Some remarks on weak fuzzy P-spaces

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Abstract
In this paper, weak fuzzy P-spaces are characterized by means of fuzzy regular \( G_\delta \)-sets and fuzzy regular \( F_\sigma \)-sets. It is established that fuzzy regular \( G_\delta \)-sets in weak fuzzy P-spaces are fuzzy open but not fuzzy dense sets. Also it is obtained that fuzzy regular \( G_\delta \)-sets and fuzzy regular \( F_\sigma \)-sets in weak fuzzy P-spaces are fuzzy somewhere dense sets and fuzzy regular \( G_\delta \)-sets are not fuzzy simply open sets in weak fuzzy P-spaces. It is established by means of fuzzy regular \( G_\delta \)-sets that the weak fuzzy P-spaces are not fuzzy hyperconnected spaces.

Keywords
Fuzzy regular open set, fuzzy regular \( G_\delta \)-set, fuzzy regular \( F_\sigma \)-set, Fuzzy dense set, fuzzy some where dense set, fuzzy P-space, fuzzy hyperconnected space.

AMS Subject Classification
54A40, 03E72.

1. Introduction
The concept of fuzzy sets as a new approach for modeling uncertainties was introduced by L. A. Zadeh [18] in the year 1965. In 1968, the concept of fuzzy topological spaces was introduced by C.L. Chang [3]. Chang’s works paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

The concepts of regular \( G_\delta \)-sets was introduced by J. Mack [5]. The concept of P-spaces in fuzzy setting was introduced by G.Balasubramanian [7]. The concept of weak fuzzy P-spaces, is introduced and studied in [12]. The notions of regular \( G_\delta \)-sets and regular \( F_\sigma \)-sets in fuzzy setting was introduced in [10]. In this paper, weak fuzzy P-spaces are characterized by means of fuzzy regular \( G_\delta \)-sets and fuzzy regular \( F_\sigma \)-sets. It is established that fuzzy regular \( G_\delta \)-sets in weak fuzzy P-spaces are fuzzy open but not fuzzy dense sets. Also it is obtained that fuzzy regular \( G_\delta \)-sets and fuzzy regular \( F_\sigma \)-sets in weak fuzzy P-spaces are fuzzy somewhere dense sets and fuzzy regular \( G_\delta \)-sets are not fuzzy simply open sets in weak fuzzy P-spaces. It is proved by an example that weak fuzzy P-spaces are fuzzy irresolvable spaces. It is established by means of fuzzy regular \( G_\delta \)-sets that the weak fuzzy P-spaces are not fuzzy hyperconnected spaces.

2. Preliminaries
Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by \((X, T)\) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval \([0,1]\). A fuzzy set \( \lambda \) in X is a mapping from X into I. The fuzzy set \( 0_\lambda \) is defined as \( 0_\lambda (x) = 0 \), for all \( x \in X \) and the fuzzy set \( 1_\lambda \) is defined as \( 1_\lambda (x) = 1 \), for all \( x \in X \).

Definition 2.1 ([3]). Let \((X, T)\) be a fuzzy topological space and \( \lambda \) be any fuzzy set in \((X, T)\). The interior, the closure and the complement of \( \lambda \) are defined respectively as follows:

(i) \( \text{int}(\lambda) = \bigvee \{ \mu/\mu \leq \lambda, \mu \in T \} \)

(ii) \( \text{cl}(\lambda) = \bigwedge \{ \mu/\lambda \leq \mu, 1 - \mu \in T \} \)

(iii) \( \lambda'(x) = 1 - \lambda(x) \), for all \( x \in X \)

For a family \( \{ \lambda_i/i \in I \} \) of fuzzy sets in \((X, T)\), the union \( \psi = V_i(\lambda_i) \) and intersection \( \delta = \bigwedge_i(\lambda_i) \), are defined respectively as
(iv) $\psi(x) = \sup \{\lambda_i(x) / x \in X\}$

(v) $\delta(x) = \inf \{\lambda_i(x) / x \in X\}$

Lemma 2.2 [(1)]. For a fuzzy set $\lambda$ of a fuzzy topological space $X$,
(i) $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$ and
(ii) $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$.

