



# On $(5, k_1)$ –regular and totally $(5, k_1)$ regular fuzzy grid

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## Abstract

In 1965, Loft A. Zadeh led the helm of a fuzzy set of a crew as a skill for signifying the marvels of insecurity in genuine realm state of doings. Nagoor Gani and Radha hosted fixed fuzzy grids, total degree and totally fixed fuzzy grids. Alison Northup willful some stuffs on  $(5, k_1)$  – regular graphs in her free view. They lead  $(r, 5, k_1)$  – regular graphs and willful some stuffs on  $(r, 5, k_1)$  – regular graphs. Throughout this broadsheet, we have a trend to shape  $d_5$  – degree and total degree of a peak in fuzzy grids. Any we have a trend to study  $(5, k_1)$  – predictability and totally  $(5, k_1)$  – regularity of fuzzy grids and also the relation between  $(5, k_1)$  – regularity and totally  $(5, 1)$  – regularity. Conjointly we have a trend to learning  $(5, k_1)$  – regularity on trial on six peaks and cycle  $c_n (n \geq 5)$  with some exact association tasks.

## Keywords

$d_5$  – degree and total  $d_5$ - degree of a vertex in fuzzy grids,  $(5, k_1)$  – regular fuzzy grids, totally  $(5, k_1)$  – regular fuzzy grids.

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## 1. Introduction

In 1965, Loft A. Zadeh led the helm of a fuzzy set of a crew as a skill for signifying the marvels of insecurity in genuine realm state of doings. Nagoor Gani and Radha hosted fixed fuzzy grids, total degree and totally fixed fuzzy grids. Alison Northup willful some stuffs on  $(5, k_1)$  – regular graphs in her free view. They lead  $(r, 5, k_1)$  – regular graphs and willful some stuffs on  $(r, 5, k_1)$  – regular graphs. Throughout this broadsheet, we have a trend to shape  $d_5$  – degree and total degree of a peak in fuzzy grids. Any we have a trend to study  $(5, k_1)$  – predictability and totally  $(5, k_1)$  – regularity of

fuzzy grids and also the relation between  $(5, k_1)$  – regularity and totally  $(5, k_1)$  – regularity. Conjointly we have a trend to learning  $(5, k_1)$  – regularity on trial on six peaks and  $c_n$  with some exact association tasks.

## 2. Preliminaries

**Definition 2.1.** Grid  $G_1$ , the  $d_5$ -degree of a peak  $V$  in  $G_1$ , meant by  $d_5(V)$  means number of peaks at a space five rapt as of  $V$ .

**Definition 2.2.** A grid  $G_1$  is alleged to be  $(5, k_1)$  regular ( $d_5$ -regular) if  $d_5(V) = k_1$ , for all  $V$  in  $G_1$ . We observe that  $(5, k_1)$  regular graph and semi-regular graphs and  $d_5$ -regular graphs are the identical.

**Definition 2.3.** The gift of connectedness between two peaks  $u$  and  $v$  is

$$\mu^\infty(uv) = \sup\{\mu^{k_1}(uv) | k = 1, 2, \dots\}$$

$$\mu(uv) = \sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \dots \wedge \mu(u_{k_1-1}v) |$$

$$u, u_1, u_2, \dots, u_{k_1-1} \in V$$

may be a path connecting  $u$  and  $v$  of length  $k_1$  }

**Definition 2.4.** Let  $G : (\sigma, \mu)$  be a fuzzy grid on  $G_1^* : (V, E)$ . If  $d(v) = k_1$  for all  $v \in V$ , then  $G_1$  is alleged to be regular fuzzy graph of degree  $k_1$ .

**Definition 2.5.** Let  $G_1 : (\sigma, \mu)$  be a fuzzy grid on  $G_1^* : (V, E)$ . The total degree of a vertex  $u$  is defined as

$$td(u) = \Sigma\mu(uv) + \sigma(u) = d(u) + \sigma(u), u, v \in E$$

If each vertex of  $G_1$  has the same total degree  $k_1$ , then  $G_1$  is said to be totally regular fuzzy graph of degree  $k_1$  or  $k_1$ -totally regular fuzzy graph.

