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Some translation theorems in bipolar valued multi fuzzy subnearring of a nearring

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Abstract

In this paper, some translations of bipolar valued multi fuzzy subnearring of a nearing are introduced and using these translations, some theorems are stated and proved.

Keywords

Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subnearring, translations, intersection.

AMS Subject Classification

97H40, 03B52, 03E72.

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Contents

1. Introduction

In 1965, Zadeh [13] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc. Lee [7] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [-1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7, 8]. Fuzzy group was introduced by Azriel Rosenfeld [4] and fuzzy algebraic structure was extended by many authors [2, 5, 11]. Anitha.M.S., et al.[1] defined a bipolar valued fuzzy subgroups of a group. After that many algebraic structure have been extended by [3, 6, 12]. Shanthi.V.K and G.Shyamala[10] have introduced the bipolar valued multi fuzzy subgroups of a group. Bipolar valued multi fuzzy subnearring of a nearing has been introduced by S.Muthukumaran and B.Anandh [9]. In this paper, the concept translation of bipolar valued multi fuzzy subnearring of a nearing is introduced and established some results.

2. Preliminaries

Definition 2.1 ([7]). *A bipolar valued fuzzy set (BVFS) B in X is defined as an object of the form*

$$B = \left\{ < x, B^{+}(u), B^{-}(u) > / x \in X \right\},$$

where $B^{+} : X \to [0,1]$ and $B^{-} : X \to [-1,0]$. The positive membership degree $B^{+}(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set B and the negative membership degree $B^{-}(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set B.

Example 2.2. $B = \{< m, 0.4, -0.3 >, < n, 0.8, -0.6 >, < o, 0.9, -0.3 >\}$ *is a BVFS of* $X = \{m, n, o\}$

Definition 2.3 ([9]). *A bipolar valued multi fuzzy set (BVMFS)*

A in X is defined as an object of the form

$$B = \{\langle x, B_1^+(u), B_2+(u), \ldots, B_n^+(u), B_1^-(u), B_2^-(u), \ldots, B_n^-(u)\rangle / x \in X\}$$

where $B_i^+ : X \to [0,1]$ and $B_i^- : X \to [-1,0]$, for all i. The positive membership degrees $B_i^+(u)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar valued multi fuzzy set B and the negative membership degrees $B_i^-(u)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set B.

Example 2.4. $B = \{< m, 0.7, 0, 5, 0.2, -0.2, -0.4, -0.8 >, < n, 0.4, 0.9, 0.2, -0.7, -0.5, -0.2 >, < 0, 0.9, 0.4, 0.7, -0.4, -0.6, -0.2 >\}$ *is a BVMFS of* $X = \{m, n, o\}$

Definition 2.5 ([9]). *Let $(N, +, -)$ be a nearring. A BVMFS B of N is said to be a bipolar valued multi fuzzy subnearring of N (BVMFSNR) if the following conditions are satisfied, for all i,*

(i) $B_i^+(u-v) \geq \min\{B_i^+(u), B_i^+(v)\}$

(ii) $B_i^+(uv) \geq \min\{B_i^+(u), B_i^+(v)\}$

(iii) $B_i^-(u-v) \leq \max\{B_i^-(u), B_i^-(v)\}$

(iv) $B_i^-(uv) \leq \max\{B_i^-(u), B_i^-(v)\}, \forall u, v \in N$

Example 2.6. *Let $N = Z_3 = \{0,1,2\}$ be a nearring with respect to addition modulo 3 and multiplication modulo 3 . Then* $B = \{< 0, 0.7, 0.9, 0.5, -0.8, -0.9, -0.7 >, < 1, 0.5, 0.7, 0.3\ -0.6, -0.5, -0.4 >, < 2, 0.5, 0.7, 0.3, -0.6, -0.5, -0.4 >\}$ *is a BVMFSNR of N*

Definition 2.7 ([9]). *Let $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ and $B = \langle B_1^+, B_2^+, \ldots\ B_n^+, B_1^-, B_2^-, \ldots, B_n^-\rangle$ be two bipolar valued multi fuzzy subsets with degree n of a set X We define the following relations and operations:*

(i) $A \subset B$ *if and only if for all i,* $A_i^+(u) \leq B_i^+(u)$ *and* $A_i^-(u) \geq B_i^-(u), \forall u \in X$.

