Application of intuitionistic multi-fuzzy set in crop selection

R. Muthuraj1* and S. Yamuna2

Abstract
In this paper, the information carried by the membership degree and the non-membership degree in Atanassov's intuitionistic fuzzy sets (IFSs) as a vector representation with the three elements are considered. To illustrate the efficiency of the proposed cosine similarity measure for fuzzy sets and the cosine similarity measures are applied to crops selection in agriculture.

Keywords
Fuzzy Set, intuitionistic Fuzzy set (IFS) Intuitionistic multi-fuzzy set (IMFS), Cosine Similarity Measures.

1. Introduction
Atanassov K.T [1, 2] proposed the Intuitionistic Fuzzy sets (IFS) as the generalization of the Fuzzy set (FS) introduced by L.A. Zadeh [31]. The Fuzzy set allows the object to partially belong to a set with a membership degree ($\mu$) between 0 and 1 whereas IFSs represent the uncertainty with respect to both membership ($\mu \in [0, 1]$) and non membership ($\vartheta \in [0, 1]$) such that $\mu + \vartheta \leq 1$. The number $\pi = 1 - \mu - \vartheta$ is called the hesitation degree or intuitionistic index.

The Multi set [4] allows the repeated occurrences of any element and hence the Fuzzy Multi set (FMS) can occur more than once with the possibly of the same or the different membership values was introduced by R. R. Yager [29]. Recently, the new concept Intuitionistic Fuzzy Multi sets (IFMS) was proposed by T. K. Shinoj and Sunil Jacob John [24],[25].

The study of distance and similarity measure of IFSs gives lots of measures, each representing specific properties and behavior in real-life decision making and pattern recognition works. For measuring the degree of similarity between vague sets, Chen and Tan [7] proposed two similarity measures. The Hamming, Euclidean distance and similarity measures were introduced by Szmidt and Kacprzyk [26], [27], [28]. Using the Cotangent function, a new similarity measure was proposed by Lian & Shi[8] Wang et al [13]. Later a new fuzzy cotangent similarity measure for IFSs was introduced by Tian Maoying[14].

As the extension of the distance and similarity measure of IFSs to IFMSs [12], [14],[15], [16] are possible; In this paper we extend the fuzzy cotangent similarity measure of IFSs to IFMSs. The numerical results of the examples show that the developed similarity measures are well suited to use any linguistic variables.

2. Preliminaries

Definition 2.1 ([31]). A fuzzy set $A$ drawn from a non-empty set $Y$ is defined as $A = \{ y, \mu(y) / y \in Y \}$, where $\mu(y) : Y \rightarrow [0, 1]$ is the membership function of the fuzzy set $A$. Fuzzy set is a collection of objects with graded membership. The generalization of fuzzy sets are the Intuitionistic fuzzy set (IFS) which was proposed by Atanassov [1,2] with independent memberships and nonmemberships.

Definition 2.2 ([1,2,3]). An Intuitionistic Fuzzy Set (IFS) $A$ of a nonempty set $Y$ is an object of the form $A = \{ y, \mu(y), \vartheta(y) : y \in Y \}$, where $\mu : Y \rightarrow [0,1]$ and $\vartheta : Y \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the
element $y \in Y$ respectively with $0 \leq \mu(y) + \theta(y) \leq 1$, $\forall y \in Y$.

Furthermore, we have $\pi(y) = (1 - \mu(y) + \theta(y))$ is called the index or hesitation margin of $y$ in IFMS $A$. $\pi(y)$ is the degree of indeterminacy of $y \in Y$ to the IFMS $A$ and $\pi(y) \in [0, 1]$. That is, $\pi: Y \to [0, 1]$ and $0 \leq \pi(y) \leq 1$, $\forall y \in X$. $\pi(y)$ expresses the lack of knowledge of whether $y$ belongs to IFMS $A$ or not.

For example, let $A$ be an IFMS with $\mu(y) = 0.3$ and $\theta(y) = 0.4$ which implies that $\pi(y) = (1 - 0.3 - 0.4) = 0.3$. It can be interpreted as the degree that the object $y$ belongs to IFMS $A$ is 0.3, the degree that the object $y$ does not belong to IFMS $A$ is 0.4 and the degree of hesitancy is 0.3.

