An application of circulant fuzzy matrices for finding the disease in animals

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Abstract
In this paper focuses its attention on circulant fuzzy matrices combined with medical diagnosis for animal diseases and a new method to find the animal diseases of circulant fuzzy matrix.

Keywords
Fuzzy matrices, disease in animals, medical diagnosis.

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1. Introduction
Fuzzylogic and fuzzy set theory are important role for developing mathematical knowledge based on systematic analysis for finding medicine in various animal disease. Fuzzy matrices deal with different type of complicated disease in animal. Sanchez formulated in diagnostic method involves fuzzy matrices represented in medical knowledge between symptoms and disease in [2,3].

In [4] Diagnosis knowledge of animal disease is fuzzy matrix and the developing to representing and reasoning model of animal diagnosis knowledge based on fuzzy set theory can reflects the nature of the medical diagnosis to same expendable.

I extend Sanchez’s method [2,3] animal disease diagnosis used to the notion of circulant matrix theory and notion of circulant and their applications are explained in [1], kim and rough. The method of circulant medical diagnosis involves circulant ordered explained.

2. Preliminaries

2.1 Circulant matrix
An $m \times n$ circulant matrix has the form

$$
C = \begin{pmatrix}
C_1 & C_2 & C_{n-1} & \cdots & C_n \\
C_n & \bar{Q} & C_2 & \cdots & C_{n-1} \\
C_{n-1} & C_n & \bar{Q} & \cdots & C_{n-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_2 & C_3 & C_4 & \cdots & C_1
\end{pmatrix}
$$

2.2 Circulant fuzzy matrix
For any given $C_0$, $C_2$, \ldots, $C_{n-1} \in f_m \times n$ be the circulant fuzzy matrix $C = (c_{ij})_{m \times n}$ is defined by $(c_{ij}) = (C_{i-j}(mod n))$
A circulant matrix is the form of
\[
C_1 = \begin{pmatrix}
C_1 & C_2 & C_{n-1} & \cdots & C_n \\
C_n & C_1 & C_2 & \cdots & C_{n-1} \\
C_{n-1} & C_n & C_1 & \cdots & C_{n-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_2 & C_3 & C_4 & \cdots & C_1 
\end{pmatrix}
\]
With entries in [0,1]

2.3 Complement of circulant fuzzy matrix
Let \( C = (C_{ij})_{max} \) be a fuzzy matrix of order \( m \times n \) then the complement of \( C \) is denoted by \( C^c = (d_{ij}) \) where \( d_{ij} = 1 - c_{ij} \) for all \( i \) and \( j \).

2.4 Product of circulant fuzzy matrices
Let \( f_{mn} \) denote the set of all \( m \times n \) matrices over \( F \), set of elements are called fuzzy membership value of matrices. For \( C = (C_{ij}) \in f_{m \times n} \) and \( D = (d_{ij}) \in f_{n \times p} \) the max-min product \( C \odot D = (\sup_k \{ \inf \{ C_{ik} : d_{kj} \} \}) \in f_{mn} \)

3. Application of circulant fuzzy matrix in animal disease
We introduced an application of circulant fuzzy matrix in animal diseases. Let \( S \) is a set of symptoms, \( D \) is a set of disease and \( A \) is a set of animals. Analogous to Sanchez’s notion of medical knowledge as an circulant ordered relation \( C \) from the symptoms \( S \) to the diagnosis \( D \).

The methodology involves mainly the following 3 jobs.

1. Determination of symptoms
2. Formulation of medical knowledge based in circulant order relations.
3. Determination of diagnosis on the basis of composition of circulant order relations.

Let as consider \( S \) is a set of symptoms of certain diseases. \( D \) is a set of diseases and \( A \) is set of animals, Construct an circulant matrix \( S \) over \( D \). A relation matrix say, \( \tilde{C}_1 \) is construct from the circulant matrix \( (F,D) \) and called symptom disease matrix. To give another relation of matrix \( S \) over \( D \), say \( \tilde{C}_2 \) is called non symptom disease matrix. Analogous to Sanchez’s notion of medical knowledge, We refered to each matrices \( \tilde{C}_1 \) and \( \tilde{C}_2 \) as medical knowledge of an circulant matrix. Similarly we construct another circulant matrix \( (F,P) \) over \( S \), where \( F \) is an mapping by \( F : P \rightarrow F(S) \). This circulant fuzzy matrix gives another relation matrix \( Q \) called patient-symptom matrix.