### 3. $d_5$ -degree of a vertex in fuzzy graphs

**Definition 3.1.** Let  $G_1 : (\sigma, \mu)$  be a fuzzy grid. The  $d_5$ -degree of a vertex  $u$  in  $G_1$  is  $d_5(u) = \Sigma\mu^5(uv)$ , where  $\sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \mu(u_3u_4) \wedge \mu(u_4u_5)\}$  Also  $\mu(uv) = 0$  for  $u, v$  not in  $E$ .

The min  $d_5$ -degree of  $G$  is  $\delta_5(G) = \wedge\{d_5(v) : v \in V\}$

The max  $d_5$ -degree of  $G$  is  $\Delta_5(G) = \vee\{d_5(v) : v \in V\}$

**Example 3.2.** Consider  $G_1^* : (V, E)$  where  $V = \{u, v, w, x, y, z\}$  and  $E = \{uv, vw, wx, xy, yz, zu\}$  Define  $G_1 : (\sigma, \mu)$  by

$\sigma(u) = 0.3, \sigma(v) = 0.2, \sigma(w) = 0.2, \sigma(x) = 0.2, \sigma(y) = 0.2, \sigma(z) = 0.3, \mu(uv) = 0.2, \mu(vw) = 0.2, \mu(wx) = 0.1, \mu(xy) = 0.1, \mu(yz) = 0.2, \mu(zu) = 0.3$

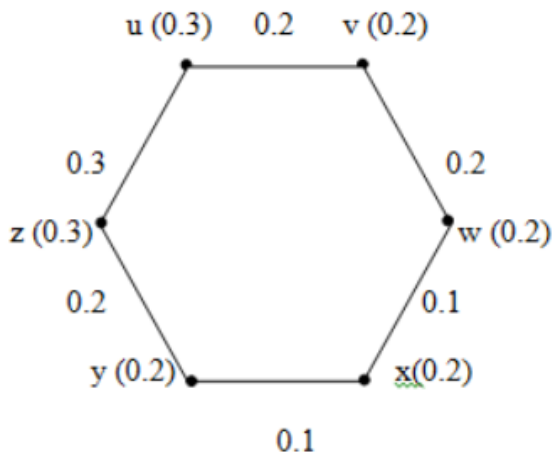


Figure 1

$$d_5(u) = \{0.2 \wedge 0.2 \wedge 0.1 \wedge 0.1 \wedge 0.2\} + \{0.3 \wedge 0.2 \wedge 0.1 \wedge 0.1 \wedge 0.2\} = \{0.1 + 0.1\} = 0.2$$

$$d_5(v) = \{0.2 \wedge 0.1 \wedge 0.1 \wedge 0.2 \wedge 0.3\} + \{0.2 \wedge 0.3 \wedge 0.2 \wedge 0.1 \wedge 0.1\} = \{0.1 + 0.1\} = 0.2$$

$$d_5(w) = \{0.1 \wedge 0.1 \wedge 0.2 \wedge 0.3 \wedge 0.2\} + \{0.2 \wedge 0.2 \wedge 0.3 \wedge 0.2 \wedge 0.1\} = \{0.1 + 0.1\} = 0.2$$

$$d_5(x) = \{0.1 \wedge 0.2 \wedge 0.3 \wedge 0.2 \wedge 0.2\} + \{0.1 \wedge 0.2 \wedge 0.2 \wedge 0.3 \wedge 0.2\} = \{0.1 + 0.1\} = 0.2$$

$$d_5(y) = \{0.2 \wedge 0.3 \wedge 0.2 \wedge 0.2 \wedge 0.1\} + \{0.1 \wedge 0.1 \wedge 0.2 \wedge 0.2 \wedge 0.3\} = \{0.1 + 0.1\} = 0.2$$

$$d_5(z) = \{0.3 \wedge 0.2 \wedge 0.2 \wedge 0.1 \wedge 0.1\} + \{0.2 \wedge 0.1 \wedge 0.1 \wedge 0.2 \wedge 0.2\} = \{0.1 + 0.1\} = 0.2$$

