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Quotient ternary seminear rings

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Abstract
We introduce the notion of Quotient ternary seminear ring by the notion of congruence in ternary seminear ring and classify their characteristics. The concept of cancellative congruence in ternary seminear ring are also defined and studied some of their interesting properties.

Keywords
Ternary seminear ring, equivalence class, congruences, quotient set, cancellative congruences

AMS Subject Classification
16Y30, 16Y99, 17A40, 18A32.

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Contents

1. Introduction

The notion of congruence was first introduced by Karl Fredrich Gauss in the beginning of the nineteenth century. Congruence is a special type of equivalence relation which plays a vital role in the study of quotient structures of different algebraic structures. Kar. S and Maity.B.K.[3] studied the quotient structures of ternary semigroup by using the notion of congruences on ternary semigroup and where the notion of quotient seminear ring was fathered by Hussain. F, Tahir. M, Abdullah. S and Sadiq. N.[2] We introduced the notion of ternary seminear rings in [5]. In this paper we introduce the notion of Quotient ternary seminear rings and derived their properties, proved Isomorphism theorems on ternary seminear rings. We also define the Cancellative congruence in ternary seminear rings and obtain their properties.

2. Preliminaries

In this section, we recall some definitions and basic results which will be used throughout the paper.

Definition 2.1. *A ternary seminear ring is a nonempty set T together with a binary operation called addition '+' and a ternary operation called ternary multiplication denoted by juxtaposition, such that $(i)(T,+)$ is a semigroup. $(ii)T$ is a ternary semigroup under ternary multiplication.*
$(iii)xy(z+u) = xyz + xyu$ for all $x,y,z,u \in T$.

Definition 2.2. *A ternary seminear ring T is said to have an absorbing zero if there exists an element $0 \in T$ such that $i)x+0 = 0+x = x$ for all $x \in T$. $ii)xy0 = x0y = 0xy = 0$ for all $x,y \in T$.*

remark 2.3. *Throughout this paper T will always stand for the ternary seminear ring will always mean that ternary seminear ring with an absorbing zero.*

Definition 2.4. *Let T and U be two ternary seminear rings and $\phi : T \to U$ be a mapping. ϕ is said to be a Homomorphism of T into U if (i) $\phi(x+y) = \phi(x) + \phi(y)$ and (ii) $\phi(xyz) = \phi(x)\phi(y)\phi(z)$ for all x,y,z $\in$ T.*

Definition 2.5. *Let T and U be two ternary seminear rings and $\phi : T \to U$ be a homomorphism*

- *If $T = U$ then ϕ is said to be an Endomorphism.*
- *If ϕ is also onto then it is called an Epimorphism.*
- *If ϕ is also one to one then it is called Monomorphism.*
- *If homomorphism ϕ is epimorphism as well as monomorphism then it is said to be an Isomorphism.*
- *A isomorphism ϕ is said to be an automorphism if T=U.*

Definition 2.6. *Let T and U be two ternary seminear rings. If $\phi : T \to U$ is an isomorphism then T is said to be isomorphic to U and it is denoted as $T \cong U$.*

Definition 2.7. *Let $(R,+,.)$ be a seminear ring. A relation ρ on R is called left compatible if for all $s,t,a \in R$ such that $(s,t) \in \rho$ implies that $(a+s,a+t),(a.s,a.t) \in \rho$. ρ is called right compatible if for all $s,t,b \in R$ such that $(s,t) \in \rho$ implies that $(s+b,t+b),(s.b,t.b) \in \rho$. Then ρ is called compatible if for all $s,t,u,v \in R$ such that $(s,t),(u,v) \in \rho$ which implies that $(s+u,t+v),(s.u,t.v) \in \rho$. A left(respectively right) compatible equivalence relation is said to be a left(respectively right) congruence relation. A compatible equivalence relation is said to be a congruence relation.*

Definition 2.8. *If ρ is a congruence on a ternary semigroup S, then we can define a ternary multiplication on the quotient set S/ρ by $(a/\rho)(b/\rho)(c/\rho) = (abc)/\rho$ for all $a,b,c \in S$.*

Definition 2.9. *A congruence ρ on a ternary semigroup S is called a cancellative congruence on S if S/ρ is a cancellative ternary semigroup.*

3. Quotient Ternary Seminear Rings

In this section the concept of **quotient ternary seminear rings** is defined and their characterization are obtained.