(ii) $A = B$ *if and only if for all i,* $A_i^+(u) = B_i^+(u)$ *and* $A_i^-(u) = B_i^-(u), \forall u \in X$.

(iii) $A \cap B = \{\langle u, \min(A_1^+(u), B_1^+(u)), \min(A_2^+(u), B_2+(u)), \ldots, \min(A_n^+(u), B_n{}^+(u))\ \max(A_1^-(u), B_1^-(u)), \max(A_2^-(u), B_2^-(u)), \ldots, \max(A_n^-(u), B_n^-(u))\rangle / u \in X\}$

Definition 2.8. *Let $A = \langle A_1^+, A_2{}^+, \ldots, A_n^+, A_1{}^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subset of X. Then the following translations are defined as*

(i) $?(A) = \langle ?\,A_1^+, ?\,A_2^+, \ldots, ?\,A_n^+, ?\,A_1^-, ?\,A_2^-, \ldots, ?\,A_n^-\rangle$, *wh-ere* $?\,A_i^+(x) = \min\{1/2,\ A_i^+(x)\}$ *and*

$$?\,A_i^-(x) = \max\{-1/2,\ A_i^-(x)\},$$

for all x in X and for all i.

(ii) $!(A) = \langle !A_1^+, !A_2{}^+, \ldots .A_n^+, !A_1^-, !A_2^-, \ldots, !A_n^-\rangle$, *where* $!A_i^+(x) = \max\{1/2,\ A_i^+(x)\}$ *and*

$$!A_i^-(x) = \min\{-1/2,\ A_i - (x)\},$$

for all x in X

(iii) $Q_{\alpha,\beta}(A) = \langle Q_{\alpha,\beta}(A_1)^+, Q_{\alpha,\beta}(A_2)^+, \ldots, Q_{\alpha,\beta}(A_n)^+, Q_\alpha, \beta(A_1)^-, Q_{\alpha_0}, (A_2)^-, \ldots, Q_{\alpha,\beta}(A_n)^-\rangle$ *where* $Q_{\alpha,\beta}(A_i)^+(x) = \min\{\alpha_i,\ A_i^+(x)\}$ *and* $Q_{\alpha,\beta}(A_i)^-(x) = \max\{\beta_i,\ A_i^-(x)\}$, *for all x in X and α_i in [0,1] and β_i in [-1,0] and for all i.*

(iv) $P_{\alpha,\beta}(A) = \langle P_{\alpha,\beta}(A_1)^+, P_{\alpha,\beta}(A_2)^+, \ldots, P_{\alpha,\beta}(A_n)^+, P_{\alpha,\beta}(A_1)^-, P_{\alpha,\beta}(A_2)^-, \ldots, P_{\alpha,\beta}(A_n)^-\rangle$ *where* $P_{\alpha\beta}(A_i)^+(x) = \max\{\alpha_i,\ A_i^+(x)\}$ *and*

$$P_{\alpha_\infty}(A_i)^-(x) = \min\{\beta_i,\ A_i^-(x)\},$$

for all x in X and α_i in [0,1] and β_i in [-1,0] and for all i.

(v) $G_{\alpha,\beta}(A) = \langle G_{\alpha,\beta}(A_1)^+, G_{\alpha,\beta}(A_2)^+, \ldots, G_{\alpha,\beta}(A_n)^+, G_{\alpha,\beta}(A_1)^-, G_{\alpha,\beta}(A_2)^-, \ldots, G_{a,\beta}(A_n)^-\rangle$ *where* $G_{\alpha,\beta}(A_i)^+(x) = \alpha_i A_i^+(x)$ *and* $G_{\alpha,\beta}(A_i)^-(x) = -\beta_i A_i^-(x)$, *for all x in X and α_i in [0,1] and β_i in [-1,0] and for all i.*