**Example 3.3.** Consider  $G_1^* : (V, E)$  where  $V = \{s, u, v, w, x, y, z\}$  and  $E = \{su, uv, vw, wx, xy, yz, zs\}$  Define  $G_1 : (\sigma, \mu)$  by

$\sigma(s) = 0.3, \sigma(u) = 0.3, \sigma(v) = 0.4, \sigma(w) = 0.6, \sigma(x) = 0.7, \sigma(y) = 0.5, \sigma(z) = 0.4, \mu(su) = 0.3, \mu(uv) = 0.4, \mu(vw) = 0.2, \mu(wx) = 0.3, \mu(xy) = 0.3, \mu(yz) = 0.4, \mu(zs) = 0.3$

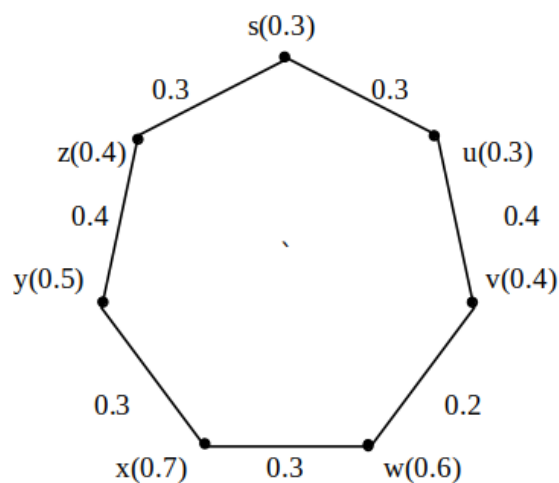


Figure 2

$$d_5(s) = \sup\{0.3 \wedge 0.4 \wedge 0.2 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.4 \wedge 0.3 \wedge 0.3 \wedge 0.2\} = \sup\{0.2, 0.2\} = 0.2$$

$$d_5(u) = \sup\{0.4 \wedge 0.2 \wedge 0.3 \wedge 0.3 \wedge 0.4, 0.3 \wedge 0.3 \wedge 0.4 \wedge 0.3 \wedge 0.3\} = \sup\{0.2, 0.3\} = 0.3$$

$$d_5(v) = \sup\{0.2 \wedge 0.3 \wedge 0.3 \wedge 0.4 \wedge 0.3, 0.4 \wedge 0.3 \wedge 0.3 \wedge 0.4 \wedge 0.3\} = \sup\{0.2, 0.3\} = 0.3$$



$$d_5(w) = \sup\{0.3 \wedge 0.3 \wedge 0.4 \wedge 0.3 \wedge 0.3, 0.2 \wedge 0.4 \wedge 0.3 \wedge 0.3 \wedge 0.4\} = \sup\{0.3, 0.2\} = 0.3$$

$$d_5(x) = \sup\{0.3 \wedge 0.4 \wedge 0.3 \wedge 0.3 \wedge 0.4, 0.3 \wedge 0.2 \wedge 0.4 \wedge 0.3 \wedge 0.3\} = \sup\{0.3, 0.2\} = 0.3$$

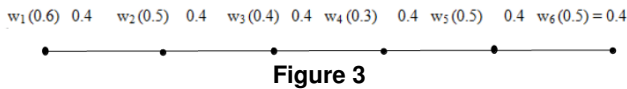
$$d_5(y) = \sup\{0.4 \wedge 0.3 \wedge 0.3 \wedge 0.4 \wedge 0.2, 0.3 \wedge 0.3 \wedge 0.2 \wedge 0.4 \wedge 0.3\} = \sup\{0.2, 0.2\} = 0.2$$

$$d_5(z) = \sup\{0.3 \wedge 0.3 \wedge 0.4 \wedge 0.2 \wedge 0.3, 0.4 \wedge 0.3 \wedge 0.3 \wedge 0.2 \wedge 0.4\} = \sup\{0.2, 0.2\} = 0.2$$

#### 4. $(5, k_1)$ - regular and totally $(5, k_1)$ - regular graphs

**Definition 4.1.** Let  $G : (\sigma, \mu)$  be a fuzzy grid on  $G_1^* : (V, E)$ . If  $d_5 = k_1$  for all  $v \in V$ , then  $G$  is so-called to be  $(5, k_1)$  - regular fuzzy graph.