Algorithm

Step:1
Input the circulant fuzzy matrix value the set of symptoms \( S \) over diseases \( D \) and write the input value over the symptoms \( S \) over \( D \) denoted by the knowledge matrix \( \tilde{C}_1 \) and \( \tilde{C}_2 \) respectively.

Step:2
Input the circulant fuzzy matrix over the set \( A \) of animals over \( S \) and write its relation \( Q \).

Step:3
Compute the relation matrices under the composition (0)

(i) \( \tilde{T}_1 = Q \odot \tilde{C}_1 \)
(ii) \( \tilde{T}_2 = Q \odot \tilde{C}_2 \)
(iii) \( \tilde{T}_3 = Q \odot (J - \tilde{C}_1) \)

here \( J \) is the matrix with all its entries \( I \), and \( I \) is the greatest element of \( F \).

(iv) \( T_4^2 = Q_0 (J - C_2) \)
(v) Compute the diagnosis score \( SC_1 \) and \( SC_2 \)

\[
ST_1 = \max \{ T_1 (a_i, d_j), T_3 (a_i, d_j) \} \text{ for } i = 1,2,3; j = 1,2,3
\]

\[
ST_2 = \max \{ T_2 (a_i, d_j), T_4 (a_i, d_j) \} \text{ for } i = 1,2,3; j = 1,2,3
\]

Step:4
Find \( S_k = \max [ST_1 (a_i, d_j) - ST_2 (a_i, d_j)] \) then we conclude the animals \( a_i \) is suffering from the disease \( d_k \)

Step:5
If \( S_k \) has more than one value then go step 1 and repeat the process by reassessing the symptoms for the animals.

Case study
Suppose there are three dog’s \( A_1, A_2, A_3 \) in a hospital with symptoms hair loss, itching, reness and possible diseases related to the above symptoms be Demodicosis, Canine scabies, Walking dandruff.

Step:1
\[
\tilde{C}_1 = \begin{pmatrix}
s_1 & 0.9 & 1 & 0.6 \\
s_2 & 1 & 0.6 & 0.9 \\
s_3 & 0.9 & 0.6 & 1
\end{pmatrix}, \quad \tilde{C}_2 = \begin{pmatrix}
s_1 & 0.3 & 0.6 & 0.9 \\
s_2 & 0.9 & 0.6 & 0.3 \\
s_3 & 0.6 & 0.9 & 0.3
\end{pmatrix}
\]

Step:2
\[
Q = \begin{pmatrix}
A_1 & 1 & 0.6 & 0.7 \\
A_2 & 0.7 & 1 & 0.6 \\
A_3 & 0.6 & 0.7 & 1
\end{pmatrix}
\]

Step:3
\[
\tilde{T}_1 = \begin{pmatrix}
A_1 & 1 & 0.7 & 0.9 \\
A_2 & 0.9 & 1 & 0.7 \\
A_3 & 0.7 & 0.9 & 1
\end{pmatrix}, \quad \tilde{T}_2 = \begin{pmatrix}
A_1 & 0.7 & 0.8 & 0.6 \\
A_2 & 0.6 & 0.7 & 0.8 \\
A_3 & 0.8 & 0.6 & 0.7
\end{pmatrix}
\]
We applied Sanchez’s method of medical diagnosis for the decision making method of circulant fuzzy matrix. In this method is an attempt to improve the section.

4.1 Fuzzy relations
A fuzzy relation Q in \( U_1 \times U_2 \times \ldots \times U_n \) is defined as \( \{ (u_1 \times u_2 \times \ldots \times u_n), \mu_Q(u_1 \times u_2 \times \ldots \times u_n) \} \) \( f(u_1 u_2 \times \ldots \times u_n) \in (u_1 u_2 \times \ldots \times u_n) \rightarrow [0, 1] \) Where \( \mu_0:(u_1 u_2 \times \ldots \times u_n) \rightarrow [0, 1] \)

4.2 Max-min composition
Let \( A \) be a fuzzy relation in \( X \times Y \) and \( B \) be a fuzzy relation in \( Y \times Z \). The max-min composition of \( A \) and \( B \) is a fuzzy relation in \( X \times Z \), such that

\[
A \circ B = \max \{ \min \{ \mu_R(x, y), \mu_S(y, z) \} \} / (x, z)
\]