2.1 Properties

Theorem 2.9 ([9]). *If $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^-\rangle$ and $C = \langle C_1^+, C_2^+, \ldots, C_n^+\ \ C_1^-, C_2^-, \ldots, C_n^-\rangle$ are two bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then their intersection $B \cap C$ is a bipolar valued multi fuzzy subnearring of R.*

Theorem 2.10. *If $A = \langle A_1{}^+, A_2{}^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then $?(A) = \langle ?(A_1^+), ?(A_2^+), \ldots, ?(A_n^+), ?(A_1^-)\ ?(A_2^-), \ldots, ?(A_n^-)\rangle$ is a bipolar valued multi fuzzy subnearring of R.*

Proof. For every u and v in R, for each i,

$$\begin{aligned} ?(A_i^+)(u-v) &= \min\{1/2,\ A_i^+(u-v)\} \\ &\geq \min\{1/2, \min\{A_i^+(u), A_i^+(v)\}\} \\ &= \min\{\min\{1/2, A_i+(u)\}, \min\{1/2, A_i^+(v)\}\} \\ &= \min\{?(A_i^+)(u), ?(A_i^+)(v)\} \text{ for all } i \end{aligned}$$

Therefore for all i, $?(A_i^+)(u-v) \geq \min\{?(A_i^+)(u), ?(A_i^+)(v)\}$ for all u, v in R. Also,

$$\begin{aligned} ?(A_i^+)(uv) &= \min\{1/2,\ A_i^+(uv)\} \\ &\geq \min\{1/2, \min\{A_i^+(u), A_i^+(v)\}\} \\ &= \min\{\min\{1/2,\ A_i^+(u)\}, \min\{1/2,\ A_i^+(v)\}\} \\ &= \min\{?(A_i^+)(u), ?(A_i^+)(v)\}. \end{aligned}$$

Therefore for all $i, ?\left(A_i^+\right)(uv) \geq \min\left\{?\left(A_i^+\right)(u), ?\left(A_i^+\right)(v)\right\}$ for all u, v in R. And,

$$\begin{aligned} ?\left(A_i^-\right)(u-v) &= \max\left\{-1/2,\ A_i^-(u-v)\right\} \\ &\leq \max\left\{-1/2, \max\left\{A_i-(u), A_i^-(v)\right\}\right\} \\ &= \max\left\{\max\left\{-1/2,\ A_i^-(u)\right\}, \max\left\{-1/2,\ A_i^-(v)\right\}\right\} \\ &= \max\left\{?\left(A_i^-\right)(u), ?\left(A_i^-\right)(v)\right\}. \end{aligned}$$

Therefore for all $i, ?\left(A_i^-\right)(u-v) \leq \max\left\{?\left(A_i^-\right)(u), ?\left(A_i^-\right)(v)\right\}$ for all u, v in R. Also,

$$\begin{aligned} ?\left(A_i^-\right)(uv) &= \max\left\{-1/2,\ A_i^-(uv)\right\} \\ &\leq \max\left\{-1/2, \max\left\{A_i^-(u), A_i^-(v)\right\}\right\} \\ &= \max\left\{\max\left\{-1/2,\ A_i^-(u)\right\}, \max\left\{-1/2,\ A_i^-(v)\right\}\right\} \\ &= \max\left\{?\left(A_i^-\right)(u), ?\left(A_i^-\right)(v)\right\} \end{aligned}$$

Therefore for all $i, ?\left(A_i^-\right)(uv) \leq \max\left\{?\left(A_i^-\right)(u), ?\left(A_i^-\right)(v)\right\}$ for all u, v in R. Hence $?(A)$ is a bipolar valued multi fuzzy subnearring of R. □

Theorem 2.11. *If $A = \left\langle A_1{}^+, A_2{}^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\right\rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then $!(A) = \left\langle !\left(A_1{}^+\right), !\left(A_2{}^+\right), \ldots, !\left(A_n^+\right), !\left(A_1^-\right) !\left(A_2^-\right), \ldots, !\left(A_n^-\right)\right\rangle$ is a bipolar valued multi fuzzy subnearring of R.*

