Malaya Journal of Matematik, Vol. S, No. 1, 12-16, 2019

https://doi.org/10.26637/MJM0S01/0004

μ_{ij}-semi open sets in bigeneralized topological space

M. Anees Fathima [1]* and R. Jamuna Rani[2]

Abstract
In this paper, we introduce μ_{ij}-semi open sets and study its properties in bigeneralized topological spaces.

Keywords
Generalized topology, bigeneralized topology, μ_i-open set, μ_i- closed set, μ_{ij}-semi open set, μ_{ij}-semi closed set.

AMS Subject Classification
18B30.

[1] *Research Scholar [Reg. No:19121172092012], Research Center, Department of Mathematics, Rani Anna Government College for Women, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.*

[2] *Assistant Professor, Department of Mathematics, Rani Anna Government College for Women, Tirunelveli-627008, Tamil Nadu, India.*

***Corresponding author**: [1] afiseyan09@gmail.com; [2] jamunarani1977@gmail.com

Article History: Received **21** December **2018**; Accepted **11** February **2019**

Contents

1. Introduction

A. Csaszar[2] introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In [7] Sivagami studied the properties of $\gamma-$Semi open sets. C. Boonpok [1] introduced the concept of bigeneralized topological spaces and studied (m,n)-closed sets and (m,n)-open sets in bigeneralized topological spaces.

In this paper we introduce the notions of μ_{ij}-semi open sets in bigeneralized topological spaces and study some of their properties. Also we investigate some of their characterizations.

2. Preliminaries

We recall some basic definitions and notations. Let X be a set and denotes $expX$ the power set of X. A subset μ of $expX$ is said to be generalized topology (briefly GT) on X if $\phi \in \mu$ and an arbitrary union of elements of μ belongs to μ[3]. Let μ be a GT on X, the elements of μ are called $\mu-$open sets and the complements of $\mu-$open sets are called μ-closed sets. Let M_μ denote the union of all elements of μ. We say μ is strong [4] if $M_\mu = X$. If $A \subseteq X$ then $i_\mu(A)$ denotes the union of all $\mu-$open sets contained in A and $c_\mu(A)$ is the intersections of all μ-closed sets containing A[5]. According to [7], for $A \subseteq X$ and $x \in X$ we have $x \in c_\mu(A)$ if and only if $x \in M \in \mu$ implies $M \cap A \neq \phi$. Let (X,μ) be a generalized topological space. A subset M of X is said to be $\mu-$semi open if and only if $M \subseteq c_\mu(i_\mu(M))$, μ-preopen if and only if $M \subseteq i_\mu(c_\mu(M))$, $\mu-\alpha$-open if and only if $M \subseteq i_\mu(c_\mu(i_\mu(M)))$, finally $\mu-\beta$-open if and only if $M \subseteq c_\mu(i_\mu(c_\mu(M)))$ [5].

Theorem 2.1. [2] Let (X,μ) be a generalized topological space. Then

(1). $c_\mu(A) = X - i_\mu(X-A)$;

(2). $i_\mu(A) = X - c_\mu(X-A)$

Proposition 2.2. [6] Let (X,μ) be a generalized topological space. For subsets A and B of X, the following properties holds:

(1). $c_\mu(X-A) = X - i_\mu(A)$ and $i_\mu(X-A) = X - c_\mu(A)$.

(2). If $(X-A) \in \mu$, then $c_\mu(A) = A$ and if $A \in \mu$, then $i_\mu(A) = A$;

(3). If $A \subseteq B$, then $c_\mu(A) \subseteq c_\mu(B)$ and $i_\mu(A) \subseteq i_\mu(B)$;

(4). $A \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq A$;

(5). $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$.

Definition 2.3. [1] Let X be a nonempty set and μ_1, μ_2 be generalized topologies on X. A triple (X, μ_1, μ_2) is said to be a bigeneralized topological space.
Let (X, μ_1, μ_2) is said to be a bigeneralized topological space and A a subset of X. The closure of A and the interior of A with respect to μ_m are denoted by $c_{\mu_m}(A)$ and $i_{\mu_m}(A)$, respectively, for $m = 1,2$.

Definition 2.4. [1] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called (m,n)-closed if and only if $c_{\mu_m}(c_{\mu_n}(A)) = A$, where $m,n = 1,2$ and $m \neq n$. The complement of (m,n)-closed set is called (m,n)-open.

