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Prime bi-interior Γ-ideals of TG-semring

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Abstract
The notation of prime bi-interior ideal, semi prime bi-interior ideal, irreducible bi-interior ideal and strongly prime bi-interior ideal of TG-Semi ring (ternary gamma semi-ring) are introduced. We study properties of these ideals and relations between them and also characterize regular TG-Semi-ring and TG-Semi-ring using prime bi-interior ideals, irreducible and strongly irreducible bi-interior ideals in this article.

Keywords
TGS, bi-interior ideal, prime ideals, prime bi-interior ideal, strong prime bi-interior ideal, semi prime bi-interior ideal and irreducible and strongly irreducible bi-interior ideals.

AMS Subject Classification
16W25, 16N60, 16U80.

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Contents

1. Introduction

We introduced the notation of prime bi-interior ideals of TG Semi rings, in this paper. G. Srinivasa Rao et.al [29-36] studied ternary semi rings.

2. Preliminaries

Let $(R,+)\&(\Gamma,+)$, commutative semi-groups. Then we call R a TG -Semi ring (TGS), if there is a mapping $R\times\Gamma\times R\times\Gamma\times R\to R$ (images of (p,a,q,b,r) will be denoted by pagbr)$\forall p,q,r\in R, a,b\in\Gamma\ni$ it satisfies the following axioms $\forall p,q,r,s,t\in R$ and $a,b,c,d\in\Gamma$

1. $pa(q+r)bs=$ pagbs + parbs
2. $(p+q)arbs=$ parbs + qarbs
3. $paqb(r+s)=$ paqbr $+paqbs$
4. pa(qbrcs)dt= paqb(rcsdt) =(paqbr)csdt.

ATGSR is said to be commutative TGS if $paqbr=parbq=qarbp=qapbr=rapbq=raqbp$, $\forall p,q,r\in R$ and $a,b\in\Gamma$. Suppose R, a TGS. If for each p in $R\exists a,b\in\Gamma$ papbe $=$ paebp $=$ eapbp $=e$, then an element $e\in R$ is called aunityelement or neutral element.

Definition 2.1. *Suppose R, a ternary Γ -semi-ring. $\emptyset\neq S$ is said to be a ternary sub $-\Gamma$ - semi-ring of R, if S is an sub-semi-group with respect to $+$ of R and $a\alpha b\beta c\in S,\forall a,b,c\in S$ and $\alpha\ \beta\in\Gamma$.*

Definition 2.2. *The* set $\emptyset\neq I\subseteq R$, *where R is a temary Γ -semi-ring is said to be left (lateral, right)ternary Γ -ideal of R, if*

1. $a,b\in I\Rightarrow a+b\in I$
2. $a,b\in R, i\in I,\alpha,\beta\in\Gamma\Rightarrow a\alpha b\beta i\in I(a\alpha i\beta b\in I,\ ia\ a\beta b\in I)$.

An ideal I is said to be ternary Γ -ideal, if it is left, lateral and right Γ -ideal of R

Example 2.3. *The set $Z = \{0, \pm1, \pm2, \pm3, \ldots\}$ and Γ, the set of even natural numbers. Then with respect to usual addition and ternary multiplication, Z is a ternary gamma semi ring.*

Example 2.4. *The set $R = \{0, \pm i, \pm 2i, \pm 3i, \ldots\}$ and $\Gamma = N$. Then R is a ternary gamma semi ring. with respect to usual addition and ternary multiplication.*

Example 2.5. *Let $Q = R$, the set of all rational numbers and Γ the set of all natural numbers. Define a mapping from $R \times \Gamma \times R \times \Gamma \times R \to R$ by usual addition and ternary multiplication defined by $(p, a, q, b, r) = paqbr, \forall p, q, r \in R, a, b \in \Gamma$ then R is a ternary gamma semi-ring.*

Definition 2.6. *Let $\phi \neq S \subseteq R$, where R be a TGS. The set S said to be a TG -sub-semi-ring of R if $(S, +)$ is a T sub-semi-group $(TSSG)$ of $(R, +)$ and $S\Gamma S\Gamma S \subseteq S$.*