Definition 2.3. A fuzzy set $\lambda$ in a fuzzy topological space $(X, T)$ is called
(i) fuzzy regular-open set in $(X, T)$ if $\lambda = \text{int cl}(\lambda)$; fuzzy regular-closed set in $(X, T)$ if $\lambda = \text{cl int}(\lambda)[1]$.
(ii) fuzzy $G_\delta$ set in $(X, T)$ if $\lambda = \Lambda_{i=1}^\infty \lambda_i$, where $\lambda_i \subseteq T$ for $i \in I$; fuzzy $F_\sigma$ set in $(X, T)$ if $\lambda = V_{\infty=1}^\infty \lambda_i$, where $1 - \lambda_i \subseteq T$ for $i \in I[2]$.
(iii) fuzzy regular $G_\delta$ set in $(X, T)$ if $\lambda = \Lambda_{i=1}^\infty \text{int}(\lambda_i)$, where $1 - \lambda_i \subseteq T$; fuzzy regular $F_\sigma$ set in $(X, T)$ if $\lambda = V_{\infty=1}^\infty \text{cl}(\mu_i)$, where $\mu_i \subseteq T[10]$.

Definition 2.4. A fuzzy set $\lambda$ in a fuzzy topological space $(X, T)$, is called a
(i) fuzzy dense set if there exists no fuzzy closed set $\mu$ in $(X, T)$ such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in $(X, T)[8]$.
(ii) fuzzy somewhere dense set if there exists a non-zero fuzzy open set $\mu$ in $(X, T)$ such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) \neq 0$, in $(X, T)[9]$.
(iii) fuzzy $\sigma$ - nowhere dense set if $\lambda$ is a fuzzy $F_\sigma$ set with $\text{int}(\lambda) = 0$, in $(X, T)[13]$.
(iv) fuzzy simply open set if $\text{Bd}(\lambda)$ is a fuzzy nowhere dense set in $(X, T)$. That is, $\lambda$ is a fuzzy simply open set in $(X, T)$ if $\text{cl}(\lambda) \land \text{cl}(1 - \lambda)$ is a fuzzy nowhere dense set in $(X, T)[14]$.
(v) fuzzy simply* open set if $\lambda = \mu \lor \delta$, where $\mu$ is a fuzzy open set and $\delta$ is a fuzzy nowhere dense set in $(X, T)[15]$.

Definition 2.5. A fuzzy topological space $(X, T)$ is called a
(i) fuzzy $P$ space if each fuzzy $G_\delta$ set in $(X, T)$ is fuzzy open set in $(X, T)[7]$.
(ii) fuzzy resolvable space if there exists a fuzzy dense set $\lambda$ in $(X, T)$ such that $\text{cl}(1 - \lambda) = 1$. Otherwise, $(X, T)$ is called a fuzzy irresolvable space [11].
(iii) fuzzy extremally disconnected space if closure of every fuzzy open set is a fuzzy open set in $(X, T)[4]$.
(iv) fuzzy hyper-connected space if every non-null fuzzy open subset of $(X, T)$ is fuzzy dense in $(X, T)[5]$.
(v) fuzzy perfectly disconnected space if for any two non-zero fuzzy sets $\lambda$ and $\mu$ defined on $X$ with $\lambda \leq 1 - \mu$, $\text{cl}(\lambda) \leq 1 - \text{cl}(\mu)$, in $(X, T)[17]$.

Theorem 2.6 [(1)]. In a fuzzy topological space,
(a) The closure of a fuzzy open set is a fuzzy regular closed set.
(b) The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.7 [(10)]. If $\lambda$ is a fuzzy regular $G_\delta$ set in a fuzzy topological space $(X, T)$ if and only if $1 - \lambda$ is a fuzzy regular $F_\sigma$ set in $(X, T)$.