Proposition 2.5. [1] Let (X, μ_1, μ_2) be a bigeneralized topological space and A a subset of X. Then A is (m,n)-closed if and only if A is both $\mu-$closed in (X, μ_m) and (X, μ_n).

Proposition 2.6. [1] Let (X, μ_1, μ_2) be a bigeneralized topological space. Then A is (m,n)-open if and only if $i_{\mu_m}(i_{\mu_n}(A)) = A$.

3. μ_{ij}-semi open sets

Definition 3.1. Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then A is said to be μ_{ij}-semi open if $A \subset C_{\mu_i}(i_{\mu_j}(A))$ where $i, j = 1,2$ and $i \neq j$. The complement of a μ_{ij}-semi open set is called a μ_{ij}-semi closed set. The collection of all μ_{ij}-semi open sets denoted by $\sigma_{ij}(\mu)$.

Example 3.2. Let $X = \{a,b,c\}$. Consider two generalized topologies $\mu_1 = \{\phi, \{a\}, \{b\}, \{a,b\}\}$ and $\mu_2 = \{\phi, \{a\}, \{c\}, \{a,c\}\}$ on X. Then $\{a\}$ is μ_{12}-semi open but $\{a,b,c\}$ is not μ_{12}-semi open.

Definition 3.3. Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the union of all μ_{ij}-semi open sets contained in A is called the μ_{ij}-semi interior of A is denoted by $i_{\sigma_{ij}}(A)$. The intersection of all μ_{ij}-semi closed sets containing A is called the μ_{ij}-semi closure of A and is denoted by $C_{\sigma_{ij}}(A)$.

The following Theorem 3.4 gives some of the properties of μ_{ij}-semi open sets in bigeneralized topological space.

Theorem 3.4. Let (X, μ_1, μ_2) be a bigeneralized topological space. Then the following hold.

(a). X is μ_{ij}-semi open and so ϕ is μ_{ij}-semi closed.

(b). Every μ_j (respectively μ_i) open set is a μ_{ij}-semi open set.

(c). Union of μ_{ij}-semi open set is μ_{ij}-semi open.

Theorem 3.5. Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following hold.

(a). $C_{\sigma_{ij}}(A)$ is the smallest μ_{ij}-semi closed set containing A.

(b). A is μ_{ij}-semi closed if and only if $A = C_{\sigma_{ij}}(A)$

(c). $x \in C_{\sigma_{ij}}(A)$ if and only if for every μ_{ij}-semi open G containing x, $G \cap A \neq \phi$.

(d). $C_{\sigma_{ij}} \in \Gamma_{012+}$.

Proof.
(a) Since union of μ_{ij}-semi open sets is μ_{ij}-semi open and by definition of $C_{\sigma_{ij}}$, the proof follows.
(b) The proof follows from (a).
(c) Suppose $x \in C_{\sigma_{ij}}(A)$. Let G be a μ_{ij}-semi open set containing x. If $G \cap A = \phi$, then $X - G$ is a μ_{ij}-semi closed set containing A and so $x \notin C_{\sigma_{ij}}(A)$, a contradiction. Therefore $G \cap A \neq \phi$.
Conversely, suppose for every μ_{ij}-semi open set G containing x, $G \cap A \neq \phi$. If $x \notin C_{\sigma_{ij}}(A)$, then there exist a μ_{ij}-semi closed set F containing A such that $x \notin F$. Then $G = X - F$ is μ_{ij}-semi open set containing x such that $G \cap A = \phi$, a contradiction. Therefore $x \in C_{\sigma_{ij}}(A)$.
(d) Since $C_{\sigma_{ij}}(A)$ is the intersection of all μ_{ij}-semi closed set containing A, then by (b), $C_{\sigma_{ij}} \in \Gamma_2$. Clearly X is μ_{ij}-semi open, ϕ is μ_{ij}-semi closed which implies that $\phi = C_{\sigma_{ij}}(\phi)$ and so $C_\sigma \in \Gamma_0$. Also ϕ is μ_i-open, ϕ is μ_{ij}-semi open. Therefore X is μ_{ij}-semi closed and $X = C_{\sigma_{ij}}(X)$ which implies that $C_{\sigma_{ij}} \in \Gamma_1$. Also $C_{\sigma_{ij}} \in \Gamma_+$ by the definition of $C_{\sigma_{ij}}$. Hence $C_\sigma \in \Gamma_{012+}$. □

The following Theorem 3.6 gives some of the properties of μ_{ij}-semi interior operator $i_{\sigma_{ij}}$.