Definition 2.7. *Let R be a TGS and $\phi \neq S \subseteq R$. The set S said to be a quasi-ideal (QI) of R if S is a TGsub-semi-ring (TGSSR) of R and $S\Gamma R\Gamma R \cap (R\Gamma S\Gamma R + R\Gamma R\Gamma S\Gamma R\Gamma R) \cap R\Gamma R\Gamma S \subseteq S$.*

Definition 2.8. *Let $\phi \neq S \subseteq R$, where R be a TGS. The set S said to be a bi-ideal (BI) of R if S is a TGSSR of R and $S\Gamma R\Gamma S\Gamma R\ \Gamma \subseteq S$.*

Definition 2.9. *Let $\phi \neq S \subseteq R$, where R be a TGS. The set S said to be an interior ideal (II) of R if S is a TGSSR of R and $R\Gamma R\Gamma S\Gamma R\Gamma R \subseteq S$.*

Definition 2.10. *Let R be a TGS and $\phi \neq S \subseteq R$. The set S said to be a rt. (medial, lt.) ideal of R if S is a TGSSR of R and $S\Gamma R\Gamma R \subseteq S (R\Gamma S\Gamma R \subseteq S, R\Gamma R\Gamma S \subseteq S)$.*

Definition 2.11. *Let $\phi \neq S \subseteq R$, where R be a TGS. The set S said to be an ideal if S is a TGSSR of R and $S\Gamma R\Gamma R \subseteq S, R\Gamma S\Gamma R \subseteq S, R\Gamma R\Gamma S \subseteq S$.*

Definition 2.12. *Let $\phi \neq S \subseteq R$, where R be a TGS. The set P said to be a bi-interior ideal (BII) of R if P is a TG subsemi ring (GSSR) of R $\Gamma R\Gamma P\Gamma R\Gamma R \cap P\Gamma R\Gamma P\Gamma R\Gamma P \subseteq P$.*

Definition 2.13. *Let $\phi \neq S \subseteq R$, where R be a TGS. The set P said to be a lt.(medial, rt.) weak-interior ideal of R if P is a GSSR of R and $R\Gamma P\Gamma P \subseteq P (P\Gamma R\Gamma P \subseteq P, P\Gamma P\Gamma R \subseteq P)$.*

Example 2.14. *Consider the TGsemi-ring $R = M_{2\times2}(W)$, and $\Gamma = M_{2\times2}(W)$ where $W = \{0, 1, 2, 3, \ldots\ldots\}$ Then R is a TG-semi-ring with $P\alpha Q\beta S$ is the ordinary ternary multiplication of matrices, $\forall P, Q, S \in R$ and $\alpha, \beta \in \Gamma$. Here*

$$U = \left\{ \begin{bmatrix} p & a \\ 0 & b \end{bmatrix} : a, b, p \in W \right\}$$

is a bi-ideal of R. Also

$$V = \left\{ \begin{bmatrix} p & 0 \\ 0 & a \end{bmatrix} : a, p \in W \right\}$$

is a bi-ideal of R.

3. Prime Bi-interior ideals in T gamma Semi rings

We study ”the notion of prime, strongly prime, semi prime, irreducible and strongly irreducible bi-interior ideals” of TG-semi-rings. And we study the properties of prime ideals and relations between them.

Definition 3.1. *A BII B of a TGS R is said to a prime bi-interior ideal (PBII) of R if $B_1\Gamma B_2\Gamma B_3 \subseteq B \Rightarrow B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$.*

Definition 3.2. *A BII P of TGS R is called a semi prime bi-interior ideal (SPBII) of R if for any bi- interior ideal P_1 of $R, P_1\Gamma P_1\Gamma P_1 \subseteq P \Rightarrow P_1 \subseteq P$.*

Definition 3.3. *A BII P of R is called a strongly prime bi-interior ideal $(STPBII)$ if $(P_1\Gamma P_2\Gamma P_3) \cap (P_1\Gamma P_2\Gamma P_3) \cap (P_1\Gamma P_2\ \Gamma P_3) \subseteq P \Rightarrow P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$, for any BI-ideals P_1, P_2 and P_3 of R.*