Theorem 2.8 [(12)]. If a fuzzy topological space $(X, T)$ is a fuzzy $P$-space, then $(X, T)$ is a weak fuzzy $P$-space.

Theorem 2.9 [(17)]. If $(\lambda_i)$’s($i = 1$ to $\infty$) are fuzzy somewhere dense sets in a fuzzy perfectly disconnected space $(X, T)$, then there exists a fuzzy $G_\delta$ set $\eta$ in $(X, T)$ such that $\eta \leq \Lambda_{i=1}^\infty \text{cl}(\lambda_i)$.

### 3. Weak fuzzy $P$-spaces

Definition 3.1 [(12)]. A fuzzy topological space $(X, T)$ is called a weak fuzzy $P$-space if the countable intersection fuzzy regular open sets in $(X, T)$ is a fuzzy regular open set in $(X, T)$. That is, $\Lambda_{i=1}^\infty \alpha_i$ is a fuzzy regular open set in $(X, T)$, where $(\alpha_i)$’s are fuzzy regular open sets in $(X, T)$.

Example 3.2. Let $X = \{a, b, c\}$. Consider the fuzzy sets $\alpha, \beta, \gamma, \alpha \lor \beta, \alpha \land \beta, \alpha \land \gamma, \beta \land \gamma \land 1$ is a fuzzy topology on $X$. On computation, $\text{cl}(\alpha) = 1 - \gamma = \alpha$; $\text{cl}(\beta) = 1 - \beta = \beta; \text{cl}(\gamma) = 1 - \alpha = \gamma; \text{cl}(\alpha \lor \beta) = 1 - [\beta \land \gamma] = \alpha \lor \beta; \text{cl}(\alpha \land \gamma) = 1 - [\alpha \land \gamma] = \alpha \land \gamma$. Then, $T = \{\alpha, \beta, \gamma, \alpha \lor \beta, \alpha \land \gamma, \beta \land \gamma, 1\}$ is a fuzzy topology on $X$. On computation, $\text{cl}(\alpha) = 1 - \gamma = \alpha$; $\text{cl}(\beta) = 1 - \beta = \beta; \text{cl}(\gamma) = 1 - \alpha = \gamma; \text{cl}(\alpha \lor \beta) = 1 - [\beta \land \gamma] = \alpha \lor \beta; \text{cl}(\alpha \land \gamma) = 1 - [\alpha \land \gamma] = \alpha \land \gamma$. Then, $T = \{\alpha, \beta, \gamma, \alpha \lor \beta, \alpha \land \gamma, \beta \land \gamma, 1\}$ is a fuzzy topology on $X$. On computation, $\text{cl}(\alpha) = 1 - \gamma = \alpha$; $\text{cl}(\beta) = 1 - \beta = \beta; \text{cl}(\gamma) = 1 - \alpha = \gamma; \text{cl}(\alpha \lor \beta) = 1 - [\beta \land \gamma] = \alpha \lor \beta; \text{cl}(\alpha \land \gamma) = 1 - [\alpha \land \gamma] = \alpha \land \gamma; \text{cl}(\beta \land \gamma) = 1 - [\alpha \lor \beta] = \beta \land \gamma; \text{cl}(\delta) = \beta \lor \gamma \neq \delta$.
Thus \( \alpha, \beta, \gamma, \alpha \lor \beta, \alpha \lor \gamma, \beta \lor \gamma \) are fuzzy regular open sets in \((X, T)\). But \(\beta\) and \(\delta\) are not fuzzy regular open sets in \((X, T)\). Now \(\alpha \lor \beta \lor \gamma \lor (\alpha \lor \beta) \lor (\alpha \lor \gamma) \lor (\beta \lor \gamma) = \alpha \lor \gamma\) and \(\alpha \lor \gamma\) is a fuzzy regular open set in \((X, T)\) implies that \((X, T)\) is a weak fuzzy \(P\)-space.

