Connected vertex–Edge dominating sets and connected vertex–Edge domination polynomials of triangular ladder

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Abstract

Let $G$ be a simple connected graph of order $n$. Let $D_{\text{cye}}(G, i)$ be the family of connected vertex - edge dominating sets of $G$ with cardinality $i$. The polynomial

$$D_{\text{cve}}(G, x) = \sum_{i = \gamma_{\text{cve}}(G)}^{n} d_{\text{cve}}(G, i)x^i$$

is called the connected vertex – edge domination polynomial of $G$ where $d_{\text{cve}}(G, i)$ is the number of vertex edge dominating sets of $G$. In this paper, we study some properties of connected vertex - edge domination polynomials of the Triangular Ladder $TL_n$. We obtain a recursive formula for $d_{\text{cve}}(TL_n, i)$. Using this recursive formula, we construct the connected vertex - edge domination polynomial

$$D_{\text{cve}}(TL_n, x) = \sum_{i = n-2}^{2n} d_{\text{cve}}(TL_n, i)x^i$$

of $TL_n$, where $D_{\text{cve}}(TL_n, i)$ is the number of connected vertex - edge dominating sets of $TL_n$ with cardinality $i$ and some properties of this polynomial have been studied.

Keywords

Triangular ladder, Connected vertex – edge dominating set, connected vertex – edge domination number, connected vertex – edge domination polynomial.

AMS Subject Classification

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1. Introduction

Let $G = (V, E)$ be a simple graph of order $n$. For any vertex $v \in V$, the open neighbourhood of $v$ is the set $N(v) = \{u \in V : uv \in E\}$ and the closed neighbourhood of $v$ is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of $S$ is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighbourhood of $S$ is $N[S] = N(S) \cup \{S\}$. A vertex $u \in V(G)$ vertex-edge dominates (ve-dominates) an edge $vw \in E(G)$ if

1. $u = v$ or $u = w(u$ is incident to $vw)$ or
2. $uv$ or $uw$ is an edge in $G(u$ is incident to an edge that is
A vertex - edge dominating set \( S \) of \( G \) is called a connected vertex - edge dominating set if the induced subgraph \( < S > \) is connected. The minimum cardinality of a connected vertex - edge dominating set of \( G \) is called the connected vertex - edge domination number of \( G \) and is denoted by \( \gamma_{cve} (G) \). A connected vertex - edge dominating set with cardinality \( \gamma_{cve} (G) \) is called \( \gamma_{cve} - \) set. We denote the set \( \{1, 2, \ldots, 2n - 1, 2n\} \) by \( [2n] \), throughout this chapter. Also we used the notation \( [x] \) for the smallest integer greater than or equal to \( x \) and \( \lfloor x \rfloor \) for the largest integer less than or equal to \( x \).

2. Connected vertex –Edge dominating sets of Triangular Ladders

Consider two paths \( u_1u_2 \ldots u_n \) and \( v_1v_2 \ldots v_n \). Join each pair of vertices \( u_i v_i \) and \( u_{i+1} v_i, i = 1, 2, \ldots, n \). The resulting graph is a Triangular Ladder. Let \( TL_n \) be a triangular ladder with \( 2n \) vertices. Label the vertices of \( TL_n \) as given in the following figure.

![Figure 1](image)

\[ V (TL_n) = \{1, 2, 3, \ldots, 2n - 2, 2n - 1, 2n\} \]
\[ E (TL_n) = \{(1, 3), (3, 5), (5, 7) \ldots (2n - 5, 2n - 3), (2, 4), (4, 6), (6, 8), \ldots (2n - 4, 2n - 2), (2n, 2), (2n - 1, 2n)\} \]

For the construction of the vertex- edge dominating sets of the Triangular Ladder \( TL_n \), we need to investigate the connected vertex - edge dominating sets of \( TL_n - \{2n\} \). In this section, we investigate the connected vertex - edge dominating sets of \( TL_n \) with cardinality \( i \). We shall find the recursive formula for \( d_{cve} (TL_n, i) \).