Definition 3.4. *A bi-interior ideal (BII) P of R is said to be an irreducible bi-interior ideal (IBII) P if $BIIsP_1, P_2, P_3$ and $P_1 \cap P_2 \cap P_3 = P \Rightarrow P_1 = P$ or $P_2 = P$ or $P_3 = P$.*

Definition 3.5. *A BII P of R, known as a strongly irreducible bi-interior ideal (SIBI) if for any BIs P_1, P_2 and P_3 of $R, P_1 \cap P_2 \cap P_3 \subseteq P \Rightarrow P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$. Clearly every SIBI is an IBI.*

Note 3.6. *(i) Every STPBII of a TGS R is a PBII of R.*

(ii) Every PBII P of a TGS R is a SPBII of R.

Theorem 3.7. *A BII P of a $TGSR$ is a prime bi-interior ideal $\Leftrightarrow K\Gamma M\Gamma L \subseteq P \Rightarrow K \subseteq P$ or $M \subseteq P$ or $L \subseteq P$ where M is a lateral ideal, K is a right ideal and L is a left ideal of R.*

Proof. Suppose that a prime bi-interior ideal P of the TGS R and $K\Gamma M\Gamma L \subseteq P$. Since K, M and L are bi- interior ideals of R, we have $K \subseteq P$ or $M \subseteq P$ or $L \subseteq P$. Again let $K\Gamma M\Gamma L \subseteq P$ where K is a right ideal, M is a lateral ideal and L is a left ideal of R. $\Rightarrow K \subseteq P$ or $M \subseteq P$ or $L \subseteq P$. Suppose $D\Gamma E\Gamma F \subseteq P, D, E$ and F are bi-interior ideals and $(d)r, (e)_m(f)_l$ are right, medial or lateral and left ideals generated by d, e and f respectively, where $d \in D$, $e \in E$ and $f \in F$. Then $(d)_r\Gamma(e)_m\rho\Gamma(f)_l = D\Gamma E\Gamma F \subseteq P, (d)_r \subseteq P$ or $(e)_m \subseteq P$ or $(f)_l \subseteq P \Rightarrow d \in P$ or $e \in P$ or $f \in P. \therefore D \subseteq P$ or $E \subseteq P$ or $F \subseteq P$. Hence a bi-interior ideal P is a prime bi- interior ideal of the TGS R. □

Theorem 3.8. *If P_1, P_2, P_3 are bi-interior ideals of a TGS R and $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) = P_1 \cap P_2 \cap P_3$ then every bi-interior ideal of a TGS R is a semi prime BI-ideal of R.*

Proof. Let P_1, P_2, P_3 be bi-interior ideals of a $TGSR \& (P_1\Gamma P_2\Gamma\ P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) = P_1 \cap P_2 \cap P_3$. Let P be a bi-interior ideal of a TGS R. To prove, P is a semi-prime ideal of R Suppose $P_1\Gamma P_1\Gamma P_1 \subseteq P_1$ Then $P_1 = P_1 \cap P_1 \cap P_1 = (P_1\Gamma P_1\Gamma P_1) \cap (P_1\Gamma P_1\Gamma P_1) \cap (P_1\Gamma P_1\ \ \Gamma P_1) \subseteq P \cap P \cap P = P$. Hence every bi-interior ideal of R is semi prime. □

Definition 3.9. *Let R be a TGS. An element $p \in R$ is said to be a regular element if there exists $x, y \in R$ and $a, b, c, d \in \Gamma$ such that $p = paxbpcydp$. Every element in TGS is a regular element then R is known as Regular TGS.*

Theorem 3.10. *R is a regular TG semi ring $\Leftrightarrow I\Gamma J\Gamma K = In$ $J \cap K$ for any left ideal K, lateral ideal J and right ideal I of R.*