Theorem 3.6. Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then

(a). $i_{\sigma_{ij}}(A)$ is the largest μ_{ij}-semi open set contained in A.

(b). A is μ_{ij}-semi open if and only if $A = i_{\sigma_{ij}}(A)$

(c). $x \in i_{\sigma_{ij}}(A)$ if and only if there exist μ_{ij}-semi open G containing x such that $G \subset A$.

(d). $i_{\sigma_{ij}} \in \Gamma_{012-}$.

Theorem 3.7. Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following hold.

(a). $(i_{\sigma_{ij}})^* = C_{\sigma_{ij}}$

(b). $(C_{\sigma_{ij}})^* = i_{\sigma_{ij}}$

(c). $i_{\sigma_{ij}}(X - A) = X - C_{\sigma_{ij}}(A)$ for every subset A of X.

(d). $C_{\sigma_{ij}}(X - A) = X - i_{\sigma_{ij}}(A)$ for every subset A of X

Proof.
(a). Let A be a subset of X. Then $(i_{\sigma_{ij}})^*(A) = X - i_{\sigma_{ij}}(X - A)$. Since $i_{\sigma_{ij}}(X - A)$ is the largest μ_{ij}-semi open set contained in $X - A$, $X - i_{\sigma_{ij}}(X - A)$ is the smallest μ_{ij}-semi closed set containing A and so $X - i_{\sigma_{ij}}(X - A) = C_{\sigma_{ij}}(A)$. Hence $(i_{\sigma_{ij}})* = C_{\sigma_{ij}}$.
(b). $(C_{\sigma_{ij}})^* = ((i_{\sigma_{ij}})^*)^* = i_{\sigma_{ij}}$. This proves (b).

(c). Let A be a subset of X. Then $(i_{\sigma_{ij}})^* = X - i_{\sigma_{ij}}(X-A)$ and by (b), $C_{\sigma_{ij}}(A) = X - i_{\sigma_{ij}}(X-A)$ which implies that $i_{\sigma_{ij}}(X-A) = X - C_{\sigma_{ij}}(A)$ for every subset A of X.
(d). The proof is similar to the proof of (c). □

Theorem 3.8. Let (X,μ_1,μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following are equivalent.

(a). A is μ_{ij}-semi open

(b). $A \subset C_{\mu_i}(i_{\mu_j}(A))$

(c). $C_{\mu_i}(A) = C_{\mu_i}(i_{\mu_j}(A))$

Proof. $(a) \Rightarrow (b)$.
Suppose A is μ_{ij}-semi open. Then there exist a μ_{ij}-open set G such that $G \subset A \subset C_{\mu_i}(G)$. Since G is μ_{ij}-open, $G = i_{\mu_j}(G)$ and so $A \subset C_{\mu_i}(i_{\mu_j}(G))$. Since $G \subset A$, $i_{\mu_j}(G) \subset i_{\mu_j}(A)$. Then $C_{\mu_i}(i_{\mu_j}(G)) \subset C_{\mu_i}(i_{\mu_j}(A))$. Therefore, $A \subset C_{\mu_i}(i_{\mu_j}(G)) \subset C_{\mu_i}(i_{\mu_j}(A))$. Thus $A \subset C_{\mu_i}(i_{\mu_j}(A))$.
$(b) \Rightarrow (c)$.
Suppose $A \subset C_{\mu_i}(i_{\mu_j}(A))$. Then $C_{\mu_i}(A) \subset C_{\mu_i}(C_{\mu_i}(i_{\mu_j}(A))) = C_{\mu_i}(i_{\mu_j}(A))$. That is $C_{\mu_i}(A) \subset C_{\mu_i}(i_{\mu_j}(A))$. Since $i_{\mu_j}(A) \subset A$, $C_{\mu_i}(i_{\mu_j}(A)) \subset C_{\mu_i}(A)$. Thus $C_{\mu_i}(A) = C_{\mu_i}(i_{\mu_j}(A))$.
$(c) \Rightarrow (a)$.
Suppose $C_{\mu_i}(A) = C_{\mu_i}(i_{\mu_j}(A))$. Since $i_{\mu_j}(A)$ is a μ_j-open set such that $i_{\mu_j}(A) \subset A \subset C_{\mu_i}(i_{\mu_j}(A))$. Therefore there exist μ_{ij}-open set $i_{\mu_j}(A)$ such that $i_{\mu_j}(A) \subset A \subset C_{\mu_i}(i_{\mu_j}(A))$. Hence A is μ_{ij}-semi open. □

Theorem 3.9. Let (X,μ_1,μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following hold.

