

Definition 4.1. *A function $g_\tau : X_\tau \to Y_\tau$ is called the group acting on $AC\beta O$ if the image of each group acted on r egular open set in X_τ is β-closed in Y_τ.*

Example 4.2. *Let*

$$X_\tau = Y_\tau = \{1,2,3\},$$
$$\tau_1 = \{\emptyset, \{1\}, \{2\}, \{1,2\}, X\}$$
$$\tau_2 = \{\emptyset, \{1\}, \{2\}, Y\}.$$

Let $g_\tau : X_\tau \to Y_\tau$ be defined $g_\tau(1) = 2, g_\tau(2) = 3, g_\tau(3) = 1$. Then g_τ is group acted on $AC\beta O$, $C\beta O$.

Example 4.3. *Let*

$$X_\tau = Y_\tau = \{1,2,3\},$$
$$\tau_1 = \{\emptyset, \{1\}, \{2\}, \{1,2\}, X\}$$
$$\tau_2 = \{\emptyset, \{1\}, \{1,2\}, Y\}.$$

Let $g_\tau : X_\tau \to Y_\tau$ be defined $g_\tau(1) = 1, g_\tau(2) = 3, g_\tau(3) = 2$. Then g_τ is group acted on not $AC\beta O$, $C\beta O$.

Theorem 4.4. *Let G is a topological group then group acted on all $C\beta O$ map is $AC\beta O$ but conversely is not true.*

Proof. Let $E_G \subseteq X_\tau$ be regular open implies that $g_\tau(E_G)$ is group acted on β in Y. Since $g_\tau : X_\tau \to Y_\tau$ is $C\beta O$. Therefore g_τ is group acted on $C\beta O$. □

Theorem 4.5. *If $g_\tau : X_\tau \to Y_\tau$ is group acted on $AC\beta O$ and $E_G \subseteq X_\tau$ is open $g_\tau(E_G)$ is τ_β closed in Y_τ.*

Proof. Let $E_G \subseteq X_\tau$ and $g_\tau : X_\tau \to Y_\tau$ exist group acted on $AC\beta O$, implies that $\beta(g_\tau(E_G)) \subseteq g_\tau(\overline{E_G})$, implies that

$$\beta(\overline{g_\tau}(E_G)) \subseteq g_\tau(E_G).$$

Since, $g_\tau(E_G) = g_\tau(\overline{E_G})$ like E_G is open. However $g_\tau(E_G) \subseteq \beta\left(\overline{g_\tau(E_G)}\right)$, we have $g_\tau(E_G) \subseteq \beta\left(\overline{g_\tau(E_G)}\right)$. Therefore $g_\tau(E_G)$ is τ_β closed in Y_τ. □

Theorem 4.6. *Let G is a topology Group if g is open $(r\mathrm{-}open)$ and g_1 is group acted on $C\beta O$, then $g_1 \circ g_2$ is group acted on $AC\beta O$.*

Proof. Let $R_G \subseteq X_\tau$ be r-open implies $g_2(R_G)$ is open $(r\mathrm{-}open)$ in Y_τ implies that $g_1(g_2(R_G)) = g_1 \circ g_2(R_G)$ is β closed in G. Hence $g_1 \circ g_2$ is group acted on $AC\beta O$. □

Theorem 4.7. *If g_2 is group acted on ACO or [group acted on CrO], g_1 is group acted on β closed then $g_1 \circ g_2$ is group acted on almost contra β open.*

Proof. Let $E_G \subseteq X_\tau$ be group acted on r-open in $X_\tau \Rightarrow g_2(E_G)$ is closed $[r-$ closed $]$ in

$$Y_\tau \Rightarrow g_1(g_2(E_G)) = g_1 \circ g_2(E_G),$$

is group acted on β closed in G. Hence $g_1 \circ g_2$ is group acted on almost contra β open. □

Theorem 4.8. *If $g_\tau : X_\tau \to Y_\tau$ group acted on almost contra β open then $\beta\left(\overline{g_\tau(E_G)}\right) \subseteq g_\tau(E_G)$.*

Proof. Let $E_G \subseteq X_\tau, g_\tau : X_\tau \to Y_\tau$ be group acted on almost contra β open then $g_\tau\left(\overline{E_G}\right)$ is β closed in Y_τ and $g_\tau(E_G) = g_\tau\left(\overline{E_G}\right)$. This implies

$$\beta\left(\overline{g_\tau(E_G)}\right) \subseteq \beta\left(\overline{g_\tau\left(\overline{E_G}\right)}\right). \tag{4.1}$$

Since $g_\tau\left(\overline{E_G}\right)$ is group acted on β open in Y_τ

$$\beta\left(\overline{g_\tau\left(\overline{E_G}\right)}\right) = g_\tau\left(\overline{E_G}\right). \tag{4.2}$$

Therefore $\beta\left(\overline{g_\tau(E_G)}\right) = g_\tau\left(\overline{E_G}\right)$ for every subset E_G of X_τ. □

Example 4.9. *Let*

$$\begin{aligned} X_\tau =& Y_\tau = \{1,2,3\} \\ \tau_1 =& \{\emptyset, \{1\}, \{1,2\}, X\} \\ \tau_2 =& \{\emptyset, \{1\}, \{2\}, \{1,2\}, Y\}. \end{aligned}$$

Let $g_\tau : X_\tau \to Y_\tau$ be the identity map then $\beta\left(\overline{g_\tau(E_G)}\right) = g_\tau\left(\overline{E_G}\right)$, for every subset E_G of X_τ. However g_τ is group acted on not $C\beta O$, since $g_\tau(\{1,2\}) = \{1,2\}$ is not β closed.