**Example 4.2.** Reflect  $G_1^* : (V, E)$  anywhere  $V = \{w_1, w_2, w_3, w_4, w_5, w_6\}$  and  $E = \{w_1w_2, w_2w_3, w_3w_4, w_4w_5, w_5w_6\}$ . Define  $G_1 : (\sigma, \mu)$  by  $\sigma(w_1) = 0.6, \sigma(w_2) = 0.5, \sigma(w_3) = 0.4, \sigma(w_4) = 0.3, \sigma(w_5) = 0.5, \sigma(w_6) = 0.6$  and  $\mu(w_1w_2) = 0.4, \mu(w_2w_3) = 0.4, \mu(w_3w_4) = 0.4, \mu(w_4w_5) = 0.4, \mu(w_5w_6) = 0.4$ .



Here  $d_5(w_1) = 0.4, d_5(w_2) = 0.4, d_5(w_3) = 0.4, d_5(w_4) = 0.4, d_5(w_5) = 0.4$  and  $d_5(w_6) = 0.4$ . This graph could be a  $(5, 0.4)$  - regular fuzzy graph.

**Definition 4.3.** Let  $G : (\sigma, \mu)$  be a fuzzy grid on  $G_1^* : (V, E)$ . The entire  $d_5$  degree of a vertex  $u \in V$  is defined as

$$d_5(u) = \sum \mu^5(uv) + \sigma(u) = d_5(u) + \sigma(u)$$

**Definition 4.4.** If all peak of  $G_1$  has the same total  $d_5$ -degree  $k_1$ , then  $G_1$  is sued to be totally  $(5, k_1)$  - regular fuzzy grid.

**Example 4.5.** A  $(5, k_1)$  - regular fuzzy grid needn't be a  $(5, k_1)$  - totally regular fuzzy graph. Reflect  $G_1^* : (V, E)$  anywhere  $V = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\}$  and  $E = \{s_1s_2, s_2s_3, s_3s_4, s_4s_5, s_5s_6, s_6s_7, s_7s_8, s_8s_9, s_9s_{10}, s_{10}s_1\}$ .

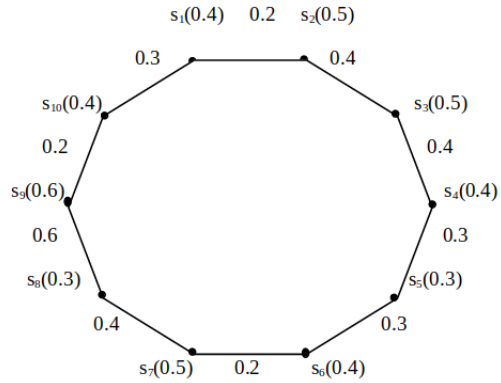


Figure 4

Here,  $d_5(s_1) = 0.4, d_5(s_2) = 0.4, d_5(s_3) = 0.4, d_5(s_4) = 0.4, d_5(s_5) = 0.4, d_5(s_6) = 0.4, d_5(s_7) = 0.4, d_5(s_8) = 0.4, d_5(s_9) = 0.4, d_5(s_{10}) = 0.4$  and  $td_5(s_1) = 0.8, td_5(s_2) = 0.9, td_5(s_3) = 0.9, td_5(s_4) = 0.8, td_5(s_5) = 0.7, td_5(s_6) = 0.8, td_5(s_7) = 0.9, td_5(s_8) = 0.7, td_5(s_9) = 1.0, td_5(s_{10}) = 0.8$

Each vertex has same  $d_5$ -degree 0.4. So  $G_1$  is  $(5, 0.4)$  regular fuzzy grid. But  $G_1$  isn't a totally  $(5, k_1)$  regular fuzzy grid.