Now \(\text{int } (\alpha - 1) = \; \gamma; \text{int } (\beta - 1) = \; \beta; \text{int } (\gamma - 1) = \; \alpha; \; \text{int } (\gamma - 1) = \; \gamma\). Hence \(\text{int } (\gamma - 1) = \; \gamma\) and \(\text{int } (\gamma - 1) = \; \gamma\). Thus \(\text{int } (\gamma - 1) = \; \gamma\) and \(\text{int } (\gamma - 1) = \; \gamma\) are fuzzy closed sets in \((X, T)\). Hence \(\alpha \lor \gamma\) and \(\beta\) are fuzzy open sets in \((X, T)\).

**Proposition 3.3.** If \(\lambda\) is a fuzzy regular \(G_\delta\)-set in a weak fuzzy \(P\)-space \((X, T)\), then

(i) \(\lambda\) is a fuzzy regular open set in \((X, T)\).

(ii) \(\lambda\) is a fuzzy open set in \((X, T)\).

(iii) \(\lambda\) is a fuzzy somewhere dense set in \((X, T)\).

**Proof.**

(i) Let \(\lambda\) be a fuzzy regular \(G_\delta\)-set in \((X, T)\). Then, \(\lambda = \biglor_{i=1}^{\infty} (\text{int } (\lambda_i))\), where \((1 - \lambda_i)\) are \(\in T\). Since \((1 - \lambda_i)\) are fuzzy open sets, \((\lambda_i)\) are fuzzy closed sets in \((X, T)\) and hence, by theorem 2.1, \(\text{int } (\lambda_i)\) are fuzzy regular open sets in \((X, T)\). Since \((X, T)\) is a weak fuzzy \(P\)-space, \(\lambda = \biglor_{i=1}^{\infty} (\text{int } (\lambda_i))\) is a fuzzy regular open set in \((X, T)\).

(ii) Since each fuzzy regular open set is a fuzzy open set in a fuzzy topological space, the fuzzy regular open set \(\lambda\) is a fuzzy open set in \((X, T)\).

(iii) Since \(X, T\) is a weak fuzzy \(P\)-space, by (ii), the fuzzy regular \(G_\delta\)-set \(\lambda\) is a fuzzy open set in \((X, T)\). Hence \(\text{int } (\lambda) \neq 0\) in \((X, T)\).

Now \(\text{int } (\lambda) \leq \text{cl } (\lambda)\) implies \(\text{int } \text{cl } (\lambda) \neq 0\) in \((X, T)\). Hence \(\lambda\) is a fuzzy somewhere dense set in \((X, T)\).

**Proposition 3.4.** If \(\mu\) is a fuzzy regular \(F_\sigma\)-set in a weak fuzzy \(P\)-space \((X, T)\), then

(i) \(\mu\) is a fuzzy regular closed set in \((X, T)\).

(ii) \(\mu\) is a fuzzy closed set in \((X, T)\).

**Proof.**

(i) Let \(\mu\) be a fuzzy regular \(F_\sigma\)-set in \((X, T)\). Then, by theorem 2.2, \(1 - \mu\) is a fuzzy regular \(G_\delta\)-set in \((X, T)\). Since \((X, T)\) is a weak fuzzy \(P\)-space, by proposition 3.1 (i), \(1 - \mu\) is a fuzzy regular open set in \((X, T)\) and hence \(\mu\) is a fuzzy regular closed set in \((X, T)\).

(ii) Since each fuzzy regular closed set is a fuzzy closed set in a fuzzy topological space, the fuzzy regular closed set \(\mu\) is a fuzzy closed set in \((X, T)\).

**Remark 3.5.** From the propositions 3.1 and 3.2, one will have the following: ”Fuzzy regular \(G_\delta\)-sets are fuzzy open sets and fuzzy regular \(F_\sigma\)-sets are fuzzy regular closed sets”.