Definition 3.1. *Let $(T,+,.)$ be a ternary seminear ring. An equivalence relation ρ on T is called*
(i) Left Congruence if for all $x,y,u,v \in T, (x,y) \in \rho$ implies that
$(u+x,u+y) \in \rho$ *and*
$(uvx,uvy) \in \rho$.
(ii) Right Congruence if for all $x,y,u,v \in T,(x,y) \in \rho$ implies that
$(x+v,y+v) \in \rho$ *and*
$(xuv,yuv) \in \rho$.
(iii) Lateral Congruence if for all $x,y,u,v \in T,(x,y) \in \rho$ implies that
$(uxv,uyv) \in \rho$.
(iv) Congruence if for all $x,x',y,y',z,z' \in T$, $(x,x') \in \rho,(y,y') \in \rho$ and $(z,z') \in \rho$ implies that
$(x+z,x'+z') \in \rho$ *and*
$(xyz,x'y'z') \in \rho$.

Theorem 3.2. *An equivalence relation ρ on a ternary seminear ring T is a congruence relation if and only if it is a left, a right and a lateral congruence relation on T.*

Proof. Let ρ be an equivalence relation on a ternary seminear ring T. Let ρ be a congruence relation. Let $x,y,u,v \in T$. If $(x,y) \in \rho$, then $(u+x,u+y) \in \rho$ and $(uvx,uvy) \in \rho$. It follows that ρ is a left congruence relation. Also if $(x,y) \in \rho$, then $(x+v,y+v) \in \rho$ and $(xuv,yuv) \in \rho$. ρ is a right congruence relation. Similarly it can be seen that $(uxv,uyv) \in \rho$ and hence ρ is a lateral congruence relation .

Conversely suppose that ρ is a left, a right and a lateral congruence relation on T. Choose $x,x',y,y',z,z' \in T$ such that $(x,x'),(y,y'),(z,z') \in \rho$ which implies that $(x+z,x'+z) \in \rho$ and $(xyz,x'yz) \in \rho$ where ρ is a right congruence on T.Also $(x'yz,x'y'z) \in \rho$ by the definition of lateral congruence.Moreover $(x'+z,x'+z') \in \rho$ and $(x'y'z,x'y'z') \in \rho$ since ρ is a left congruence on T. It follows that $(x+z,x'+z') \in \rho$ and $(xyz,x'y'z') \in \rho$ by transitivity. Thus ρ is a congruence relation on T. □

Theorem 3.3. *If ϕ is a homomorphism from a ternary seminear ring $(T,+,.)$ to a ternary seminear ring $(U,+,.)$, then ϕ defines a congruence relation ρ on T given by $(x,y) \in \rho$ if and only if $\phi(x) = \phi(y)$.*

Proof. Let $\phi : T \to U$ be a ternary seminear ring homomorphism. We show that ρ is a congruence relation on T. Assume $\phi(x) = \phi(x)$ clearly for all $x \in T$,$(x,x) \in \rho$ and so the relation ρ is reflexive. If $(x,y) \in \rho$ for some $x,y \in T$ then $\phi(x) = \phi(y)$ and this implies that $\phi(y) = \phi(x)$. Thus $(y,x) \in \rho$ and so the relation ρ is symmetric. Now if $(x,y) \in \rho$ *and* $(y,z) \in \rho$ then by the definition $\phi(x) = \phi(y)$ and $\phi(y) = \phi(z)$ and this gives us $\phi(x) = \phi(z)$. It follows that $(x,z) \in \rho$ and the relation ρ is transitivity.