Lemma 2.1. (i) For every \( n \in N \) and \( n \geq 4 \), \( \gamma_{cve} (TL_n) = n - 1 \).

(ii) For every \( n \in N \) and \( n \geq 5 \), \( \gamma_{cve} (TL_n - \{2n\}) = n - 2 \).

(iii) \( D_{cve} (TL_n, i) = \varphi \) iff \( i < n - 10 \) or \( i > 2n \).

(iv) \( D_{cve} (TL_n - \{2n\}, i) = \varphi \) iff \( i < n - 2 \) or \( i > 2n - 1 \).

Proof. (i) Clearly, \( \{3, 5, 7, 9, \ldots, 2n - 3\} \) is a minimum connected vertex - edge dominating set for \( TL_n \). If \( n \) is even or odd, it contains \( n - 1 \) elements. Hence, \( \gamma_{cve} (TL_n) = n - 10 \).

(ii) \( D_{cve} (TL_n, i) = \varphi \) iff \( i < n - 10 \) or \( i > 2n \).

(iii) \( D_{cve} (TL_n - \{2n\}, i) = \varphi \) iff \( i < n - 2 \) or \( i > 2n - 1 \).

Lemma 2.2. (i) If \( D_{cve} (TL_n - \{2n\}, i - 1) = \varphi \), \( D_{cve} (TL_n - \{2n - 2\}, i - 1) = \varphi \) and \( D_{cve} (TL_n - \{2n - 2\}, i - 1) = \varphi \) then \( D_{cve} (TL_n, i - 1) = \varphi \).

(ii) If \( D_{cve} (TL_n - \{2n\}, i - 1) \neq \varphi \), \( D_{cve} (TL_n - \{2n - 2\}, i - 1) \neq \varphi \) and \( D_{cve} (TL_n - \{2n - 2\}, i - 1) \neq \varphi \) then \( D_{cve} (TL_n, i) \neq \varphi \).

Proof. (i) Since, \( D_{cve} (TL_n - \{2n\}, i - 1) = \varphi \), \( D_{cve} (TL_n - \{2n - 2\}, i - 1) = \varphi \) and \( D_{cve} (TL_n - \{2n - 2\}, i - 1) = \varphi \) then \( D_{cve} (TL_n, i) = \varphi \).

(ii) Since, \( D_{cve} (TL_n - \{2n\}, i - 1) \neq \varphi \), \( D_{cve} (TL_n - \{2n - 2\}, i - 1) \neq \varphi \) and \( D_{cve} (TL_n - \{2n - 2\}, i - 1) \neq \varphi \) then \( D_{cve} (TL_n, i) \neq \varphi \).
Lemma 2.3. Suppose that \( D_{cve}(TL_n, i) \neq \emptyset \), then for every \( n \in \mathbb{N} \),

(i) \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \) and \( D_{cve}(TL_{n-1}, i - 1) = \emptyset \) if \( i = 2n - 2 \).

(ii) \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \) and \( D_{cve}(TL_{n-1}, i - 1) \neq \emptyset \) if \( i = 2n - 1 \).

(iii) \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \) if \( D_{cve}(TL_{n-1}, i - 1) \neq \emptyset \) and \( D_{cve}(TL_{n-2}, i - 1) = \emptyset \) if \( i = 2n - 2 \).

Proof. Assume that \( D_{cve}(TL_n, i) \neq \emptyset \). Then \( n - 1 \leq i \leq 2n - 1 \).

(i) \( \iff \) since \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \), by lemma 2.1 (iv), we have \( n - 2 \leq i - 1 \leq 2n - 1 \). Therefore, \( n - 1 \leq i \leq 2n - 2 \).

Also since \( D_{cve}(TL_{n-1}, i - 1) = \emptyset \), by lemma 2.1 we have \( i - 1 < n - 2 \) or \( i - 1 > 2n - 2 \). If \( i - 1 < n - 2 \), then \( i - 1 < n - 1 \) which implies \( D_{cve}(TL_n, i) = \emptyset \), a contradiction. Therefore \( i - 1 > 2n - 2 \). Therefore \( i - 1 \geq 2n - 1 \). This implies \( i \geq 2n \). Therefore, \( i = 2n \). \( \iff \) follows from Lemma 2.1 (iii) and (iv).