It is to be noted that the fuzzy \(G_\delta\)-sets in weak fuzzy \(P\)-spaces need not be fuzzy regular \(G_\delta\)-sets. For consider the following example:

**Example 3.6.** Let \(X = \{a, b, c\}\). Consider the fuzzy sets \(\lambda, \mu\) and \(\gamma\) defined on \(X\) as follows:

\[
\lambda : X \to [0, 1] \text{ is defined as } \lambda(a) = 0.4; \lambda(b) = 0.5; \lambda(c) = 0.6 \\
\mu : X \to [0, 1] \text{ is defined as } \mu(a) = 0.6; \mu(b) = 0.4; \mu(c) = 0.5 \\
\gamma : X \to [0, 1] \text{ is defined as } \gamma(a) = 0.7; \gamma(b) = 0.6; \gamma(c) = 0.4,
\]

Then,

\[
T = \{0, \lambda, \mu, \gamma, \lambda \lor \mu, \lambda \lor \gamma, \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \lambda \lor \mu \lor \gamma, 1\}
\]

is a fuzzy topology on \(X\). On computation \(\lambda, \lambda \lor \mu, \lambda \lor \gamma, \lambda \lor \mu \lor \gamma, \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \gamma, \lambda \lor \mu \lor \gamma\) are fuzzy regular open sets in \((X, T)\). Hence \(\lambda \lor \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \gamma, \lambda \lor \mu \lor \gamma\) are fuzzy regular open sets in \((X, T)\) implies that \((X, T)\) is a weak fuzzy \(P\)-space. Also \(\gamma, \lambda \lor \mu \lor \gamma, \lambda \lor \mu \lor \gamma\) is a fuzzy \(G_\delta\)-set but not a fuzzy regular \(G_\delta\)-set in \((X, T)\).

The following propositions characterize the weak fuzzy \(P\)-spaces in terms of fuzzy regular \(G_\delta\)-sets and fuzzy regular \(F_\sigma\)-sets.

**Proposition 3.7.** A fuzzy topological space \((X, T)\) is a weak fuzzy \(P\)-space if and only if each fuzzy regular \(G_\delta\)-set is a fuzzy regular open set in \((X, T)\)

**Proof.** Suppose that \((X, T)\) is a weak fuzzy \(P\)-space and \(\lambda\) be a fuzzy regular \(G_\delta\)-set in \((X, T)\). Then, by proposition 3.1, the fuzzy regular \(G_\delta\)-set \(\lambda\) is a fuzzy regular open set in \((X, T)\).

Conversely, let each fuzzy regular \(G_\delta\)-set be a fuzzy regular open set in the fuzzy topological space \((X, T)\). Suppose that \(\lambda_i\) are fuzzy regular open sets in \((X, T)\). Then int \(\text{cl } (\lambda_i) = \lambda_i\) in \((X, T)\). Let \(\text{cl } (\lambda_i) = \mu_i\). Then \(\mu_i\) are fuzzy closed sets in \((X, T)\). Hence \(\text{int } \text{cl } (\lambda_i) \neq 0\) in \((X, T)\). Hence \(\lambda_i\) is a fuzzy somewhere dense set in \((X, T)\).

**Proposition 3.8.** A fuzzy topological space \((X, T)\) is a weak fuzzy \(P\)-space if and only if each fuzzy regular \(F_\sigma\)-set is a fuzzy regular closed set in \((X, T)\).
Proof. Let \((X, T)\) be a weak fuzzy \(P\)-space and \(\mu\) be a regular \(F_\sigma\) -set in \((X, T)\). Then, by theorem 2.2, \(1 - \mu\) is a fuzzy regular \(G_\delta\) -set in \((X_2, T)\). By proposition 3.3, \(1 - \mu\) is a fuzzy regular open set in \((X, T)\) and hence \(\mu\) is a fuzzy regular closed set in \((X, T)\).

