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A network communication through McGee graph and Antimagic labeling

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Abstract

Let G be a simple graph with p nodes and q links. A one to one correspondence between the set of links and the set of integers $\{1,2,\ldots,q\}$ is called the Antimagic labeling if the sum of the link labels incident with a node is different for all nodes. If the Antimagic labeling is assignable on a graph, it is termed as an Antimagic graph. In this paper, on the McGee graph, the Antimagic, the Even Antimagic and the Odd Antimagic graphs can be allotted is proved. Through the McGee graph and Antimagic labeling, an application for Network Communication is presented.

Keywords

McGee graph, Antimagic labeling, Even Antimagic labeling, Odd Antimagic labeling, Bus topology and Star topology.

AMS Subject Classification

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1. Introduction

The Authors of this paper were inspired by the Dynamic Survey of Graph Labelings by J.A.Gallian [1] to take up their Research work on labelings. Having presented a coding technique with a combination of McGee Graph and a Prime Cordial labeling [2], they were looking for another labeling. They were motivated to choose the Antimagic labeling from the result," a non-bipartite, regular graph of at least degree three is an Antimagic graph" [3]. As the McGee Graph is non-bipartite and a regular cubic graph, they worked on McGee Graph [4], Antimagic labeling and provided a solution for a network problem using Bus topology and Star topology [5].

2. Pre – Requisites

Definition 2.1 (McGee Graph)**.** *The McGee graph contains 24 nodes and 36 links. It is a special graph as it is the smallest 3-regular graph with girth 7.*

Definition 2.2 (Antimagic Labeling)**.** *For a graph G with p nodes, q links and without any isolated vertex, an Antimagic edge labeling is a bijection $f : E \to \{1,2,\ldots,q\}$, such that the induced vertex sum $f^+ : V \to N$ given by $f^+(u) = \{\Sigma f(uv) : uv \in E\}$ is injective.*

A graph which admits the Antimagic labeling is called Antimagic graph.

Definition 2.3 (Even Antimagic Labeling)**.** *A graph G is said to be an Even Antimagic graph if there is a bijection $f : E \to \{2,4,\ldots,2q\}$ and $f^+ : \{v_1,v_2,\ldots v_p\} \to N$ such that $f^+(u) = \{\Sigma f(uv) : uv \in E\}$ is injective.*

A graph which admits the Even Antimagic labeling is called Even Antimagic graph.

Definition 2.4 (Odd Antimagic Labeling)**.** *An Odd Antimagic edge labeling is a one to one correspondence between the set of edges and the set $\{1,3,\ldots,2q-1\}$ such that no two vertex sum is the same.*

A graph with Odd Antimagic labeling is called Odd Antimagic graph.

Definition 2.5 (Bus Topology)**.** *Bus topology is used to LAN (Local Area Network). All the computers are connected to a single communication line. So all the computers can receive the information. If the communication line fails, the entire network is lost.*

Definition 2.6 (Star Topology)**.** *All the stations in a star topology are connected to a central unit called hub. The hub refers to a common connection for all the stations on the network. Each station has its own direct cable connection to the hub.*

3. Results

Theorem 3.1. *The McGee graph is an Antimagic graph.*

Proof. Let $\{v_1, v_2, \ldots, v_{24}\}$, be the vertices and $\{e_1, e_2, \ldots, e_{36}\}$, be the edges of the McGee graph G. Define the labeling function f as follows:

$$f : E(G) \to \{1, 2, \ldots, q\}$$

$$f(e_i) = \begin{cases} i & \text{for } i = 1-12; 15, 16, 19, 20; \\ & 23-28; 31-36 \\ i+1 & \text{for } i = 13, 17, 21, 29 \\ i-1 & \text{for } i = 14, 18, 22, 30 \end{cases}$$

Verification of vertex labeling:

The above edge labeling pattern gives different label sum at each vertex and it is verified here. $S(v_i)$ is the total sum of the labelings of the edges with v_i as an end vertex for $i = 1-24$.

1. e_1, e_{24}, e_{25} are incident at v_1. Therefore
   $S(v_1) = f(e_1) + f(e_{24}) + f(e_{25}) = 1 + 24 + 25 = 50$
2. At v_{14}, the edges e_{13},e_{14} and e_{35} are incident and they have different rules for the labeling.
   For e_{13} '$i+1$'; for e_{14} '$i-1$'and for e_{35} 'i'.Therefore
   $S(v_{14}) = f(e_{13}) + f(e_{14}) + f(e_{35}) = (13+1) + (14-1) + 35 = 62$
   and so on.

It is easily seen that $S(v_i) \neq S(v_j)$ for $i \neq j$ where $i, j \in \{1, 2, \ldots 24\}$.