**Definition 2.3** ([9,13,14,17]). A Multi-Fuzzy Set (MFS) $A$ of a non-empty set $Y$ is defined as $A = \{ (y, \mu_A(y)) : y \in Y \}$ where $\mu_A(y) = (\mu_1(y), \mu_2(y), \ldots, \mu_k(y))$ and $\mu_i: Y \to [0, 1]$, $\forall i = 1, 2, \ldots, k$. Here $k$ is the finite dimension of $A$. Also note that, for all $i$, $\mu_i(y)$ is a decreasingly ordered sequence of elements. That is, $\mu_i(y) \geq \mu_{i+1}(y) \geq \cdots \geq \mu_k(y), \forall y \in Y$.

**Definition 2.4** ([18,19,20]). Let $A = \{ (y, \mu_A(y), \theta_A(y)) : y \in Y \}$ where $\mu_A(y) = (\mu_1(y), \mu_2(y), \ldots, \mu_k(y))$ and $\theta_A(y) = (\theta_1(y), \theta_2(y), \ldots, \theta_k(y))$ such that $0 \leq \mu_i(y) + \theta_i(y) \leq 1$, for all $i$, $\forall y \in Y$. Also for each $i = 1, 2, \ldots, k$, $\mu_i: Y \to [0, 1]$ and $\theta_i: Y \to [0, 1]$. Here, $\mu_i(y) \geq \mu_{i+1}(y) \geq \cdots \geq \mu_k(y), \forall y \in Y$. That is $\mu_i$‘s are decreasingly ordered sequence. That is, $0 \leq \mu_i(y) + \theta_i(y) \leq 1, \forall y \in Y$ for all $i = 1, 2, \ldots, k$. Then the set $A$ is said to be an Intuitionistic Multi-Fuzzy Set (IMFS) with dimension $k$ of $Y$.

**Definition 2.5** ([21,22,23]). The cardinality of the membership function $M(y)$ and the non membership function $NM(y)$ is the length of an element $y$ in an IMFS denoted as $\eta$, defined as $\eta = |M(y)| = |NM(y)|$.

If $A, B, C$ are the IFM S defined on $X$, then their cardinality $\eta = \max\{\eta(A), \eta(B), \eta(C)\}$.

**Definition 2.6** ([22,23,30]). $S(A, B)$ is said to be the similarity measure between $A$ and $B$, where $A, B \in X$ and $X$ is an IFMS, as $S(A, B)$ satisfies the following properties

1. $S(A, B) \in [0, 1]$
2. $S(A, B) = 1$ if and only if $A = B$
3. $S(A, B) = S(B, A)$
4. If $A \subseteq B \subseteq C \subseteq X$, then $S(A, C) \leq S(A, B)$ and $S(A, B) \leq S(B, C)$
5. $S(A, B) = 0$ if and only if $A = \phi$ and $B = \bar{A}$ or $A = B$ and $B = \phi$.

### 3. Proposed cosine similarity measures of IFMSs

**Definition 3.1.** The similarity measure for IMFS based on cosine function with two parameters membership and non-

The new similarity measure for IMFS based on cosine function with three parameters membership, non-membership and hesitation function is

$$CS_{IMFS}(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left[ \frac{1}{k} \sum_{i=1}^{k} \frac{\mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j) + \mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j)}{\sqrt{\mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j) + \mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j)}} \right]$$

**Proposition 3.2.** The defined new similarity measure $CS_{IFMS}(A, B)$ between IFMS $A$ and $B$ satisfies the following properties

1. **P1** $0 \leq CS_{IFMS}(A, B) \leq 1$
2. **P2** $CS_{IFMS}(A, B) = 1$ if and only if $A = B$
3. **P3** $CS_{IFMS}(A, B) = CS_{IFMS}(B, A)$
4. **P4** If $A \subseteq B \subseteq C$, then $CS_{IFMS}(A, C) \leq CS_{IFMS}(A, B)$ and $CS_{IFMS}(A, C) \leq CS_{IFMS}(B, C)$.