4.3 Maximum product composition
Let \( A \) be a fuzzy relation \( X \times Y \) and \( B \) be a fuzzy relation in \( Y \times Z \). The maximum product composition of \( A \) and \( B \) denoted by \( A \circ B \) is a fuzzy relation in \( X \times Z \), such that

\[
A \circ B = \max \{ \mu_R(x, y), \mu_S(y, z) \} / (x, z)
\]

4.4 Relativity function
Let \( X \) and \( Y \) be variables defined on a universal set \( X \). The relativity function is denoted by \( f(x/y) \) where, \( f(x/y) = \frac{\mu_x(x) - \mu_y(y)}{\max \{ \mu_y(x), \mu_x(y) \} } \) Where \( \mu_y(x) \) is the membership function of \( x \) with respect to \( y \).

4.5 Composition matrix
Let \( A = \{x_1, x_2, \ldots, x_{n-1}, x_n \} \) be a set of \( n \) variables defined on a universal set \( X \). Form a matrix of relativity values \( f(x_i/y_j) \) where \( x_i \)'s (for \( i = 1 \ to \ n \)) are \( n \) variables defined on a universe \( X \). The square matrix \( R = (r_{ij}) \) is called the composition matrix with \( f(x_i/y_j) = \mu_{ij}(x_i) \).

Proposed Algorithm
Let \( S \) be the set of symptoms of some diseases, \( D \) is a set of diseases and \( A \) is a set of animals.

Step:1
Construct a symptom disease circulant fuzzy matrix \( C = (c_{ij}) \) of order \( m \times n \).

Step:2
Construct a patient-symptom circulant fuzzy matrix \( Q = (q_{ij}) \) of order \( m \times n \).

Step:3
Compute the complement of both circulant fuzzy matrices \( C \) and \( Q \). Let it be \( C^c \) and \( Q^c \).

Step:4
Compute the relational matrix under the maximum product composition (.) \( T_1 = C^c \circ Q, T_2 = C^c \circ Q^c \).

Step:5
Compute \( T = T_1 - T_2 \) where “-” is the maximum of \( T_1 \) and \( T_2 \).

Step:6
Calculating the relativity values \( f(p_i/d_i) \) and form the comparison circulant fuzzy matrix

\[
W = (w_{ij})_{m \times n} = [f(p_i/d_i)]_{i=1,2,3}
\]

Then maximum value in each of the rows of the \( W \)-circulant fuzzy matrix will have the maximum possibility for ranking purposes. This gives the solution to the required problem.

5. Illustrative examples
Suppose there are three dog’s \( A_1, A_2 \) and \( A_3 \) in a veterinary hospital with symptoms Hairloss, Itching, Reness. Let the possible diseases relating to the above symptoms be Demodicosis, Canine scabies, Walking dandruff.

Step:1
Consider a symptom-disease circulant fuzzy matrix \( C \) of order \( 3 \times 3 \) such that.

\[
C = \begin{pmatrix}
1 & 0.6 & 0.9 \\
0.9 & 1 & 0.6 \\
0.6 & 0.9 & 1
\end{pmatrix}
\]

Step:2
Consider a animal-symptom circulant fuzzy matrix \( Q \) as

\[
Q = \begin{pmatrix}
1 & 0.6 & 0.7 \\
0.7 & 1 & 0.6 \\
0.6 & 0.7 & 1
\end{pmatrix}
\]
6. Conclusion

We clarify the theory of circulant fuzzy matrices in the field of animal diseases for medical diagnosis. We develop some new notions such as complement to the max of circulant fuzzy matrix. There are hundreds of diseases for animals, and the special degree is relatively lower in veterinary, any veterinarian have to solved internal medicine sickness, the obstetrical disease, surgical sickness, parasitic disease and so on. Therefore, the expert system for animal disease must conform to the objective law of veterinarian clinical. Hence in this paper, Fuzzy set structure has been utilized is several different approaches to model the medical diagnostic process and decision making process. At the conclusion from the above analysis it is obvious that, Dog (A₁) suffer from walking dandruff (d₁), Dog(A₂) suffer from demodicosis (d₁) and dog(A₃) suffer from d₂ (Canine Scabies).

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