Conversely, let each fuzzy regular \(F_\sigma\) -set \(\delta\) be a fuzzy regular closed set in the fuzzy topological space \((X, T)\). Suppose that \((\lambda_1)\)'s are fuzzy regular open sets in \((X, T)\). Then, \((1 - \lambda_1)\)'s are fuzzy regular closed sets in \((X, T)\). Then \(\delta\) implies that \((1 - \lambda_1) = 1 - \lambda_1\). Let \((1 - \lambda_1) = \gamma\). Then \((\gamma)\)'s are fuzzy open sets in \((X, T)\). Then, \(V_{1=1}^\gamma \mu\) is a fuzzy regular \(F_\sigma\) -set in \((X, T)\). By hypothesis, \(V_{1=1}^\gamma \mu\) is a fuzzy regular closed set in \((X, T)\). But \(1 - V_{1=1}^\gamma \mu\) is a fuzzy regular open set in \((X, T)\). Then, \(1 - V_{1=1}^\gamma \mu\) is a fuzzy regular open set in \((X, T)\). Thus, \(1 - V_{1=1}^\gamma \mu\) is a fuzzy regular open set in \((X, T)\). Then, \(1 - V_{1=1}^\gamma \mu\) is a fuzzy regular open set in \((X, T)\).

Proposition 3.9. \(\text{If } \lambda\) is a fuzzy regular \(G_\delta\) -set in a weak fuzzy \(P\)-space \((X, T)\), then \(\lambda\) is not a fuzzy dense set in \((X, T)\).

Proof. Let \(\lambda\) be a fuzzy regular \(G_\delta\) -set in the weak fuzzy \(P\)-space \((X, T)\). Then, \(\lambda = \Lambda_{1=1}^{\infty} \int (\lambda_i)\), where \((1 - \lambda_i) \in T\) and \(\lambda_i \neq 1\). Suppose that \(\lambda\) is a fuzzy dense set in \((X, T)\). Then, \(\int (\lambda) = 1\), implies that \(\int (\lambda_i) = 1\). Since \(\int (\lambda_i) = 1\), \(\lambda\) implies that \(\Lambda_{1=1}^{\infty} \int (\lambda_i) = 1\). Hence \(\lambda\) is not a fuzzy dense set in \((X, T)\).

Remark 3.10. In view of the proposition 3.1 and 3.3, one will have the following: "Fuzzy regular \(G_\delta\) -sets in weak fuzzy \(P\)-spaces are fuzzy open but not fuzzy dense sets".

Proposition 3.11. \(\text{If } \mu\) is a fuzzy regular \(F_\sigma\) -set in a weak fuzzy \(P\)-space \((X, T)\), then \(\int (\mu) \neq 0\) in \((X, T)\).

Proof. Let \(\mu\) be a fuzzy regular \(F_\sigma\) -set in \((X, T)\). Suppose that \(\int (\mu) = 0\) in \((X, T)\). Since \(\mu\) is a fuzzy regular \(F_\sigma\) -set, by theorem 2.2, \(1 - \mu\) is a fuzzy regular \(G_\delta\) -set in a weak fuzzy \(P\)-space \((X, T)\). Now \(\int (1 - \mu) = 1 - \int (\mu) = 1 - 0 = 1\). This implies that \(1 - \mu\) is a fuzzy dense set in \((X, T)\), which is a contradiction by proposition 3.5, that fuzzy regular \(G_\delta\) -sets are not fuzzy dense sets in a weak fuzzy \(P\)-space \((X, T)\). Hence \(\int (\mu) \neq 0\) in \((X, T)\).

Proposition 3.12. \(\text{If } \mu\) is a fuzzy regular \(F_\sigma\) -set in a weak fuzzy \(P\)-space \((X, T)\), then \(\mu\) is not a fuzzy \(\sigma\) -nowhere dense set in \((X, T)\).