Hence the McGee graph is found to admit the Antimagic labeling. So, the McGee graph is an Antimagic graph is proved. □

Theorem 3.2. *The McGee graph is an Even Antimagic Graph.*

Proof. Let $v_1, v_2, \ldots, v_{24}\}$, be the vertices and $\{e_1, e_2, \ldots, e_{36}\}$, be the edges of the McGee graph G. Define the labeling function f as follows:

$$f : E(G) \to \{2, 4, \ldots, 2q\}$$

$$f(e_i) = \begin{cases} 2i & \text{for } i = 1-12; 15, 16, 19, 20; \\ & 23-28; 31-36 \\ 2i+2 & \text{for } i = 13, 17, 21, 29 \\ 2i-2 & \text{for } i = 14, 18, 22, 30 \end{cases}$$

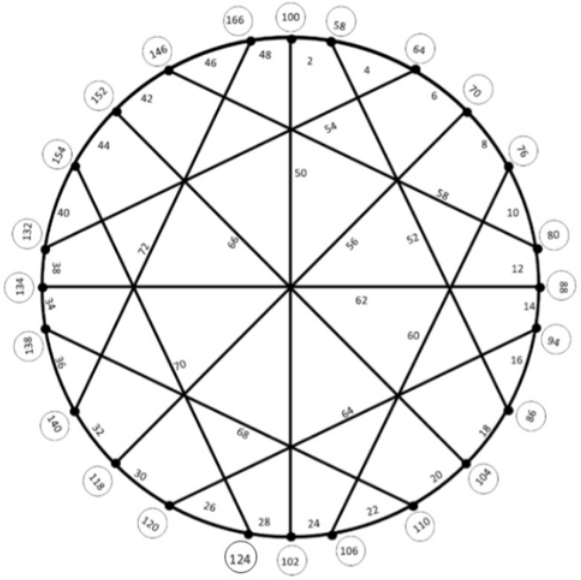

Figure 1. McGee graph with Even Antimagic labeling

From the labelled graph it is noted that $S(v_i) \neq S(v_j)$ for $i \neq j$ where $i, j \in \{1, 2, \ldots 24\}$. Hence the McGee graph is found to admit the Even Antimagic labeling. So, the McGee graph is an Even Antimagic graph is proved. □

Theorem 3.3. *The McGee graph is an Odd Antimagic Graph.*

Proof. Let $\{v_1, v_2, \ldots, v_{24}\}$, be the vertices and $\{e_1, e_2, \ldots, e_{36}\}$, be the edges of the McGee graph G. Define the labeling function f as follows:

$$f : E(G) \to \{1, 3, \ldots, 2q-1\}$$

$$f(e_i) = \begin{cases} 2i-1 & \text{for } i = 1-12; 15, 16, 19, 20; \\ & 23-28; 31-36 \\ 2i+1 & \text{for } i = 13, 17, 21, 29 \\ 2i-3 & \text{for } i = 14, 18, 22, 30 \end{cases}$$

Verification of vertex labeling:

The above edge labeling pattern gives different label sum at each vertex and it is verified here. $S(v_i)$ is the total sum of the labelings of the edges with v_i as an end vertex for $i = 1-24$.

1. At v_2, the edges e_1, e_2, e_{26} are incident. Therefore
   $S(v_2) = f(e_1) + f(e_2) + f(e_{26}) = 1 + 3 + 51 = 55$
2. At v_{18}, the edges e_{17},e_{18} and e_{34} are incident and they have different rules for the labeling.

For e_{17} '$2i+1$'; for e_{18} '$2i-3$'and for e_{34} '$2i-1$'.Therefore

$S(v_{18}) = f(e_{17}) + f(e_{18}) + f(e_{34}) = (34+1) + (36-3) + (68-1) = 135$

and so on.

From the graph $S(v_i) \neq S(v_j)$ for $i \neq j$ where $i, j \in \{1, 2, \ldots 24\}$. Hence the McGee graph is found to admit the Odd Antimagic labeling. So, the McGee graph is an Odd Antimagic Graph is proved.

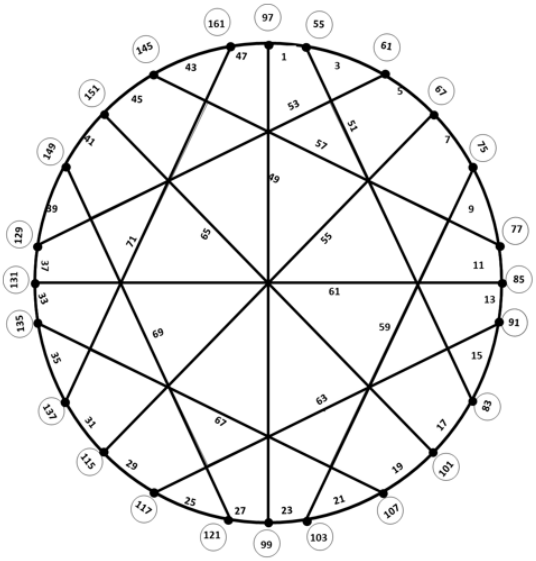

Figure 2. McGee graph with Odd Antimagic labeling

□

4. Applications

The following is a problem posed and solved using McGee graph, Antimagic labelling in providing the network connection.