**Proof.** **P1** $0 \leq CS_{IFMS}(A, B) \leq 1$.

Since the values of membership, non-membership and hesitation functions of the intuitionistic fuzzy multiset are lying in the interval $[0, 1]$, the similarity measure based on cosine function $CS_{IFMS}(A, B)$ is lying between 0 and 1.

**P2** $CS_{IFMS}(A, B) = 1$ if and only if $A = B$.

(i) If the two IFMS $A$ and $B$ are equal, then $\mu_i^2(x_j) = \mu_i^2(x_j)$ and $\mu_i^2(x_j) = \mu_i^2(x_j)$, $j$, which implies that

$$|\mu_i^2(x_j) - \mu_i^2(x_j)| = 0, \quad |\theta_i^2(x_j) - \theta_i^2(x_j)| = 0,$$

$$|\Pi_i^2(x_j) - \Pi_i^2(x_j)| = 0.$$

Hence $CS_{IFMS}(A, B) = 1$.

(ii) Let $CS_{IFMS}(A, B) = 1$. Then $|\mu_i^2(x_j) - \mu_i^2(x_j)| = 0$

$$|\theta_i^2(x_j) - \theta_i^2(x_j)| = 0, \quad |\Pi_i^2(x_j) - \Pi_i^2(x_j)| = 0.$$  This implies that $\mu_i^2(x_j) = \mu_i^2(x_j), \theta_i^2(x_j) = \theta_i^2(x_j)$ and $\Pi_i^2(x_j) = \Pi_i^2(x_j)$ for all $j$ Thus $A = B$.

**P3.** $CS_{IFMS}(A, B) = CS_{IFMS}(B, A)$

$$CS_{IFMS}(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left[ \frac{1}{k} \sum_{i=1}^{k} \frac{\mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j) + \mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j)}{\sqrt{\mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j) + \mu_i^2(x_j) + \theta_i^2(x_j) + \pi_i^2(x_j)}} \right]$$
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Table 1. Crops vs soil features

<table>
<thead>
<tr>
<th>TOPOGRAPHY</th>
<th>CLIMATE</th>
<th>CHEMICAL PROPERTIES</th>
<th>PHYSICAL PROPERTIES</th>
<th>BIOTIC PROPERTIES</th>
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<tr>
<td>C1</td>
<td>(0.3,0.6,0.1)</td>
<td>(0.1,0.7,0.2)</td>
<td>(0.8,0.1,0.1)</td>
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<td>(0.8,0.2,0)</td>
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<tr>
<td>C4</td>
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<td>(0.5,0,4,0,1)</td>
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<td>(0.3,0,6,0,1)</td>
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</table>

Table 2. Soil features vs crop selection factors

<table>
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<tr>
<th>TOPOGRAPHY</th>
<th>PROFITABILITY</th>
<th>MARKETABILITY</th>
<th>TECHNOLOGY</th>
<th>SECURITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.7,0,3,0)</td>
<td>(0.3,0,4,0,3)</td>
<td>(0.8,0,2,0)</td>
<td>(0.2,0,6,0,2)</td>
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<tr>
<td>CLIMATE</td>
<td>(0.6,0,2,0,2)</td>
<td>(0.2,0,8,0)</td>
<td>(0.6,0,4,0)</td>
<td>(0.3,0,6,0,1)</td>
</tr>
<tr>
<td>CHEMICAL PROPERTIES</td>
<td>(0.1,0,7,0,2)</td>
<td>(0.7,0,2,0,1)</td>
<td>(0.3,0,6,0,1)</td>
<td>(0.3,0,7,0)</td>
</tr>
<tr>
<td>BIOTIC PROPERTIES</td>
<td>(0.5,0,2,0,3)</td>
<td>(0.5,0,2,0,3)</td>
<td>(0.5,0,3,0,2)</td>
<td>(0.3,0,7,0)</td>
</tr>
</tbody>
</table>

P4 If \( A \subseteq B \subseteq C \), then, \( CS_{FMS}(A,C) \leq CS_{FMS}(A,B) \) and \( CS_{FMS}(A,C) \leq CS_{FMS}(B,C) \).