Proof. For every u and v in R, for each i,

$$\begin{aligned} !\left(A_i^+\right)(u-v) &= \max\left\{1/2,\ A_i^+(u-v)\right\} \\ &\geq \max\left\{1/2, \min\left\{A_i^+(u), A_i^+(v)\right\}\right\} \\ &= \min\left\{\max\left\{1/2,\ A_i^+(u)\right\}, \max\left\{1/2,\ A_i^+(v)\right\}\right\} \\ &= \min\left\{!\left(A_i^+\right)(u), !\left(A_i^+\right)(v)\right\}. \end{aligned}$$

Therefore for each $i, !\left(A_i^+\right)(u-v) \geq \min\{!\left(A_i^+\right)(u), !\left(A_i^+\right)(v)\}$ for all u, v in R. And

$$\begin{aligned} !\left(A_i^+\right)(uv) &= \max\left\{1/2,\ A_i^+(uv)\right\} \\ &\geq \max\left\{1/2, \min\left\{A_i^+(u), A_i^+(v)\right\}\right\} \\ &= \min\left\{\max\left\{1/2,\ A_i^+(u)\right\}, \max\left\{1/2,\ A_i^+(v)\right\}\right\} \\ &= \min\left\{!\left(A_i^+\right)(u), !\left(A_i^+\right)(v)\right\}. \end{aligned}$$

Therefore for each i, $!\left(A_i^+\right)(uv) \geq \min\left\{!\left(A_i^+\right)(u), !\left(A_i^+\right)(v)\right\}$ for all u, v in R. Also,

$$\begin{aligned} !\left(A_i^-\right)(u-v) &= \min\left\{-1/2,\ A_i^-(u-v)\right\} \\ &\leq \min\left\{-1/2, \max\left\{A_i^-(u), A_i^-(v)\right\}\right\} \\ &= \max\left\{\min\left\{-1/2,\ A_i^-(u)\right\}, \min\left\{-1/2,\ A_i^-(v)\right\}\right\} \\ &= \max\left\{!\left(A_i^-\right)(u), !\left(A_i^-\right)(v)\right\} \end{aligned}$$

For each i, thus $!\left(A_i^-\right)(u-v) \leq \max\left\{!\left(A_i^-\right)(u), !\left(A_i^-\right)(v)\right\}$ for all u, v in R. And ,

$$\begin{aligned} !\left(A_i^-\right)(uv) &= \min\left\{-1/2,\ A_i^-(uv)\right\} \\ &\leq \min\left\{-1/2, \max\left\{A_i^-(u), A_i^-(v)\right\}\right\} \\ &= \max\left\{\min\left\{-1/2,\ A_i^-(u)\right\}, \min\left\{-1/2,\ A_i^-(v)\right\}\right\} \\ &= \max\left\{!\left(A_i^-\right)(u), !\left(A_i^-\right)(v)\right\}. \end{aligned}$$

Therefore for all $i, !\left(A_i^-\right)(uv) \leq \max\left\{!\left(A_i^-\right)(u), !\left(A_i^-\right)(v)\right\}$ for all u, v in R. Hence $!(A)$ is a bipolar valued multi fuzzy subnearring of R. □

Theorem 2.12. *If $A = \left\langle A_1^+, A_2^+, \ldots, A_n+, A_1^-, A_2^-, \ldots, A_n^-\right\rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then*

$$\begin{aligned} & Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A) \\ & = \langle Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^+\right), \\ & Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_2^+\right), \ldots, \\ & Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_n^+\right), \\ & Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^-\right), \\ & Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_2^-\right), \ldots, \\ & Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_n^-\right)\rangle \end{aligned}$$

is a bipolar valued multi fuzzy subnearring of R.