Proof. Let I, J, K be a rt. ideal, medial and a lt. ideal of a regular TGS R respectively. Suppose R be a regular TGS. Clearly $I\Gamma J\Gamma K \subseteq I \cap J \cap K$. It enough to show $I \cap J \cap K \subseteq I\Gamma J\Gamma K$. Let $p \in I \cap J \cap K$. Since R is a regular TG semi ring, there exist $a, b, c, d \in \Gamma$ and $x, y \in R$ such that $p =$ paxbpcydp. Since paxbp $\in I$, and $pcydp \in J \Rightarrow$ paxbpcydp $\in I\Gamma J\Gamma K$. Thus $p \in I\Gamma J\Gamma K$. Hence $I\Gamma J\Gamma K = I \cap J \cap K$. Against suppose that $I\Gamma J\Gamma K = I \cap J \cap K$, for any left ideal K, for any lateral ideal or medial J and right ideal I of R. Let $p \in R$ and I be the right ideal generated by p, J be a lateral ideal generated by p and K be a left ideal generated by p. Implies $p \in I \cap J \cap K = I\Gamma J\Gamma K$. Since I is a right ideal generated by p, we have $p \in I$ implies $p =$ paxbp and also since J is a lateral ideal generated by p, we have $p \in J$ implies $p = pcydp$. Consider $p =$ paxbp $=$ paxbp$.(pcydp) =$ paxbpcydp then p is a regular element and thus R is a regular TG semiring. □

Theorem 3.11. *If $P\Gamma P\Gamma P = P$, for all bi interior ideals P of a TGS R, then TGS R is regular and*

$$P_1 \cap P_2 \cap P_3 = (P_1\Gamma P_2\Gamma P_3)\Gamma(P_2\Gamma P_3\Gamma P_1)\Gamma(P_3\Gamma P_1\Gamma P_2)$$

for any bi-interior ideals P_1, P_2 and P_3 of R.

Proof. Let $P\Gamma P\Gamma P = P$, for all BIIs P of a TGS R. Let I be a $rt.$ ideal, J be a medial ideal and L be a $lt.$ ideal of R. Then $I \cap J \cap L$ is a BII of $R \ldots (I \cap J \cap L)\Gamma(I \cap J \cap L)\Gamma(I \cap J \cap L)) = (I \cap J \cap L) \Rightarrow I \cap J \cap L \subseteq I\Gamma J\Gamma L$. We have $I\Gamma J\Gamma L \subseteq I \cap J \cap L$. Hence $I \cap J \cap L = (I\Gamma J\Gamma L)$ TGS. □

Theorem 3.12. *If P is a BII of R and $p \in R$ such that $p \in P$ then $\exists$ an $\mathbb{BIIJ}$ of $R \exists$ $P \subseteq J$ and $p \in J$.*

Theorem 3.13. *Suppose R, a regular TGS and $P\Gamma P\Gamma P = P$, for all BII P of R. Then any BII P of R is STIBII $\Leftrightarrow$ P is STPBII.*

Proof. Given R is a regular Γ -semi ring and $P\Gamma P\Gamma P = P$, for all BIIs P of a TGS R. Suppose P be a STIBII of R. Now we show that P is a STPBII. Suppose that $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq P$ then by Theorem [3.11], $P_1 \cap P_2 \cap P_3 = (P_1\Gamma P_2\Gamma P_3)\Gamma(P_2\Gamma P_3\Gamma P_1)\Gamma(P_3\Gamma P_1\Gamma P_2)$ for any bi-interior ideals P_1, P_2 and P_3 of R. $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq P \Rightarrow P_1 \cap P_2 \cap P_3 \subseteq P \Rightarrow P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$. Thus P is a strongly prime bi-interior ideal of R. Reversely assume P is a STPBII of R. Let P_1, P_2 and P_3 be BIIs of $R \exists$ $P_1 \cap P_2 \cap P_3 \subseteq P \Rightarrow (P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) = \{(P_1\Gamma P_2\Gamma P_3)\Gamma(P_2\Gamma P_3\Gamma P_1)\Gamma(P_3\Gamma P_1\Gamma P_2)\} \cap \{(P_1\Gamma P_2\Gamma P_3)\Gamma(P_2\Gamma P_3\Gamma P_1)\Gamma(P_3\Gamma P_1\Gamma P_2)\} \cap \{(P_1\Gamma P_2\Gamma P_3)\Gamma (P_2\Gamma P_3\Gamma P_1)\Gamma(P_3\Gamma P_1\Gamma P_2)\} \subseteq P_1\Gamma R\Gamma P_1 \cap P_1\Gamma P_1\Gamma R \cap P_1\Gamma P_1\Gamma R \subseteq P_1$. It is easy to prove that $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq P_2$ and $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq P_3$. Therefore $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq P_1 \cap P_2 \cap P_3 \subseteq P$. Since P is a strongly prime bi-interior ideal of R, we have $P_1 \subseteq P$ or $P_{2.} \subseteq P$ or $P_3 \subseteq P$. Hence P is a strongly irreducible bi-interior ideal of R. □