Now let $(x,x'),(y,y'),(z,z') \in \rho$. Then by the definition we get

$$\begin{aligned}\phi(x+z) &= \phi(x)+\phi(z)\\ &= \phi(x')+\phi(z')\\ \phi(x+z) &= \phi(x'+z')\end{aligned}$$

It follows that $(x+z,x'+z') \in \rho$. In the same way we can show that

$$\begin{aligned}\phi(xyz) &= \phi(x)\phi(y)\phi(z)\\ &= \phi(x')\phi(y')\phi(z')\\ \phi(xyz) &= \phi(x'y'z')\end{aligned}$$

This implies that $(xyz,x'y'z') \in \rho$. Thus ρ is a Congruence relation on T. □

Definition 3.4. *Let T be a ternary seminear ring and ρ be an equivalence relation X, then T/ρ is the quotient ternary set defined as*

$$\begin{aligned}x\rho + z\rho &= (x+z)\rho\\ x\rho y\rho z\rho &= (xyz)\rho\end{aligned}$$

Then $(T/\rho,+,.)$ is a ternary seminear ring under addition and ternary multiplication which is said to be a quotient ternary seminear ring.

Theorem 3.5. *(First isomorphism theorem on ternary seminear ring)*

Statement. *Let ρ be a congruence relation on a ternary seminear ring T. Consider the ternary seminear ring T/ρ. The mapping $\rho^\sharp : T \to T/\rho$ defined by $\rho^\sharp(x) = x\rho$, for all $x \in T$ is an epimorphism. Let $(U,+,.)$ be a ternary seminear ring and $\phi : T \to U$ be a homomorphism. Then the relation $ker\phi : \phi \circ \phi^{-1} = \{(x,z) \in T \times T : \phi(x) = \phi(z)\}$ is a congruence relation on the ternary seminear ring T and there is a monomorphism $\alpha : T/ker\phi \to U$ such that $Im\ \alpha = Im\ \phi$ and the diagram given below commutes*

$$\begin{tikzcd} T \arrow[r, "\phi"] \arrow[d, "ker\phi"'] & U \\ T/ker\phi \arrow[ur, "\alpha"'] & \end{tikzcd}$$

Proof. Let ρ be a congruence relation on a ternary seminear ring T. Then T/ρ is a ternary seminear ring below the following operations

$$x\rho + z\rho = (x+z)\rho \ and$$
$$(x\rho)(y\rho)(z\rho) = (xyz)\rho \ for\ all\ x\rho, y\rho, z\rho \in T/\rho.$$

The mapping $\rho^\sharp : T \to T/\rho$ defined by $\rho^\sharp = x\rho$ for all $x \in T$. Now choose $x,y,z \in T$. Then

$$\begin{aligned} \rho^\sharp(x+z) &= (x+z)\rho \\ &= x\rho + z\rho \\ \rho^\sharp(x+z) &= \rho^\sharp(x) + \rho^\sharp(z) \end{aligned}$$

And

$$\begin{aligned} \rho^\sharp(xyz) &= (xyz)\rho \\ &= x\rho y\rho z\rho \\ \rho^\sharp(xyz) &= \rho^\sharp(x)\rho^\sharp(y)\rho^\sharp(z) \end{aligned}$$

It follows that $\rho^\sharp$ is a homomorphism. Let $u\rho \in T/\rho$, where $u \in T$. Then $\rho^\sharp(u) = u\rho$. Therefore $\rho^\sharp$ is onto. Thus $\rho^\sharp$ is an epimorphism. Then $ker\phi$ is a congruence relation on T by the theorem 3.3. Now, we define $\alpha : T/ker\phi \to U$ by $\alpha(xker\phi) = \phi(x)$. For all $xker\phi, yker\phi, zker\phi \in T/ker\phi$.

$$\begin{aligned} xker\phi = zker\phi &\Leftrightarrow (x,z) \in ker\phi \\ &\Leftrightarrow \phi(x) = \phi(z) \\ &\Leftrightarrow \alpha(xker\phi) = \alpha(zker\phi) \end{aligned}$$

Thus α is well defined as well as one to one mapping.

$$\begin{aligned} \alpha[(xker\phi)+(zker\phi)] &= \alpha[(x+z)ker\phi] \\ &= \phi(x+z) \\ &= \phi(x)+\phi(z) \\ \alpha[(xker\phi)+(zker\phi)] &= \alpha(xker\phi)+\alpha(zker\phi)\ for\ all\ x,z \in T \end{aligned}$$

$$\begin{aligned} \alpha[(xker\phi)(yker\phi)(zker\phi)] &= \alpha[(xyz)ker\phi] \\ &= \phi(xyz) \\ &= \phi(x)\phi(y)\phi(z) \\ \alpha[(xker\phi)(yker\phi)(zker\phi)] &= \alpha(xker\phi)\alpha(yker\phi)\alpha(zker\phi) \\ &\quad for\ all\ x,y,z \in T \end{aligned}$$