(ii) \( \iff \) since \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \), and \( D_{cve}(TL_{n-1}, i - 1) \neq \emptyset \) by lemma 2.1 (ii) and (iv), we have \( n - 2 \leq i - 1 \leq 2n - 1 \) and \( n - 2 \leq i - 1 \leq 2n - 1 \). Therefore, \( n - 1 \leq i \leq 2n - 2 \). Also, since \( D_{cve}(TL_{n-1} - \{2n - 2\}, i - 1) = \emptyset \), by lemma 2.1 (iv), we have \( i - 1 < n - 3 \) or \( i - 1 > 2n - 3 \). Therefore, \( i < n - 2 \) or \( i > 2n - 2 \). If \( i < n - 2 \), then \( i < n - 1 \) holds which implies \( D_{cve}(TL_n, i) = \emptyset \), a contradiction. Therefore, \( i < 2n - 2 \). Hence \( D_{cve}(TL_{n-1}, i) \neq \emptyset \). Combining together, we have \( 2n - 2 \leq \) (\( \iff \)) follows from Lemma 2.1 (iii) and (iv).

Theorem 2.4. For every \( n \geq 4 \)

(i) If \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \) and \( D_{cve}(TL_{n-1}, i - 1) \neq \emptyset \), then \( D_{cve}(TL_n, i) \neq \emptyset \).

Proof. (i) If \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \) and \( D_{cve}(TL_{n-1}, i - 1) \neq \emptyset \), then \( D_{cve}(TL_n, i) = \{X_1 \cup \{2n\}\} \).

(ii) If \( D_{cve}(TL_n - \{2n\}, i - 1) \neq \emptyset \) and \( D_{cve}(TL_{n-1}, i - 1) \neq \emptyset \), then \( D_{cve}(TL_n, i) \neq \emptyset \).
Theorem 2.5. If $D_{cve}(T_{L_n},i)$ is the family of connected vertex–edge dominating sets of $T_{L_n}$ cardinality $i$, where $i \geq n-1$, then
\[
d_{cve}(T_{L_n},i) = d_{cve}(T_{L_n} - \{2n\},i-1) + d_{cve}(T_{L_{n-1}},i-1)
\]

Proof. We consider the two cases given in theorem 2.4 Suppose $D_{cve}(T_{L_n} - \{2n\},i-1) \neq \emptyset$ and $D_{cve}(T_{L_{n-1}},i-1) = \emptyset$. Then $i = 2n$

\[
d_{cve}(T_{L_n},i) = d_{cve}(T_{L_n} - \{2n\},i-1) = d_{cve}(T_{L_{n-1}},i-1) = 0.
\]

Therefore,
\[
d_{cve}(T_{L_n} - \{2n\},i-1) + d_{cve}(T_{L_{n-1}},i-1) = 1 + 0 = 1
\]

Therefore, in this case
\[
d(T_{L_n},i) = d_{cve}(T_{L_n} - \{2n\},i-1) + d_{cve}(T_{L_{n-1}},i-1)
\]
holds. By theorem 2.4 (ii) we have,
\[
D_{cve}(T_{L_n},i) = \{X_1 \cup \{2n-1\}, \text{ if } 2n-3 \in X_1/X_1 \in D_{cve}(T_{L_n} - \{2n\},i-1) \} \cup \{X_1 \cup \{2n\}, \text{ if } 2n-2 \text{ or } 2n-1 \in X_1/X_1 \in D_{cve}(T_{L_n} - \{2n\},i-1) \} \cup \{X_2 \cup \{2n-2\}, \text{ if } 2n-4 \text{ or } 2n-3 \in X_2/X_2 \in D_{cve}(T_{L_{n-1}},i-1) \} \cup X_2 \cup \{2n-1\}, \text{ if } 2n-2 \in X_2/X_2 \in D_{cve}(T_{L_{n-1}},i-1) \}.
\]