**Example 4.6.** An exact  $(5, k_1)$  regular fuzzy grid needn't be a  $(5, k_1)$  regular fuzzy grid. Reflect  $G_1^* : (V, E)$  anywhere  $V = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$  and  $E = \{b_1b_2, b_2b_3, b_3b_4, b_4b_5, b_5b_6, b_6b_7, b_7b_8, b_8b_1\}$ .

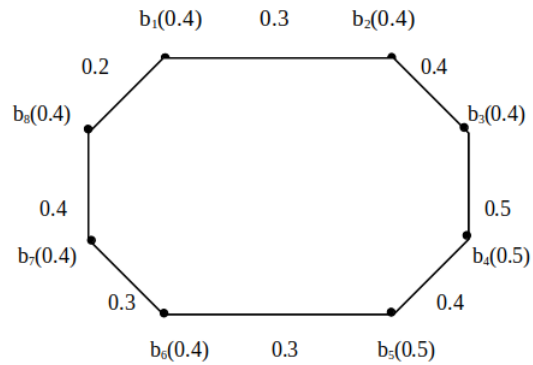
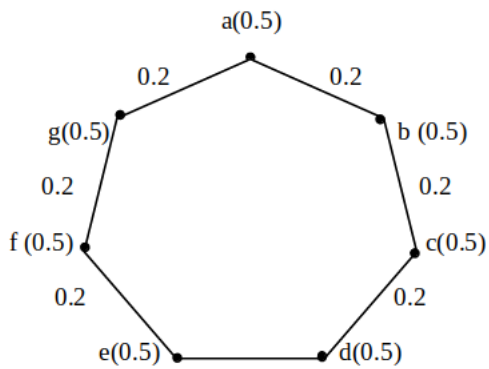


Figure 5

Here  $d_5(b_1) = 0.5, d_5(b_2) = 0.5, d_5(b_3) = 0.5, d_5(b_4) = 0.4, d_5(b_5) = 0.4, d_5(b_6) = 0.5, d_5(b_7) = 0.5, d_5(b_8) = 0.5$  and  $td_5(b_1) = 0.9, td_5(b_2) = 0.9, td_5(b_3) = 0.9, td_5(b_4) = 0.9, td_5(b_5) = 0.9, td_5(b_6) = 0.9, td_5(b_7) = 0.9, td_5(b_8) = 0.9$ . Each vertex has the identical totally  $d_5$ - degree 0.9. So  $G_1$  is totally  $(5, 0.9)$  - regular fuzzy grid. But  $G_1$  is not  $(5, k_1)$  regular fuzzy grid.

**Example 4.7.** A  $(5, k_1)$  - regular fuzzy grid which is totally  $(5, k_1)$  regular fuzzy grid. Reflect  $G_1^* : (V, E)$  anywhere  $V = \{a, b, c, d, e, f, g\}$  and  $E = \{ab, bc, cd, de, ef, fg, ga\}$ .





0.2  
Figure 6

Here  $d_5(a) = 0.2, d_5(b) = 0.2, d_5(c) = 0.2, d_5(d) = 0.2, d_5(e) = 0.2, d_5(f) = 0.2$  and  $td_5(a) = 0.7, td_5(b) = 0.7, td_5(c) = 0.7, td_5(d) = 0.7, td_5(e) = 0.7, td_5(f) = 0.7, td_5(g) = 0.7$ .

Each vertex has the identical  $d_5$ -degree 0.2. So  $G_1$  may be a  $(5, 0.2)$  regular fuzzy graph. Each vertex has the identical total  $d_5$ -degree 0.8. So  $G_1$  could be a totally  $(5, 0.7)$  regular fuzzy graph.

**Theorem 4.8.** Let  $G : (\sigma, \mu)$  be a fuzzy grid on  $G_1^* : (V, E)$ . Then  $\sigma(u) = c_1$ , for all  $u \in V$  if and as long as the later after illnesses are the same.