Theorem 3.14. *If P is an irreducible bi-interior ideal and $P\Gamma P\Gamma P = P$ of a regular TGS R then P is a strongly irreducible ideal of R.*

Proof. Let P_1, P_2 and P_3 are bi-interior ideals of R such that $P_1 \cap P_2 \cap P_3 \subseteq P$. Then by Theorem [3.11], $P_1 \cap P_2 \cap P_3 = (P_1\Gamma P_2\Gamma P_3)\Gamma(P_2\Gamma P_3\Gamma P_1)\Gamma(P_3\Gamma P_1\Gamma P_2) \Rightarrow (P_1\Gamma P_2\Gamma P_3 \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) = P_1 \cap P_2 \cap P_3 \subseteq P \Rightarrow P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$. Thus P is a strongly irreducible ideal of R. □

Theorem 3.15. *Every proper bi-interior ideal P of R is the intersection of all irreducible bi-interior ideals R containing P.*

Proof. Let $R \neq P$, BI-interior ideal of $TGSR$ and $\{B_k : k \in \Delta\}$ be the collection of irreducible bi- interior ideals and it contains $P \Rightarrow P \subseteq \bigcap_{k\in\Delta} B_k$. Assume that $\bigcap_{k\in\Delta} B_k \nsubseteq P$. Then $\exists p \in \bigcap_{k\in\Delta} B_k$ and $p \notin P$ then by the known theorem, $\exists$ an irreducible bi-interior ideal $D \exists$ $P \subseteq D$ and $p \notin D \Rightarrow p \notin \bigcap_{k\in\Delta} B_k$ it is a contradiction. Our assumption that $\bigcap_{k\in\Delta} B_k \not\subset P$ is wrong. Thus $\bigcap_{k\in\Delta} B_k \subseteq P$. Hence $\bigcap_{k\in\Delta} B_k = P$. □

Theorem 3.16. *The intersection of any family of PBIIs of TGS R is a SPBII.*

Proof. Let $\{B_k : k \in \Delta\}$ be a set of $PBIIs$ of R and $P = \bigcap_{k\in\Delta} B_k$ For any BII P of R, $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq \bigcap_{k\in\Delta} B_k = P \Rightarrow (P_1\Gamma P_2\Gamma P_3) \subseteq (P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq B_k, \forall k \in \Delta \Rightarrow (P_1\Gamma P_2\Gamma P_3) \subseteq B_k, \forall$ for all $k \in \Delta \Rightarrow (P_1\Gamma P_2\Gamma P_3) \subseteq \bigcap_{k\in\Delta} B_k, k \in \Delta = P$. Since each B_k are PBIIs, we have P is a $PBII$ of $R \Rightarrow P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$.

Therefore P is a SPBII. Hence the intersection of any family of PBIIs of TGS R is a SPBII. □

Remark 3.17. *"Family of intersection of BII of R is also a BII of R and it is the set of all BIIs of R form a complete lattice."*

Theorem 3.18. *Strongly irreducible, semi-prime bi- interior ideal of a TGS R is a strongly prime biinterior ideal.*

Proof. Let P be a strongly irreducible and semi-prime bi-interior ideal of a TGS R. For any bi-interior ideals P_1, P_2 and P_3 of R, $(P_1\Gamma P_2\Gamma P_3) \cap (P_2\Gamma P_3\Gamma P_1) \cap (P_3\Gamma P_1\Gamma P_2) \subseteq P$. Hence, by Ref.[22, Theorem 22] and [35, Theorem 22], $P_1 \cap P_2 \cap P_3$ bi-interior ideal of R. $(P_1 \cap P_2 \cap P_3)^3 = (P_1 \cap P_2 \cap P_3)\Gamma(P_2 \cap P_3 \cap P_1)\Gamma(P_3 \cap P_1 \cap P_2) \subseteq (P_1\Gamma P_2\Gamma P_3)$ and since P is strongly irreducible and semi-prime bi-interior ideal of a TGSR, we have