Theorem 4.10. *Let X_τ, Y_τ, Z_τ be the topological group and the group acting on each β closed set is r-open in Y_τ Then the group acting on $AC\beta O$ is composed of two $AC\beta O(ACrO)$ maps.*

Proof. Let $g_\tau : X_\tau \to Y_\tau$ and $h_\tau : Y_\tau \to Z_\tau$ be groups acting on a $AC\beta O$ map. Let E_G be any r-open set in $X_\tau \Rightarrow g_\tau(E_G)$ is β closed in $Y_\tau \Rightarrow g_\tau(E_G)$ is group acting on r-open in Y_τ. It means that $h_\tau(g_\tau(E_G)) = h_\tau \circ g_\tau(E_G)$ in Z is β-closed. Therefore $h_\tau \circ g_\tau$ is group acting on $AC\beta O$.

Let $g_\tau : X_\tau \to Y_\tau, \quad h_\tau : Y_\tau \to Z_\tau$ be group acting on $AC\beta O$ map. Let E_G be any r-open set in $X_\tau \Rightarrow g_\tau(E_G)$ is r-closed in $Y_\tau \Rightarrow g_\tau(E_G)$ is group acting on β-closed in $Y_\tau \Rightarrow g_\tau(E_G)$ is group acting on r-open in Y_τ. It implies that $h_\tau(g_\tau(E_G))$ in Z_τ is r-closed and in $Z_\tau h_\tau \circ g_\tau(E_G)$ is β-closed. Therefore $h_\tau \circ g_\tau$ is group acting on $AC\beta O$. □

Theorem 4.11. *If $g_\tau : X_\tau \to Y_\tau$ is group acted on $AC\beta O$ and E_G is an open set of X_τ then $g_{E_G} : (X_\tau, \tau_1(E_G)) \to (Y_\tau, \tau_2)$ is group acted on $AC\beta O$.*

Proof. Let R be an group acted on r-open set in E_G. Then $R = E_G \cap T_G$ for some open set T_G of X_τ and so R is open in $X_\tau \Rightarrow g_\tau(R)$ is group acted on β closed in Y_τ But $g_\tau(R) = g_{E_G}(R)$. Therefore g_{E_G} is group acted on $AC\beta O$. □

5. Conclusion

We introduced different functions related to CSO map and $AC\beta O$ map in this paper and analysed their properties, concepts and results available in the literature. Moreover, we introduce the concepts of group acts related to the topological spaces' CSO map, $AC\beta O$ map. Further analysis of many topics in topological space group behaviour.

References

[1] D. Andrijevic, semi-pre open sets, *Mat. vesnik*, 1(38)(1986), 24-32.

[2] C. W. Baker, Contra-open Functions and Contra-closed Functions, *Mathematics Today*, 15(1997), 19-24.

[3] S. Balasubramanian, C. Sandhya, Almost contra β open and almost contra β closed Mappings, *IJESRT*, 2(4)(2013), 811-819.

[4] N. Biswas, On some mappings in topological spaces, *Bull. Calcuta Math. Soc,* 61(1969), 127-135.

[5] J. Dontchev, Contra-continuous Functions and Strongly S-closed Space, *Internat J. Mat. Math and Math Sci,* 19(1996), 303-310.

[6] N. Levine, Semi-open sets and semi-continuty in topological spaces, *Amer. Math,* 70(1963), 36-41.

[7] G. B. Navalagi, Semi-pre neighbourhoods and generalized semi-preregular closed sets in topological spaces, *Topology Atlas*, Preprint 455.

[8] G. B. Navalagi, Semi pre continuous functions and properties of Generalized semi pre closed sets in Topological spaces, *IJMMS*, 29(2)(2002), 85-98.

[9] S. Sivakumar, P. Palanichamy, On orbit space of b-compact Group action, *Acta cieneia India,* 4(813)(2011).

[10] S. Sivakumar, T. Ramanathan, Group action on Semi-pre open set and functions, *Journal of Xi'an University of Architecture and Technology*, XII(IV)(2020), 5749-5754.

[11] S. Sivakumar, T. Ramanathan, Group action of some Topological spaces Via supra B-open and supra β-open sets, *Aegaeum Journal*, 8(6)(2020), 201-208.

⋆⋆⋆⋆⋆⋆⋆⋆⋆

ISSN(P):2319 − 3786

Malaya Journal of Matematik

ISSN(O):2321 − 5666

⋆⋆⋆⋆⋆⋆⋆⋆⋆