Proof. Let \(\mu\) be a fuzzy regular \(F_\sigma\) -set in \((X, T)\). Since \(\mu\) is a weak fuzzy \(P\)-space \((X, T)\), by proposition 3.6, \(\int (\mu) \neq 0\) in \((X, T)\). Since \(\mu\) is a fuzzy regular \(F_\sigma\) -set in \((X, T)\), by theorem 2.2, \(1 - \mu\) is a fuzzy regular \(G_\delta\) -set in \((X, T)\). Then, \(\int (\mu) \neq 0\) in \((X, T)\). This implies that \(\mu\) is not a fuzzy \(\sigma\) -nowhere dense set in \((X, T)\).
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4. Weak fuzzy P-space and other fuzzy topological spaces

**Proposition 4.1.** If \( \lambda \) is a fuzzy regular \( G_\delta \)-set in a fuzzy extremely disconnected and weak fuzzy P-space \((X,T)\), then \( \text{cl}(\lambda) \) is a fuzzy clopen set in \((X,T)\).

**Proof.** Let \( \lambda \) be a fuzzy regular \( G_\delta \)-set in \((X,T)\). Since \((X,T)\) is a weak fuzzy P-space, by proposition 3.1 (i), the fuzzy regular \( G_\delta \)-set \( \lambda \) is a fuzzy nowhere dense set in \((X,T)\). But \( \lambda \) is a fuzzy regular \( G_\delta \)-set in \((X,T)\) and hence \( \text{cl}(\lambda) \neq 1 \). Then, \( \text{cl}(\lambda) = 1 - \text{cl}(\lambda) \). Now \( \text{cl}(\lambda) = 0 \) in \((X,T)\). Hence \( \text{cl}(\lambda) \neq 1 \). Thus \( \text{cl}(\lambda) \neq 1 \). Hence \( \text{cl}(\lambda) \) is a fuzzy clopen set in \((X,T)\).

Proof. Let \( \lambda \) be a fuzzy regular \( G_\delta \)-set in \((X,T)\). Since \((X,T)\) is a weak fuzzy P-space, by proposition 3.1 (i), the fuzzy regular \( G_\delta \)-set \( \lambda \) is a fuzzy nowhere dense set in \((X,T)\). But \( \lambda \) is a fuzzy regular \( G_\delta \)-set in \((X,T)\) and hence \( \text{cl}(\lambda) \neq 1 \). Then, \( \text{cl}(\lambda) = 1 - \text{cl}(\lambda) \). Now \( \text{cl}(\lambda) = 0 \) in \((X,T)\). Hence \( \text{cl}(\lambda) \neq 1 \). Thus \( \text{cl}(\lambda) \) is a fuzzy clopen set in \((X,T)\).

**Proposition 4.2.** If \( \lambda \) is an fuzzy regular \( F_\sigma \)-set in the fuzzy extremely disconnected and weak fuzzy P-space \((X,T)\), then \( \text{int}(\lambda) \) is a fuzzy clopen set in \((X,T)\).

**Proof.** Let \( \lambda \) be a fuzzy regular \( F_\sigma \)-set in \((X,T)\). Then, \( 1 - \lambda \) is a fuzzy regular \( G_\delta \)-set in \((X,T)\). Then, \( \text{cl}(\lambda) \neq 1 \). Hence \( \text{cl}(\lambda) \) is a fuzzy clopen set in \((X,T)\).
Then \( \text{int}(\lambda) \) is a fuzzy closed set in \((X,T)\). Hence \( \text{int}(\lambda) \) is a fuzzy clopen set in \((X,T)\).