1.  $G_1 : (\sigma, \mu)$  is a  $(5, k_1)$ -regular fuzzy grid.
2.  $G_1 : (\sigma, \mu)$  may be a totally  $(5, k_1 + c_1)$ -regular fuzzy grid.

*Proof.* Expect that  $\sigma(u) = c_1$ , for all  $u \in V$ . Adopt that  $G_1 : (\sigma, \mu)$  is a  $(5, k_1)$ -regular fuzzy grid. Then  $d_5(u) = k_1$ , for all  $u \in V$ . Hence,  $td_5(u) = d_5(u) + \sigma(u)$  for all  $u \in V$ . Therefore,  $td_5(u) = k_1 + c_1$  for all  $u \in V$ . Therefore,  $G_1 : (\sigma, \mu)$  could be a totally  $(5, k_1 + c_1)$ -regular fuzzy grid. Thus  $1 \Rightarrow 2$  is proved.

Suppose  $G_1 : (\sigma, \mu)$  is a totally  $(5, k_1 + c_1)$ -regular fuzzy grid.

- $\Rightarrow td_5(u) = k_1 + c_1$  for all  $u \in V$
- $\Rightarrow d_5(u) + \sigma(u) = k_1 + c_1$  for all  $u \in V$
- $\Rightarrow d_5(u) + c_1 = k_1 + c_1$  for all  $u \in V$
- $\Rightarrow d_5(u) = k_1$  for all  $u \in V$

Therefore,  $G_1 : (\sigma, \mu)$  could be a  $(5, k_1)$ -regular fuzzy grids. Thus  $2 \Rightarrow 1$  is proved. Hence (1) & (2) are equivalent. Equally adopt that (1) & (2) are same. Let  $G_1 : (\sigma, \mu)$  could be a totally  $(5, k_1 + c_1)$ -regular fuzzy grid and  $(5, k_1)$ -regular fuzzy grid.

- $\Rightarrow td_5(u) = k_1 + c_1$  and  $d_5(u) = k_1$  for all  $u \in V$
- $\Rightarrow d_5(u) + \sigma(u) = k_1 + c_1$  and  $d_5(u) = k_1$  for all  $u \in V$
- $\Rightarrow k_1 + \sigma(u) = k_1 + c_1$  for all  $u \in V$
- $\Rightarrow \sigma(u) = c_1$  for all  $u \in V$

Hence,  $\sigma(u) = c_1$  for all  $u \in V$  □

### 5. $(5, k_1)$ -regular fuzzy graphs on a path on 6 vertices with some specific membership Functions

Throughout this segment  $(5, k_1)$ -regularity and totally  $(5, k_1)$ -regularity on fuzzy graph whose underlying crisp graph may be a path on 6 apexes is planned with some exact affiliation tasks.

**Theorem 5.1.** Tenancy  $G : (\sigma, \mu)$  be a fuzzy grid stated  $G_1^* : (V, E)$  could be a trail on six apexes. Then,  $G : (\sigma, \mu)$  is a  $(5, k_1)$ -regular fuzzy grid if  $\mu(uv) = k_1$  for all  $uv \in V$ .

*Proof.* Suppose that could be a constant function say  $\mu(uv) = k_1$  for all  $uv \in V$ , then  $d_5(v) = k_1$ , for all  $v \in V$ . Hence,  $G_1$  is a  $(5, k_1)$  regular fuzzy graph. □

**Example 5.2.** Cogitate a fuzzy grid  $G : (\sigma, \mu)$  such that  $G_1^* : (V, E)$  may be a trail on six peaks.

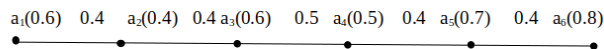


Figure 7

Here  $d_5(a_1) = 0.4, d_5(a_2) = 0.4, d_5(a_3) = 0.4, d_5(a_4) = 0.4, d_5(a_5) = 0.4$  and  $d_5(a_6) = 0.4$ . So  $G_1$  is  $(5, 0.4)$ -regular. But  $\mu$  isn't a relentless task.

### 6. $(5, k_1)$ Regularity on a cycle with some specific membership functions

During this section,  $(5, k_1)$ -regularity on cycle  $C_n$  is studied with some specific membership functions.