Therefore α is homomorphism. Now $Im\ \alpha = \{\alpha(xker\phi)/x \in T\} = \{\phi(x)/x \in T\}$. Thus $Im\ \alpha = Im\ \phi$. Since $\alpha : T/ker\phi \to U, (ker\phi)^\sharp : T \to T/ker\phi$, $\alpha \circ (ker\phi)^\sharp : T \to U$ is defined as

$$\begin{aligned} [\alpha \circ (ker\phi)^\sharp](x) &= \alpha[(ker\phi)^\sharp(x)] \\ &= \alpha(xker\phi) \\ [\alpha \circ (ker\phi)^\sharp](x) &= \phi(x) \end{aligned}$$

consequently, $\alpha \circ (ker\phi)^\sharp = \phi$ and hence the given above diagram commutes. □

Theorem 3.6. *(Second isomorphism theorem on ternary seminear ring)*

Statement. *Let $(T,+,.)$ and $(U,+,.)$ be two ternary seminear rings. Let ρ be a congruence relation on the ternary seminear ring T, and let $\phi : T \to U$ be a homomorphism such that $\rho \subseteq ker\phi$. Then there is a unique homomorphism $\beta : T/\rho \to U$ such that $Im\ \beta = Im\ \phi$ and the following diagram is commutative.*

$$\begin{tikzcd} T \arrow[r, "\phi"] \arrow[d, "\rho^\sharp"'] & U \\ T/\rho \arrow[ur, "\beta"'] & \end{tikzcd}$$

Proof. We define the mapping $\beta : T/\rho \to U$ by $\beta(x\rho) = \phi(x)$ for all $x\rho \in T/\rho$. Then β is well defined, since for all $x\rho, z\rho \in T/\rho, x\rho = z\rho$

$$\begin{aligned} &\Rightarrow (x,z) \in \rho \\ &\Rightarrow (x,z) \in ker\phi.\ (Since\ \rho \subseteq ker\phi) \\ &\Rightarrow \phi(x) = \phi(z) \\ &\Rightarrow \beta(x\rho) = \beta(z\rho) \end{aligned}$$

For all $x\rho, y\rho, z\rho \in T/\rho$,

$$\begin{aligned}\beta(x\rho + z\rho) &= \beta[(x+z)\rho]\\ &= \phi(x+z)\\ &= \phi(x) + \phi(z)\\ \beta(x\rho + z\rho) &= \beta(x\rho) + \beta(z\rho)\end{aligned}$$

Also

$$\begin{aligned}\beta(x\rho y\rho z\rho) &= \beta[(xyz)\rho]\\ &= \phi(xyz)\\ &= \phi(x)\phi(y)\phi(z)\\ \beta(x\rho y\rho z\rho) &= \beta(x\rho)\beta(y\rho)\beta(z\rho)\end{aligned}$$

Therefore β is homomorphism. Now $\beta \circ \rho^\sharp(x) = \beta(x\rho) = \phi(x)$. Then clearly $Im\,\beta = Im\,\phi$, and hence the given diagram is commutative. Now, we show that β is also unique. If possible let $\beta' : T/\rho \to U$ be an another homomorphism mapping defined by $\beta' \circ \rho^\sharp(x) = \beta'(x\rho) = \phi(x)$. Let $x \in T$. Then

$$\begin{aligned}\beta' \circ \rho^\sharp(x) = \phi(x) = \beta \circ \rho^\sharp(x) &\Rightarrow \beta'(x\rho) = \beta(x\rho)\\ &\Rightarrow \beta' = \beta.\end{aligned}$$

□

Theorem 3.7. *(Third isomorphism theorem on ternary seminear ring)*

Statement. *Let T be a ternary seminear ring. Let ρ and σ be two congruence relations on T such that $\rho \subseteq \sigma$. Then $\sigma/\rho = \{(x\rho, z\rho) \in T/\rho \times T/\rho : (x,z) \in \sigma\}$ is a congruence relation on T/ρ and $(T\rho)/(\sigma/\rho) \cong T/\sigma$.*