4. IFMS in crops selection in agriculture

Harvest choice is one of the principle issues looked by ranchers and examination into farming as a result of the vulnerabilities in different elements, for example, ecological conditions, nature of yields and so forth. IFMS hypothesis acquainted an effective demonstrating strategy with handle vulnerability. Notwithstanding the reason for cultivating and it is imperative to choose a yield and assortment with wide protection from significant vermin and infections. The utilization of helpless assortments may result to significant expense of creation or, in the case of any component. Different components ought to be considered in harvest choice. This is an essential that should be attempted before really beginning a cultivating adventure. Indeed, even without a foreordained area and site of a homestead, the harvest can be developed and chosen by them attractiveness and benefit. Nonetheless, there are numerous cases particularly in nations with horticulture based economy in which the homestead parcel is as of now accessible. It might have been gained through legacy, or by buy, or in any case moved through different methods. At all, harvest and varietal choice is the primary thought in beginning or building up the ranch. Right choice in the choice of yield or harvests to be developed, especially enduring sorts, will eventually change over into an effective cultivating adventure.

As of late, the new idea Intuitionistic Fuzzy Multi sets (IFMS) was proposed by T.K. Shinoj and Sunil Jacob John. Horticulture envelops different strengths like soil and seed the executives, water and water system and so on. The issues associated with these zones are mind boggling in view of numerous elements, for example, atmosphere conditions, area and so forth. Thus, as the intricacy expands the vulnerabilities engaged with these territories likewise increment. A few sorts of models can deal with vulnerabilities. Probably the most well known vulnerability based models that are moving presently are fluffy sets, harsh sets and delicate sets. Fluffy
sets introduced by Zadeh in 1965 have been seen as prevalent in taking care of vulnerability, and have been broadly utilized, in actuality, applications. In any case, an issue with fluffy sets is that there is no exceptional method to characterize the participation work. A few uses of fluffy rationale in agribusiness are talked about by Roseline et al., remembering use for bug the board, examination of soil, and building up a specialist framework for different yields (Rosaline, 2009). A few uses of unpleasant set models are talked about in (Jianping, 2009). These model sorts, nonetheless, need definition instruments. Here we present IFMS a primary An example of crop selection via IFMS is presented.

Let $C = \text{rice, ragi, jowar, wheat}$ be a set of crops

$D = \text{profitability, marketability, technology, security}$ be a set of factors influencing crop selection, and $S$ be the soil features such as

$S = \text{topography, climate, chemical properties, biotic properties, physical properties.}$

Now, by considering only one factor with the soil nature, we start a crop production. It is not possible to get a profit with immediate results with one result. There may be different results for different crops with soil features. Now we analyses the situation of each crops, we give in $D$.

Let us take 3 different crops and their soil features in 3 different times in a month. Now the details are as follows.

Table 1 shows that, each soil feature $S_i$ is given by three numbers: membership $\mu$, non-membership $v$ and hesitation margin $\pi$.

The goal is to identify the right crop which suits with the soil to get the maximum profit with minimum investment in calculated time. Let the crops growth be taken at three different times in a month. For every 10 days in a month.

Here the distance calculates the distance of each crop $C_i$ with the soil features $S_i$ for each factor influencing crop selection $d_k: k = 1, 2, 3, 4$. Now the first set represents the membership values obtained at three different times in a month (10 days/30 days). Now the first set represents the membership values obtained at three different intervals in a month. The second and third sets represent the related non-membership and hesitation margin.

From Table 2 and Table 3 the least distance point gives the appropriate identification of the crops. Crop C1 needs more from security and C2 needs more marketability, C3 needs more security and C4 needs marketability.

<table>
<thead>
<tr>
<th>C1</th>
<th>PROFITABILITY</th>
<th>MARKETABILITY</th>
<th>TECHNOLOGY</th>
<th>SECURITY</th>
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<td>0.76737</td>
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<td>C2</td>
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<tr>
<td>C3</td>
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<td>C4</td>
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<td>0.75383</td>
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</tbody>
</table>

### 5. Conclusion

In this paper we have introduced a mathematical technique intuitionistic fuzzy Multiset and we have analyzed the various operations and possibilities for optimum crop production in a limited time period. This paper finds an application of IFMS in agriculture. In the proposed method we have measured the distance of each crops from factors influencing the crops by considering the particular soil features with respective crops. The concept of Multiset is developed by taking the sample of different crops at different time intervals with the soil features.

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