Proof. For every u and v in R, α_i in [0,1] and β_i in $[-1,0]$, for each i, we have

$$\begin{aligned} & Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u-v) = \min\left\{\alpha_i,\ A_i^+(u-v)\right\} \\ & \geq \min\left\{\alpha_i, \min\left\{A_i^+(u), A_i^+(v)\right\}\right\} \\ & = \min\left\{\min\left\{\alpha_i,\ A_i^+(u)\right\}, \min\left\{\alpha_i,\ A_i^+(v)\right\}\right\} \\ & = \min\Big\{Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u), \\ & \quad Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\Big\} \end{aligned}$$

Therefore for each i,

$$\begin{aligned} & Q_{((c_1,\alpha_2,\ldots,a_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u-v) \\ & \geq \min\left\{Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right. \\ & \left. Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\} \end{aligned}$$

for all u, v in R. And

$$\begin{aligned} & Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(uv) = \min\left\{\alpha_i,\ A_i^+(uv)\right\} \\ & \geq \min\left\{\alpha_i, \min\left\{A_i^+(u), A_i^+(v)\right\}\right\} \\ & = \min\left\{\min\left\{\alpha_i,\ A_i^+(u)\right\}, \min\left\{\alpha_i,\ A_i^+(v)\right\}\right\} \\ & = \min\left\{Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^+\right)(u),\right. \\ & \left. Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}. \end{aligned}$$

For each i, thus

$$\begin{aligned} & Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(uv) \\ & \geq \min\left\{Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right. \\ & \left. Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\} \end{aligned}$$

for all u, v in R. Also

$$\begin{aligned} & Q_{((c_1,a_2,\ldots,a_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u-v) = \max\left\{\beta_i,\ A_i^-(u-v)\right\} \\ & \leq \max\left\{\beta_i, \max\left\{A_i^-(u), A_i^-(v)\right\}\right\} \\ & = \max\left\{\max\left\{\beta_i, A_i^-(u)\right\}, \max\left\{\beta_i, A_i^-(v)\right\}\right\} \\ & = \max\left\{Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right. \\ & \left. Q_{((a_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}. \end{aligned}$$

Therefore for each i,

$$Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u-v)$$
$$\leq \max\left\{Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}$$

for all u,v in R. And

$$Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(uv)=\max\left\{\beta_i,\ A_i^-(uv)\right\}$$
$$\leq \max\left\{\beta_i,\max\left\{A_i^-(u),A_i^-(v)\right\}\right\}$$
$$=\max\left\{\max\left\{\beta_i,\ A_i^-(u)\right\},\max\left\{\beta_i,\ A_i^-(v)\right\}\right\}$$
$$=\max\left\{Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}.$$

Therefore for all i,

$$Q_{((a_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(uv)$$
$$\leq \max\left\{Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}$$

for all u,v in R. Hence $Q_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)$ is a bipolar valued multi fuzzy subnearring of R. □

Theorem 2.13. *If $A=\left\langle A_1^+,A_2^+,\ldots A_n^+,A_1^-,A_2^-,\ldots,A_n^-\right\rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then*

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)$$
$$=\langle P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^+\right),$$
$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_2^+\right),\ldots,$$
$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_n^+\right)$$
$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^-\right),$$
$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_2^-\right),\ldots,$$
$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_n^-\right)\rangle$$

is a bipolar valued multi fuzzy subnearring of R.

Proof. For every u and v in R, α_i in [0,1] and β_i in $[-1,0]$, for each i

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^+\right)(u-v)=\max\left\{\alpha_i,\ A_i^+(u-v)\right\}$$
$$\geq \max\left\{\alpha_i,\min\left\{A_i^+(u),A_i^+(v)\right\}\right\}$$
$$=\min\left\{\max\left\{\alpha_i,\ A_i^+(u)\right\},\max\left\{\alpha_i,\ A_i^+(v)\right\}\right\}$$
$$=\min\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}$$

Therefore for each i,

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u-v)$$
$$\geq \min\left\{P_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}$$

for all u and v in R. And,

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(uv)=\max\left\{\alpha_i,A_i^+(uv)\right\}$$
$$\geq \max\left\{\alpha_i,\min\left\{A_i+(u),A_i+(v)\right\}\right\}$$
$$=\min\left\{\max\left\{\alpha_i,\ A_i+(u)\right\},\max\left\{\alpha_i,A_i^+(v)\right\}\right\}$$
$$=\min\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n)}\ \left(A_i^+\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\ \left(A_i^+\right)(v)\right\}.$$