Therefore
\[
c_{cve} D_{cve}(T_{L_n},i) = d_{cve}(T_{L_n} - \{2n\},i-1) + d_{cve}(T_{L_{n-1}},i-1).
\]

\[\Box\]

3. Connected total domination polynomials of triangular ladders

Theorem 3.1. Let $D_{cve}(T_{L_n},i)$ be the family of connected vertex–edge dominating sets of $T_{L_n}$ with cardinality $i$ and let $d_{cve}(T_{L_n},i) = |D_{cve}(T_{L_n},i)|$. Then the connected vertex edge domination polynomial $D_{cve}(T_{L_n},x)$ of $T_{L_n}$ is defined as,
\[
D_{cve}(T_{L_n},x) = \sum_{i=\gamma_{cve}(T_{L_n})}^{2n} d_{cve}(T_{L_n},i)x^i.
\]

Theorem 3.2. For every $n \geq 4$,
\[
D_{cve}(T_{L_n},x) = x[D_{cve}(T_{L_n} - \{2n\},x) + D_{cve}(T_{L_{n-1}},x)]
\]
with initial values
\[
D_{cve}(T_{L_2} - \{4\},x) = 3x^2 + x^3
\]
\[
D_{cve}(T_{L_2},x) = 5x^2 + 4x^3 + x^4
\]
\[
D_{cve}(T_{L_3} - \{6\},x) = 7x^2 + 8x^3 + 5x^4 + x^5
\]
\[
D_{cve}(T_{L_3},x) = 7x^2 + 12x^3 + 12x^4 + 6x^5 + x^6
\]
\[
D_{cve}(T_{L_4} - \{8\},x) = 5x^2 + 14x^3 + 20x^4 + 17x^5 + 7x^6 + x^7
\]
\[
D_{cve}(T_{L_4},x) = 3x^2 + 12x^3 + 26x^4 + 32x^5 + 23x^6
\]
\[+ 8x^7 + x^8
\]
\[
D_{cve}(T_{L_5} - \{10\},x) = x^2 + 8x^3 + 26x^4 + 46x^5 + 49x^6
\]
\[+ 30x^7 + 9x^8 + x^9
\]

\[\Box\]
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Table 1

| n   | → i | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15  | 16  | 17  | 18  |
|-----|-----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
| TL2 − {4} | 3  | 1  |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| TL2      | 5  | 4  | 1  |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| TL3 − {6} | 7  | 8  | 5  | 1  |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| TL3      | 7  | 12 | 12 | 6  | 1  |    |    |    |    |    |    |    |    |    |     |     |     |     |
| TL4 − {8} | 5  | 14 | 20 | 17 | 7  | 1  |    |    |    |    |    |    |    |    |     |     |     |     |
| TL4      | 3  | 12 | 26 | 32 | 23 | 8  | 1  |    |    |    |    |    |    |    |     |     |     |     |
| TL5 − {10} | 1  | 8  | 26 | 46 | 49 | 30 | 9  | 1  |    |    |    |    |    |    |     |     |     |     |
| TL5      | 0  | 4  | 20 | 52 | 78 | 72 | 38 | 10 | 1  |    |    |    |    |    |     |     |     |     |
| TL6 − {12} | 0  | 1  | 12 | 46 | 98 | 127 | 102 | 47 | 11 | 1  |    |    |    |    |     |     |     |     |
| TL6      | 0  | 0  | 5  | 32 | 98 | 176 | 199 | 140 | 57 | 12 | 1  |    |    |    |     |     |     |     |
| TL7 − {14} | 0  | 0  | 1  | 17 | 78 | 196 | 303 | 301 | 187 | 68 | 13 | 1  |    |    |     |     |     |     |
| TL7      | 0  | 0  | 0  | 6  | 49 | 176 | 372 | 502 | 444 | 244 | 80 | 14 | 1  |    |     |     |     |     |
| TL8 − {16} | 0  | 0  | 0  | 1  | 23 | 127 | 372 | 675 | 803 | 628 | 312 | 93 | 15 | 1  |    |     |     |     |
| TL8      | 0  | 0  | 0  | 0  | 7  | 72  | 303 | 744 | 1377 | 1244 | 872 | 392 | 107 | 16 | 1  |     |     |
| TL9 − {18} | 0  | 0  | 0  | 0  | 1  | 30  | 199 | 675 | 1419 | 2180 | 1872 | 1191 | 485 | 122 | 17 | 1  |     |
| TL9      | 0  | 0  | 0  | 0  | 0  | 8  | 102 | 502 | 1419 | 2796 | 3424 | 2744 | 1583 | 592 | 138 | 18 | 1  |