**Theorem 6.1.** Let  $G_1 : (\sigma, \mu)$  be a fuzzy graph such  $G_1 : (V, E)$  is phase of size  $\geq 6$ . If  $\mu$  could be a perpetual task, then  $G_1$  could be a  $(5, k_1)$ -regular fuzzy grid where  $k_1 = 2\mu(uv)$ .

*Proof.* Expect that,  $\mu$  is perpetual task say  $\mu(uv) = c_1$  for all  $uv \in E$ , then  $d_5(v) = 2c_1$ . Hence  $G_1$  stands  $(5, 2c_1)$ -regular fuzzy grid. □

**Remark 6.2.** As a model muse  $G_1 : (\sigma, \mu)$  a fuzzy grid listed  $G_1 : (V, E)$  is an odd set of size seven. Anywhere  $V = \{a, b, c, d, e, f, g\}$  and  $E = \{ab, bc, cd, de, ef, fg, ga\}$ .

Define  $G_1 : (\sigma, \mu)$  by

$\sigma(a) = 0.5, \sigma(b) = 0.5, \sigma(c) = 0.6, \sigma(d) = 0.5, \sigma(e) = 0.3, \sigma(f) = 0.7, \sigma(g) = 0.5, \mu(ab) = 0.3, \mu(bc) = 0.4, \mu(cd) = 0.3, \mu(de) = 0.3, \mu(ef) = 0.4, \mu(fg) = 0.3, \mu(ga) = 0.3$



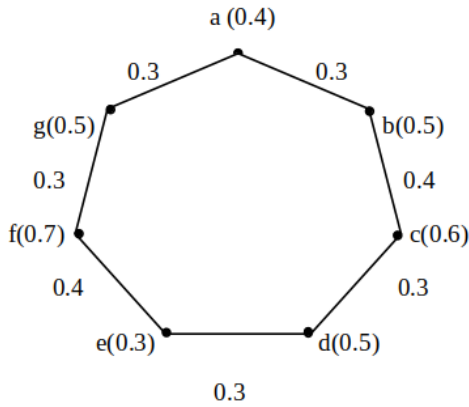


Figure 8

Here,  $d_5(a) = 0.6, d_5(b) = 0.6, d_5(c) = 0.6, d_5(d) = 0.6, d_5(e) = 0.6, d_5(f) = 0.6$  and  $d_5(g) = 0.6$ . So  $G_1$  could be a  $(5, 0.6)$ -regular fuzzy grid. But  $\mu$  isn't a harsh task.

**Theorem 6.3.** Let  $G_1 : (\sigma, \mu)$  a fuzzy grid listed  $G_1 : (V, E)$  is a superb phase. If the limits have the same affiliation prices, then  $G$  may be a  $(5, k_1)$ -regular fuzzy grid.

*Proof.* If the alternate edges have the identical membership values, then

$$\mu(e_i) = \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 & \text{if } i \text{ is even} \end{cases}$$

If  $c_1 = c_2$ , then  $\mu$  could be a continual task. So  $G_1$  could be a  $(5, 2c_1)$ -regular fuzzy grid.

If  $c_1 < c_2$ , then  $d_5(v) = 2c_1$  for all  $v \in V$ . So  $G_1$  could be a  $(5, 2c_1)$ -regular fuzzy grid.

If  $c_1 > c_2$ , then  $d_5(v) = 2c_2$  for all  $v \in V$ . So  $G_1$  may be a  $(5, 2c_1)$ -regular fuzzy grid.  $\square$