Proof. We consider the following figure

$$\begin{tikzcd} T \arrow[rr, "\sigma^\sharp"] \arrow[d, "\rho^\sharp"'] & & T/\sigma \\ T/\rho \arrow[rru, "\beta"] \arrow[rr, "(\sigma/\rho)^\sharp"'] & & (T/\rho)/(\sigma/\rho) \arrow[u, "\alpha"'] \end{tikzcd}$$

First we show that σ/ρ is a congruence relation. Let $x,y,z \in T$. Then $(x,x) \in \sigma$, since σ is reflexive. This implies that $(x\rho, x\rho) \in \sigma/\rho$, so σ/ρ is reflexive.If $(x\rho, y\rho) \in \sigma/\rho$ then $(x,y) \in \sigma$ and this implies $(y,x) \in \sigma$, since σ is symmetric. It follows that $(y\rho, x\rho) \in \sigma/\rho$, so σ/ρ is symmetric. If $(x\rho, y\rho), (y\rho, z\rho) \in \sigma/\rho$ then we get (x,y) *and* $(y,z) \in \sigma$ which implies that $(x,z) \in \sigma$ as σ is transitive. Thus σ/ρ is an equivalence relation on T/ρ. Let $x', y', z' \in T$ such that $(x\rho, x'\rho) \in \sigma/\rho$, $(y\rho, y'\rho) \in \sigma/\rho$ and $(z\rho, z'\rho) \in \sigma/\rho$. Then by definition $(x,x'), (y,y')$ *and* $(z,z') \in \sigma$. It follows that $(x+z, x'+z'), (xyz, x'y'z') \in \sigma$, since σ is congruence on T and this implies that $[(x+z)\rho, (x'+z')\rho], [(xyz)\rho, (x'y'z')\rho] \in \sigma/\rho$. Hence σ/ρ is a congruence on T/ρ. Now define a mapping $\beta : T/\rho \to T/\sigma$ by $\beta(x\rho) = x\sigma$. Choose $x\rho, y\rho, z\rho \in T/\rho$ then

$$\begin{aligned}\beta(x\rho + z\rho) &= \beta[(x+z)\rho]\\ &= (x+z)\sigma\\ &= x\sigma + z\sigma\\ \beta(x\rho + z\rho) &= \beta(x\rho) + \beta(z\rho)\end{aligned}$$

$$\begin{aligned}\beta(x\rho y\rho z\rho) &= \beta[(xyz)\rho]\\ &= (xyz)\sigma\\ &= x\sigma y\sigma z\sigma\\ \beta(x\rho y\rho z\rho) &= \beta(x\rho)\beta(y\rho)\beta(z\rho)\end{aligned}$$

It follows that β is homomorphism. Let $\sigma x \in T/\sigma$, *where* $x \in T$. *Then* $\rho x \in T/\rho$. *Now* $\beta(\rho x) = \sigma(x)$. Thus β is onto. From theorem 3.7, there is a monomorphism $\alpha : T/\rho | ker\beta \to T/\sigma$ defined by $\alpha((x\rho)ker\beta) = x\sigma$ for all $x \in T$. Therefore it is onto. Thus $T/\rho | ker\beta \cong T/\sigma$. Now,

$$\begin{aligned}ker\beta &= \{(x\rho, z\rho) \in T/\rho \times T/\rho : \beta(x\rho) = \beta(z\rho)\}\\ &= \{(x\rho, z\rho) \in T/\rho \times T/\rho : x\sigma = z\sigma\}\\ &= \{(x\rho, z\rho) \in T/\rho \times T/\rho : (x,z) \in \sigma\}\\ &= \sigma/\rho\end{aligned}$$

Therefore $(T/\rho)/(\sigma/\rho) \cong T/\sigma$. □

Theorem 3.8. *Let T be a ternary seminear ring. If ρ_1 and ρ_2 are two left congruences(respectively right congruences, lateral congruences, congruences) on T, then $\rho_1 \circ \rho_2$ is a left congruence(respectively right congruence, lateral congruence, congruence) of T.*