A fuzzy weak fuzzy P-space need not be a fuzzy resolvable space. For consider the following example:

**Example 4.3.** Let \( X = \{a,b,c\} \). Consider the fuzzy sets \( \lambda, \mu \) and \( \gamma \) defined on \( X \) as follows: \( \lambda : X \to [0,1] \) is defined as \( \lambda(a) = 0.5; \lambda(b) = 0.7; \lambda(c) = 0.3 \), \( \mu : X \to [0,1] \) is defined as \( \mu(a) = 0.6; \mu(b) = 0.5; \mu(c) = 0.8 \), \( \gamma : X \to [0,1] \) is defined as \( \gamma(a) = 0.7; \gamma(b) = 0.3; \gamma(c) = 0.6 \).

Then, \( \text{int}\{\lambda \} = \{0, \lambda, \mu, \gamma, \lambda \lor \mu, \lambda \lor \gamma, \mu \lor \gamma, \lambda \land \mu, \lambda \land \gamma, \mu \land \gamma, \lambda \lor \mu \land \gamma, \lambda \lor \mu \lor \gamma, \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \mu \lor \gamma, \lambda \lor \mu \lor \gamma, \mu \lor \gamma \} \) is a fuzzy topology on \( X \). On computation, \( \lambda, \lambda \land \mu, \lambda \land \gamma \) are fuzzy regular open sets in \((X,T)\). Now \( [\lambda \land (\lambda \land \mu) \lor (\lambda \land \gamma)] = \lambda \land \gamma \) and \( \lambda \land \gamma \) is a fuzzy regular open set in \((X,T)\) implies that \((X,T)\) is a weak fuzzy P-space. The fuzzy dense sets in \((X,T)\) are \( \mu, \gamma, \lambda \lor \mu, \lambda \lor \gamma, \mu \lor \gamma, \lambda \lor [\mu \land \gamma], \mu \lor [\lambda \lor \gamma], \lambda \lor [\lambda \lor \gamma] \lor [\mu \lor \gamma]. \) Now \( \text{cl}(\mu) = 1 \) and \( \text{cl}(1-\mu) = 1 - \mu \neq 1 \), implies that \((X,T)\) is a fuzzy irresolvable space and \((X,T)\) is not a fuzzy resolvable space.

**Proposition 4.4.** If \((X,T)\) is a weak fuzzy P-space, then \((X,T)\) is not a fuzzy hyperconnected space.

**Proof.** Let \((X,T)\) be a weak fuzzy P-space and \( \lambda \) be a fuzzy regular \( G_\delta \)-set in \((X,T)\). By proposition 3.1 (ii), the fuzzy regular \( G_\delta \)-set \( \lambda \) is a fuzzy open set in \((X,T)\). By proposition 3.5, the fuzzy regular \( G_\delta \)-set \( \lambda \) in the weak fuzzy P-space \((X,T)\), is not a fuzzy dense in \((X,T)\) and hence the fuzzy open set \( \lambda \) is not a fuzzy dense in \((X,T)\), implies that \((X,T)\) is not a fuzzy hyperconnected space.

**Proposition 4.5.** If \( (\lambda_i) \) \( i = 1 \) to \( \infty \) are fuzzy regular \( G_\delta \)-sets in a fuzzy perfectly disconnected and weak fuzzy P-space \((X,T)\), then there exists a fuzzy \( G_\delta \)-set \( \eta \) in \((X,T)\) such that \( \eta \leq \wedge_{i=1}^{\infty} [\text{cl}(\lambda_i)] \).

**Proof.** Let \( (\lambda_i) \) \( i = 1 \) to \( \infty \) be fuzzy regular \( G_\delta \)-sets in \((X,T)\). Since \((X,T)\) is a weak fuzzy P-space, by proposition 3.1(i), the fuzzy regular \( G_\delta \)-sets \( (\lambda_i) \) \( i = 1 \) to \( \infty \) are fuzzy somewhere dense sets in the fuzzy perfectly disconnected space \((X,T)\). Then, by theorem 2.4, there exists a fuzzy \( G_\delta \)-set \( \eta \) in \((X,T)\) such that \( \eta \leq \wedge_{i=1}^{\infty} [\text{cl}(\lambda_i)] \).

**References**