**Example 6.4.** Reflect  $G_1 : (\sigma, \mu)$  a fuzzy grid specified  $G_1 : (V, E)$  is offbeat set of size

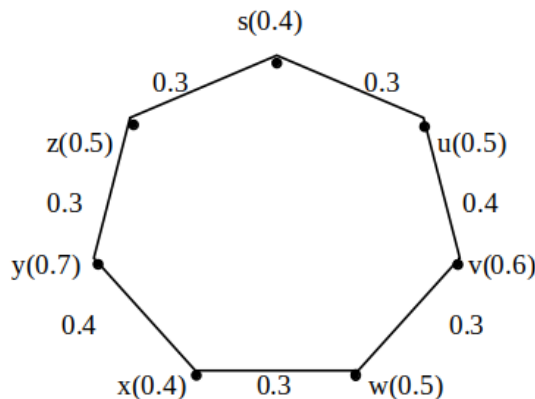


Figure 9

Here  $d_5(s) = 0.6, d_5(u) = 0.6, d_5(v) = 0.6, d_5(w) = 0.6, d_5(x) = 0.6, d_5(y) = 0.6$  and  $d_5(z) = 0.6$ . So  $G_1$  is  $(5, 0.6)$ -regular grid.

**Theorem 6.5.** Let  $G_1 : (\sigma, \mu)$  be a fuzzy grid such  $G_1^* : (V, E)$  is cycle of length  $\geq 6$ . Tenancy

$$\mu(e_i) = \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 \geq c_1 & \text{if } i \text{ is even} \end{cases}$$

Then  $G_1$  may be a  $(5, k_1)$ -regular fuzzy graph.

*Proof.* Tenancy

$$\mu(e_i) = \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 \geq c_1 & \text{if } i \text{ is even} \end{cases}$$

**Case (i):**

Tenancy  $G_1 : (\sigma, \mu)$  be a fuzzy grid such  $G_1^* : (V, E)$  is a fair phase of size  $\leq 6$ .

$$d_5(v_i) = \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1$$

for all  $v_i \in V$ . So  $G_1$  capability is a  $(5, 2c_1)$ -regular fuzzy grid.

**Case (ii):**

Let  $G_1 : (\sigma, \mu)$  be a fuzzy grid such  $G_1^* : (V, E)$  is an odd cycle of length  $\geq 7$ .

Let  $e_1, e_2, e_3, \dots, e_{2n+1}$  be perimeters of an odd cycle of  $G_1^*$  in this order.

$$d_5(v_1) = \{\mu(e_1) \wedge \mu(e_2)\} + \{\mu(e_{2n}) \wedge \mu(e_{2n+1})\} \\ = \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1$$

$$d_5(v_2) = \{\mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_2) \wedge \mu(e_3)\} \\ = \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1$$

for  $i = 3, 4, \dots, 2n$

$$d_5(v_i) = \{\mu(e_{i-1}) \wedge \mu(e_{i-2})\} + \{\mu(e_{i+1}) \wedge \mu(e_{i+2})\} \\ = \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1$$

$$d_5(v_{2n}) = \{\mu(e_1) \wedge \mu(e_{2n+1})\} + \{\mu(e_{2n}) \wedge \mu(e_{2n-1})\} \\ = \{c_1 \wedge c_2\} + \{c_2 \wedge c_1\} = c_1 + c_1 = 2c_1$$

$$\therefore d_5(v_i) = 2c_1 \text{ for all } v_i \in V$$

So  $G_1$  may be a  $(5, 2c_1)$ -regular fuzzy grid.  $\square$

**Remark 6.6.** Tenancy  $G_1 : (\sigma, \mu)$  be a fuzzy grid such  $G_1^* : (V, E)$  could be a phase of size  $\geq 6$ . Even if

$$\mu(e_i) = \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 \geq c_1 & \text{if } i \text{ is even} \end{cases}$$

$G_1$  needn't be a very  $(5, k_1)$ -regular fuzzy grid. Since if  $\sigma$  isn't a permanent task then  $G_1$  exists not an exact  $(5, k_1)$ -regular fuzzy grid.



## 7. Conclusion

In this paper,  $(5, k_1)$  – regular fuzzy graphs and totally  $(5, k_1)$  regular fuzzy graphs are compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also we provide  $(5, k_1)$  regular fuzzy graphs and totally  $(5, k_1)$  – regular fuzzy graphs in which underlying crisp graphs are a path on six vertices, and a cycle of length  $\geq 5$  is studied with some specific membership function. The results discussed may be used to study about various fuzzy graphs.

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