For each i, thus

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(uv)$$
$$\geq \min\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}$$

for all u and v in R. Also,

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u-v)=\min\left\{\beta_i,\ A_i^-(u-v)\right\}$$
$$\leq \min\left\{\beta_i,\max\left\{A_i^-(u),A_i^-(v)\right\}\right\}$$
$$=\max\left\{\min\left\{\beta_i,\ A_i^-(u)\right\},\min\left\{\beta_i,\ A_i^-(v)\right\}\right\}$$
$$=\max\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}.$$

Therefore for all i,

$$P_{((a_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u-v)$$
$$\leq \max\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}$$

for all u and v in R. And,

$$P_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(uv)=\min\left\{\beta_i,\ A_i^-(uv)\right\}$$
$$\leq \min\left\{\beta_i,\max\left\{A_i^-(u),A_i^-(v)\right\}\right\}$$
$$=\max\left\{\min\left\{\beta_i,\ A_i^-(u)\right\},\min\left\{\beta_i,\ A_i^-(v)\right\}\right\}$$
$$=\max\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n)(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}.$$

For each i,

$$P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(uv)$$
$$\leq \max\left\{P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}$$

for all u and v in R. Hence $P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)$ is a bipolar valued multi fuzzy subnearring of R. □

Theorem 2.14. *If $A=\left\langle A_1^+,A_2^+,\ldots,A_n^+,A_1^-,A_2^-,\ldots,A_n^-\right\rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a* *nearring R, then*

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)$$
$$=\langle G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^+\right),$$
$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_2^+\right),\ldots,$$
$$G_{(\alpha_1,\alpha_2,\ldots,\alpha_n),\beta_1,\beta_2,\ldots,}$$
$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_1^-\right),$$
$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_2^-\right),\ldots,$$
$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_n^-\right)\rangle$$

is a bipolar valued multi fuzzy subnearring of R.

Proof. For every u and v in R, α_i in [0,1] and β_i in $[-1,0]$,

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u-v)=\alpha_i\,A_{i^+}(u-v)$$
$$\geq \alpha_i\left(\min\left\{A_i^+(u),A_i^+(v)\right\}\right)$$
$$=\min\left\{\alpha_i\,A_i^+(u),\alpha_i\,A_i^+(v)\right\}$$
$$=\min\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}.$$

Therefore for each i,

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u-v)$$
$$\geq\min\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.G_{((a_1,\alpha_2,\ldots,a_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}$$

for all u and v in R. And

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(uv)=\alpha_i\,A_i^+(uv)$$
$$\geq \alpha_i\left(\min\left\{A_i^+(u),A_i^+(v)\right\}\right)$$
$$=\min\left\{\alpha_i\,A_i^+(u),\alpha_i\,A_i^+(v)\right\}$$
$$=\min\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n)((\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}.$$

For each i, thus

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(uv)$$
$$\geq\min\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(u),\right.$$
$$\left.G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^+\right)(v)\right\}$$

for all u and v in R. Also

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u-v)=-\beta_i\,A_i^-(u-v)$$
$$\leq -\beta_i\left(\max\left\{A_i^-(u),A_i^-(v)\right\}\right)$$
$$=\max\left\{-\beta_i\,A_i^-(u),-\beta_i\,A_i^-(v)\right\}$$
$$=\max\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}.$$

Therefore for all i,

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u-v)$$
$$\leq\max\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.G_{((c_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}$$

for all u and v in R. And,

$$G_{((\alpha_1,\alpha_2,\ldots,a_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(uv)=-\beta_i\,A_i^-(uv)$$
$$\leq -\beta_i\left(\max\left\{A_i^-(u),A_i^-(v)\right\}\right)$$
$$=\max\left\{-\beta_i\,A_i^-(u),-\beta_i\,A_i^-(v)\right\}$$
$$=\max\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}.$$

Therefore for all i,

$$G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(uv)$$
$$\leq\max\left\{G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(u),\right.$$
$$\left.G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(A_i^-\right)(v)\right\}$$

for all u and v in R. Hence $G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)$ is a bipolar valued multi fuzzy subnearring of R. □