Proof. Let ρ_1 and ρ_2 be two left congruences of T. Suppose $(x,y) \in \rho_1 \circ \rho_2$ holds. Then there exists $z \in T$ such that $(x,z) \in \rho_1$ and $(z,y) \in \rho_2$ hold. Since ρ_1, ρ_2 are left congruence of T, it follows that $(u+x, u+z) \in \rho_1$, $(uvx, uvz) \in \rho_1$ and $(u+z, u+y) \in \rho_2, (uvz, uvy) \in \rho_2$ hold for $u,v \in T$. This implies that $(u+x, u+y) \in \rho_1 \circ \rho_2, (uvx, uvy) \in \rho_1 \circ \rho_2$ hold for all $u,v \in T$ and hence $\rho_1 \circ \rho_2$ is a left congruence on T. Similarly, it is easy to show $\rho_1 \circ \rho_2$ is a right congruence of T. Let ρ_1 and ρ_2 be lateral congruence on T. If $(x,y) \in \rho_1 \circ \rho_2$ then there exists $z \in T$ such that $(x,z) \in \rho_1$ and $(z,y) \in \rho_2$ holds. Since ρ_1 and ρ_2 are lateral congruence of T, it follows that $(uxv, uzv) \in \rho_1$ and $(uzv, uyv) \in \rho_2$ hold for $u,v \in T$. This implies that $(uxv, uyv) \in \rho_1 \circ \rho_2$ hold for all $u,v \in T$ and hence $\rho_1 \circ \rho_2$ is a lateral congruence on T. Similarly, we can prove that $\rho_1 \circ \rho_2$ is a congruence on T. □

4. Cancellative Ternary Seminear Rings

This section discribes the **cancellative ternary seminear rings** and their characteristics.

Definition 4.1. *Let $(T,+,.)$ be a ternary seminear ring. For all $x,y,u,v \in T$, T is called*
i) Left cancellative if $u+x = u+y \Rightarrow x = y$ and $uvx = uvy \Rightarrow x = y$
ii) Right cancellative if $x+v = y+v \Rightarrow x = y$ and $xuv = yuv \Rightarrow x = y$
iii) Lateral cancellative if $uxv = uyv \Rightarrow x = y$
iv) Cancellative if T is left, right and laterally cancellative.

Definition 4.2. *A congruence ρ on a ternary seminear ring T is said to be a cancellative congruence on T if T/ρ is a cancellative ternary seminear ring.*

Theorem 4.3. *Let T be a ternary seminear ring. Then a congruence ρ on T is a cancellative congruence if and only if (i)$(u+x,u+y) \in \rho \Rightarrow (x,y) \in \rho$ and $(uvx,uvy) \in \rho \Rightarrow (x,y) \in \rho$,(ii) $(x+v,y+v) \in \rho \Rightarrow (x,y) \in \rho$ and $(xuv,yuv) \in \rho \Rightarrow (x,y) \in \rho$ and (iii) $(uxv,uyv) \in \rho \Rightarrow (x,y) \in \rho$ for all $x,y,u,v \in T$.*

Proof. Let T be a ternary seminear ring and ρ be a cancellative congruence on T. Then T/ρ is a cancellative ternary seminear ring, for all $x,y,u,v \in T$. Suppose that $(u+x,u+y) \in \rho$ *and* $(uvx,uvy) \in \rho$ hold. Then

$$\Rightarrow (u+x)\rho = (u+y)\rho$$
$$\Rightarrow (u\rho)+(x\rho) = (u\rho)+(y\rho)$$
$$\Rightarrow x\rho = y\rho(by\ left\ cancellative\ law)$$
$$\Rightarrow (x,y) \in \rho$$

Next

$$\Rightarrow (uvx)\rho = (uvy)\rho$$
$$\Rightarrow (u\rho)(v\rho)(x\rho) = (u\rho)(v\rho)(y\rho)$$
$$\Rightarrow x\rho = y\rho(by\ left\ cancellative\ law)$$
$$\Rightarrow (x,y) \in \rho$$

Similarly, it can be shown that $(x+v,y+v) \in \rho \Rightarrow (x,y) \in \rho$,$(xuv,yuv) \in \rho$ which implies that $(x,y) \in \rho$ and $(uxv,uyv) \in \rho$ which also implies that $(x,y) \in \rho$. □

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