Example 3.3.

\[
D_{cve} (TL_5, x) = 4x^3 + 20x^4 + 52x^5 + 78x^6 + 72x^7 + 38x^8 + 10x^9 + x^{10}
\]

\[
D_{cve} (TL_6 - \{12\}, x) = x^3 + 12x^4 + 46x^5 + 98x^6 + 127x^9 + 102x^8 + 47x^9 + 11x^{10} + x^{11}
\]

By theorem 3.2, we have

\[
D_{cve} (TL_6, x) = x \left[ 4x^3 + 20x^4 + 52x^5 + 78x^6 + 72x^7 + 38x^8 + 10x^9 + x^{10} + 12x^4 + 46x^5 + 98x^6 + 127x^9 + 102x^8 + 47x^9 + 11x^{10} + x^{11} \right]
\]

\[
= 5x^4 + 32x^5 + 98x^6 + 176x^7 + 199x^8 + 140x^9 + 57x^{10} + 12x^{11} + x^{12}
\]

We obtain, \( d_{cve} (TL_n, i) \) and \( d_{cve} (TL_n - \{2n\}, i) \) for \( 2 \leq n \leq 9 \) as shown in table 1.

In the following theorem we obtain some properties of \( d_{cve} (TL_n, i) \).

Theorem 3.4. The following properties hold for the coefficients of \( D_{cve} (TL_n, x) \) and \( D_{cve} (TL_n - \{2n\}, x) \) for all \( n \).

(i) \( d_{cve} (TL_n, 2n) = 1 \) for all \( n \geq 2 \).
(ii) \( d_{cve} (TL_n, 2n - 1) = 2n \) for all \( n \geq 2 \).
(iii) \( d_{cve} (TL_n - \{2n\}, 2n - 1) = 1 \) for all \( n \geq 2 \).
(iv) \( d_{cve} (TL_n - \{2n\}, 2n - 2) = 2n - 1 \) for all \( n \geq 2 \).
(v) \( d_{cve} (TL_n, 2n - 2) = 2n^2 - 3n + 3 \) for all \( n \geq 2 \).
(vi) \( d_{cve} (TL_n - \{2n\}, 2n - 3) = 2n^2 - 5n + 5 \) for all \( n \geq 3 \).
(vii) \( d_{cve} (TL_n - \{2n\}, n - 3) = 1 \) for all \( n \geq 5 \).
(viii) \( d_{cve} (TL_n, n - 2) = n - 1 \) for all \( n \geq 4 \).

Proof.

(i) Since \( D_{cve} (TL_n, 2n) = \{2n\} \), we have the result.

(ii) Since, \( D_{cve} (TL_n, 2n - 1) = \{2n - \{\} \} \), we have \( d_{cve} (TL_n, 2n - 1) = 2n \).

(iii) Since, \( D_{cve} (TL_n - \{2n\}, 2n - 1) = \{2n - 1\} \), we have the result.

(iv) To prove \( d_{cve} (TL_n - \{2n\}, 2n - 2) = 2n - 1 \), for every \( n \geq 2 \), we apply induction on \( n \).

When \( n = 2 \),

\[ \text{L.H.S} = d_{cve} (TL_2 - \{4\}, 2) = 3 \] (From table : 1) and

\[ \text{R.H.S} = 2 \times 2 - 1 = 3 \].