Theorem 2.15. *If* $A=\left\langle A_1{}^+,A_2{}^+,\ldots,A_n^+,A_1^-,A_2^-,\ldots,A_n^-\right\rangle$ *and* $B=\left\langle B_1{}^+,B_2{}^+,\ldots,B_n^+,B_1^-,\ B_2^-,\ldots,B_n^-\right\rangle$ *are bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then* $!(A\cap B)=!(A)\cap!(B)$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the Theorem 2.1 and 2.3. □

Theorem 2.16. *If* $A=\left\langle A_1^+,A_2^+,\ldots,A_n^+,A_1^-,A_2^-,\ldots,A_n^-\right\rangle$ *and* $B=\left\langle B_1^+,B_2{}^+,\ldots,B_n^+,B_1^-\ \ B_2^-,\ldots,B_n^-\right\rangle$ *are bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then* $?(A\cap B)=?(A)\cap?(B)$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the theorem 2.1 and 2.2. □

Theorem 2.17. *If* $A=\left\langle A_1{}^+,A_2^+,\ldots,A_n^+,A_1^-,A_2^-,\ldots,A_n^-\right\rangle$ *is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then* $!(?(A))=?(!(A))=\langle 1/2,1/2,\ldots,1/2,-1/2,$ $-1/2,\ldots,\ -1/2\rangle$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the theorem 2.2 and 2.3. And for every x in R and for all i i. $?\,A_i^+(x)=\min\left\{1/2,\ A_i^+(x)\right\}\leq 1/2$ and $!A_i^+(x)=\max\left\{1/2,\ A_i^+(x)\right\}\geq 1/2$, so $!\left(?\left(A_i^+\right)\right)=?\left(!\left(A_i^+\right)\right)=1/2$.
And $?A_i^-(x)=\max\left\{-1/2,A_i^-(x)\right\}\geq -1/2$ and $!A_i^-(x)=\min\left\{-1/2,A_i^-(x)\right\}\leq -1/2$, so $!\left(?\left(A_i^-\right)\right)=?\left(!\left(A_i^-\right)\right)=-1/2$.
Hence $!(?(A))=?(!(A))=\langle 1/2,1/2,\ldots,1/2,-1/2,-1/2,\ldots,-1/2\rangle$ is a bipolar valued multi fuzzy subnearring of R. □

Theorem 2.18. *If* $A=\left\langle A_1^+,A_2^+,\ldots,A_n^+,A_1^-,A_2^-,\ldots,A_n^-\right\rangle$ *and* $B=\left\langle B_1^+,B_2^+,\ldots,B_n^+,B_1^-\ \ B_2^-,\ldots,B_n^-\right\rangle$ *are bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then* $P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A\cap B)=P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)\cap P_{((\alpha_1,\alpha_2,\ldots,a_n),(\beta_1,\beta_2,\ldots,\beta_n))}(B)$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the theorem 2.1 and 2.5 . □

Theorem 2.19. *If* $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ *and* $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^- \ B_2^-, \ldots, B_n^- \rangle$ *are bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then* $Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A \cap B) = Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A) \cap Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(B)$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the theorem 2.1 and 2.4. □

Theorem 2.20. *If* $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2-, \ldots, A_n^- \rangle$ *is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then*
$P_{((\alpha_1,a_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(Q_{((\alpha_1,\alpha_2,\ldots,\alpha_i),(\beta_1,\beta_2,\ldots,\beta_i))}(A)\right)$
$= Q_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}\left(P_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A)\right)$
$= \langle \alpha_1, \alpha_2, \ldots, \alpha_n, \beta_1, \beta_2, \ldots, \beta_n \rangle$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the theorem 2.4 and 2.5. □

Theorem 2.21. *If* $A = \langle A_1^+, A_2{}^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ *and* $B = \langle B_1^+, B_2{}^+, \ldots, B_n^+, B_1^- \ B_2^-, \ldots, B_n^- \rangle$ *are bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then* $G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A \cap B) = G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(A) \cap G_{((\alpha_1,\alpha_2,\ldots,\alpha_n),(\beta_1,\beta_2,\ldots,\beta_n))}(B)$ *is also a bipolar valued multi fuzzy subnearring of R.*

Proof. The proof follows from the theorem 2.1 and 2.6. □

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