$(P_1 \cap P_2 \cap P_3)^3 \subseteq (P_2 \Gamma P_3 \Gamma P_1), (P_1 \cap P_2 \cap P_3)^3 \subseteq (P_3 \Gamma P_1 \Gamma P_2)$. Therefore

$$(P_1 \cap P_2 \cap P_3)^3 = (P_1 \Gamma P_2 \Gamma P_3) \cap (P_2 \Gamma P_3 \Gamma P_1) \cap (P_3 \Gamma P_1 \Gamma P_2) \subseteq P.$$

Since P is a semi-prime bi-interior ideal of $R, P_1 \cap P_2 \cap P_3 \subseteq P$ and also since P is a strongly irreducible bi-interior ideal, we have $P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$. Hence P is a strongly prime bi- interior ideal of R. □

Theorem 3.19. *Let R be a TG-semi ring. Prove the following:*

1. *The family of BI ideals of R is totally ordered set with respect to set inclusion* $\Leftrightarrow$
2. *Every BII of R is strongly irreducible* $\Leftrightarrow$ (3) *Every BII of R is irreducible.*

Proof. Given R is a TG-semi ring. Suppose, the set of BI ideals of R is a totally ordered set with respect to $\subseteq$. Now we show that each BII of R is strongly irreducible. Let P be any *BI* ideal of R. It is enough to show P is a STIBI ideal of R. Let P_1, P_2 and P_3 be BI ideals of R such that $P_1 \cap P_2 \cap P_3 \subseteq P$. From the hypothesis, we have either $P_1 \subseteq P_2, P_1 \subseteq P_3$ or $P_2 \subseteq P_3, P_2 \subseteq P_1$ or $P_3 \subseteq P_1, P_3 \subseteq P_2 \ldots P_1 \cap P_2 \cap P_3 = P_1$ or $P_1 \cap P_2 \cap P_3 = P_2$ or $P_1 \cap P_2 \cap P_3 = P_3$. Hence $P_1 \subseteq P$ or $P_2 \subseteq P$ or $P_3 \subseteq P$. Thus P is a STIBI ideal of R. Hence $(1) \Rightarrow (2)$.
$(2) \Rightarrow (3)$: Let, every BII of R be STI. To show every BI ideal of R is irreducible. Let B be any *BII* of $R \ni B_1 \cap B_2 \cap B_3 = B, \forall BIIsB_1$, B_2 and B_3 of R. Hence from our assumption (2), we have $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$. As $B \subseteq B_1$ and $B \subseteq B_2$ and $B \subseteq B_3$, we have $B_1 = B$ or $B_2 = B$ or $B_3 = B$. Therefore B is an IBII of R.
$(2) \Rightarrow (1)$: Let each BII of R is an IBI. Let P_1, P_2 and P_3 be any BIIs of R. Then by the remark, $P_1 \cap P_2 \cap P_3$ is also a BII of R. Hence $P_1 \cap P_2 \cap P_3 = P_1 \cap P_2 \cap P_3 \Rightarrow P_1 = P1 \cap P_2 \cap P_3$ or $P_2 = P1 \cap P_2 \cap P_3$ or $P_3 = P_1 \cap P_2 \cap P_3$ by our assumption. $P_1 \subseteq P_2, P_1 \subseteq P_3$ or $P_2 \subseteq P_3, P_2 \subseteq P_1$ or $P_3 \subseteq P_1, P_3 \subseteq P_2$. The collection of all BIIs of R is a totally ordered set under $\subseteq$. Hence given conditions are equivalent. □

4. Conclusion

Generalization of ideals of algebraic structures and ordered algebraic structure plays a very remarkable role and also necessary for further advance studies and applications of various algebraic structures. We introduced the notion of prime bi-interior ideal, semi prime bi-interior ideal, irreducible bi-interior ideal and strongly prime bi-interior ideal of TGSR and explained axioms and relations between them and also characterized regular TGSR and TGSR using PBI ideals.

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