Therefore, the result is true for \( n = 2 \). Now, suppose that the result is true for all natural numbers less than \( n \) and we prove it for \( n \). By theorem 2.5, we have,

\[ d_{cve} (TL_n - \{2n\}, 2n - 2) = d_{cve} (TL_{n-1}, 2n - 3) + d_{cve} (TL_{n-1}, \{2n - 2\}, 2n - 3) \]

\[ = 2(n - 1) + 1 \]

That is, \( d_{cve} (TL_n, \{2n\}, 2n - 2) = 2n - 1 \).

Hence, the result is true for all \( n \).

(v) To prove \( d_{cve} (TL_n, 2n - 2) = 2n^2 - 3n + 3 \), for every \( n \geq 2 \), we apply induction on \( n \).

When \( n = 2 \),

\[ \text{L.H.S} = d_{cve} (TL_2, 2) = 5 \] (from table 1) and

\[ \text{R.H.S} = 2 \times 4 - 3 \times 2 + 3 = 5 \].

Therefore, the result is true for \( n = 2 \). Now, suppose that the result is true for all numbers less than \( 'n' \) and we prove it for
n. By theorem 2.5, we have
\[
d_{cve}(TL_n, 2n-2) = d_{cve}(TL_n - \{2n\}, 2n-3) + d_{cve}(TL_n, 2n-3)
\]

\[
= d_{cve}(TL_{n-1}, 2n-4) + d_{cve}(TL_{n-1} - \{2n-2\}, 2n-4)
\]

\[
+ d_{cve}(TL_{n-1}, 2n-3)
\]

\[
= 2(n-1)^2 - 3(n-1) + 3 + 2(n-1) - 1 + 2(n-1)
\]

\[
= 2(n^2 - n + 1) - 3n + 3 + 2n - 2 - 1
\]

\[
= 2n^2 - 4n + 2 + n + 1
\]

\[
= 2n^2 - 3n + 3
\]

Hence the result is true for all n.

(vi) To prove \(d_{cve}(TL_n - \{2n\}, 2n-3) = 2n^2 - 5n + 5\), for every \(n \geq 3\), we apply induction on n.

When \(n = 3\), L.H.S = \(d_{cve}(TL_3 - \{6\}, 3) = 8\) (From table 1)

R.H.S = \(2 \times 9 - 5 \times 3 + 5 = 8\).

Therefore, the result is true for \(n = 3\). Now, suppose that the result is true for all numbers less than \(n\) and prove it for \(n\). By theorem 2.5, we have

\[
d_{cve}(TL_3 - \{2n\}, 2n-3)
\]

\[
= d_{cve}(TL_3 - \{2n\} - \{2n\}, 2n-3) + d_{cve}(TL_3 - \{2n\}, 2n-3)
\]

\[
= 2(n-1)^2 - 3(n-1) + 3 + 2(n-1) - 1
\]

\[
= 2(n^2 - 2n + 1) - 3n + 3 + 2n - 2 - 1
\]

\[
= 2n^2 - 4n + 2 - n + 3
\]

\[
d_{cve}(TL_3 - \{2n\}, 2n-3) = 2n^2 - 5n + 5
\]

Hence, the result is true for all n.

(vii) Since, \(D_{cve}(TL_n - \{2n\}, n-3) = \{3, 5, 7, \ldots, 2n-1\}\), we have the result.

(viii) To prove \(d_{cve}(TL_n, n-2) = n - 1\) for all \(n \geq 4\), we apply induction on n.

When, \(n = 4\), \(d_{cve}(TL_4, 2) = 3\)

R.H.S = \(n - 1 = 4 - 1 = 3\).

Therefore, the result is true for \(n = 4\) By theorem 2.5, we have,

\[
d_{cve}(TL_n, n-2) = d_{cve}(TL_n - \{2n\}, n-3)
\]

\[
+ d_{cve}(TL_{n-1}, n-3)
\]

\[
= 1 + n - 2
\]

\[
= n - 1
\]

Hence, the result is true for all \(n\) by mathematical induction.

References