NATIVI, the most famous builders of the city is in the completion of Sha-She Gardens consisting of 24 massive buildings, each with 100 apartments. The builders face a problem in providing a network connection which has to satisfy the following conditions.

- All buildings must be connected.
- Every building must be connected to only three other buildings directly.
- The number of connections between two buildings varies from a single connection, 2 connections and so on with a maximum number of 36 connections.
- No two buildings should have the same number of connections and the total number of connections to any building should not exceed 100.

The Builders entrusts the work to Alb-Sam, the network consultant. The network consultant visualises the problem.

The 24 buildings, each connected to 3 buildings imply that 72 connections are required. But connection between B_i to B_j is not different from B_j to B_i. So, the 72 connections get reduced to 36 connections.

The number of buildings 24 and the number of connections 36 make the person at solving, strike at the McGee graph, realising the role of the Antimagic labeling and provide the solution for connection as shown below.

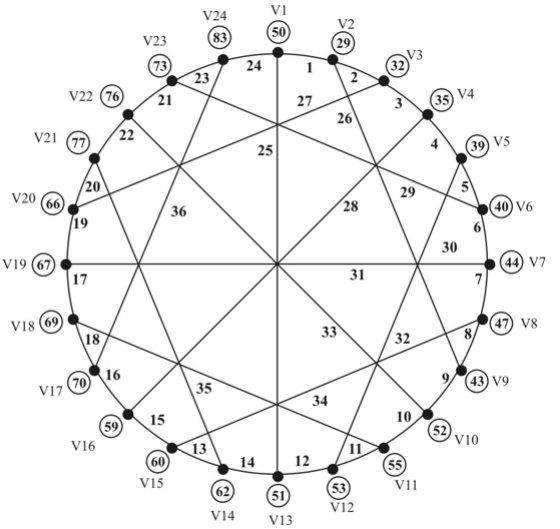

Figure 3. McGee graph with Antimagic labeling

Here, the Bus topology and Star topology are made use of

- A single connection to 24 buildings through a Bus topology.

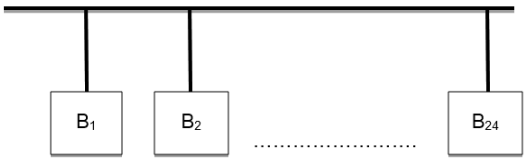

Figure 4. Bus Topology between the buildings

- A Star topology with the hub at each building connected to three buildings as mentioned above.
- Another Star topology between the apartments in two different buildings.

Any 24 apartments of B_{24} to be connected to an apartment of B_1. This can be considered as a star topology with $B_1(A_i)$ as the hub, where i, i_1, i_2, …, i_{24} take any value between 1 to 100. As required by the Builders the network connection is given.

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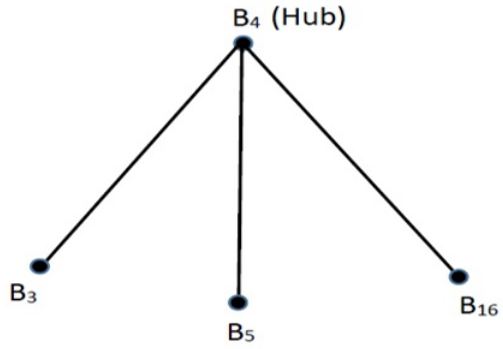

Figure 5. Star topology between the buildings

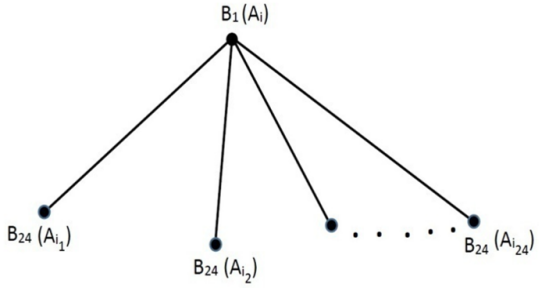

Figure 6. Star topology between the Apartments

5. Conclusion

The problem given by the Builders looks complicated. The Network Connection problem is simplified with a graph and a suitable graph labeling. Here a heterogeneous connectivity problem in Networks is given a solution with the knowledge of graph theory.

It is expected that more and more graphs with a variety of graph labelings may provide solutions for much complicated network problems.

References

[1] J.A.Gallian: A dynamic survey of graph labelling, *The Electronic Journal of Combinatorics* 6(2010), ≠D56.

[2] D.Sathiya, G.Margaret Joan Jebarani and A.Merceline Anita: Coding Messages through Prime Cordial Labeling on McGee graph, *Advances in Mathematics: Scientific Journal* 9 (8) (2020), 6169 – 6175.

[3] F Chang, YC Liang, Z Pan and X Zhu, Antimagic labeling of regular graphs, *J. Graph Theory*, Wiley Online Library, (2016).

[4] F. Harary, *Graph Theory*, Addison – Wesley, Reading, Mass, (1972).

[5] P.Rizwan Ahamed, *Network Security and Cryptography*, Margham Publications, Chennai, (2013).

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