(a). A is μ_{ij}-semi open if and only if A is $C_{\mu_i}i_{\mu_j}$-open if and only if $A = i_{C_{\mu_i}i_{\mu_j}}(A)$

(b). $i_{\sigma_{ij}} = i_{C_{\mu_i}i_{\mu_j}}$ and $C_{\sigma_{ij}} = C_{C_{\mu_i}i_{\mu_j}}$

(c). $i_{\sigma_{ij}}(A) = A \cap C_{C_{\mu_i}i_{\mu_j}}(A)$

(d). $C_{\sigma_{ij}}(A) = A \cap i_{\mu_j}(C_{\mu_j}(A))$

Proof. (a) follows from Theorem 3.8 (a) and (b).
(b) If $x \in i_{\sigma_{ij}}(A)$ then there exist a μ_{ij}-semi open set B such that $x \in B \subset A$. Then by (a) B is $C_{\mu_i}i_{\mu_j}$-open and $x \in i_{C_{\mu_i}i_{\mu_j}}(A)$. Hence $i_{\sigma_{ij}}(A) \subset i_{C_{\mu_i}i_{\mu_j}}(A)$. Similarly, we can prove that $i_{C_{\mu_i}i_{\mu_j}} \subset i_{\sigma_{ij}}(A)$. Therefore $i_{C_{\mu_i}i_{\mu_j}} = i_{\sigma_{ij}}(A)$. Again $C_{\sigma_{ij}} = (i_{C_{\mu_i}i_{\mu_j}})^*$ and so $C_{\sigma_{ij}} = (i_{C_{\mu_i}}i_{\mu_j})^* = C_{C_{\mu_i}}i_{\mu_j}$.
(c) Since $i_{\mu_j}(i_{\mu_j}(A)) = i_{\mu_j}(A)$ and $i_{\mu_j}(A) \subset C_{\mu_i}i_{\mu_j}(A)$, we have $i_{C_{\mu_i}i_{\mu_j}}(A) = A \cap C_{C_{\mu_i}i_{\mu_j}}(A)$. Also by (b) $i_{\sigma_{ij}}(A) = i_{C_{\mu_i}i_{\mu_j}}(A)$. Therefore, $i_{\sigma_{ij}}(A) = A \cap C_{C_{\mu_i}i_{\mu_j}}(A)$
(d) By (c), $i_{C_{\mu_i}i_{\mu_j}}(A) = A \cap C_{\mu_i}(i_{\mu_j}(A)$. Then $(i_{C_{\mu_i}i_{\mu_j}}(A))^* = (A \cap C_{\mu_i}(i_{\mu_j}(A))^*$ implies $C_{C_{\mu_i}i_{\mu_j}}(A) = A \cap i_{\mu_j}(C_{\mu_i}(A))$. Also by (b), $C_{\sigma_{ij}}(A) = A \cup i_{\mu_j}(C_{\mu_j}(A))$. □

Theorem 3.10. Let (X,μ_1,μ_2) be a bigeneralized topological space. Let $A \subset B \subset C_{\mu_i}(A)$ and A is μ_{ij}-semi open. Then B is μ_{ij}-semi open.

Proof. Suppose A is μ_{ij}-semi open. Then by Theorem 3.8(c), $C_{\mu_i}(A) = C_{\mu_i}(i_{\mu_j}(A))$. Since $B \subset C_{\mu_i}(A)$, $B \subset C_{\mu_i}(i_{\mu_j}(B))$ and so by Theorem 3.8 (b), B is μ_{ij}-semi open. □

Theorem 3.11. Let (X,μ_1,μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following are equivalent.

(a). A is μ_{ij}-semi closed

(b). $i_{\mu_i}C_{\mu_j}(A) \subset A$

(c). $i_{\mu_i}C_{\mu_j}(A) = i_{\mu_i}(A)$

(d). There exist a μ_j closed set F (resp. μ_i-closed sets) such that $i_{\mu_j}(F) \subset A \subset F$

Proof. $(a) \Rightarrow (b)$.
If A is μ_{ij}-semi closed, then $X-A$ is μ_{ij}-semi open and so by Theorem 3.8(b)
$X-A \subset C_{\mu_i}(i_{\mu_j}(X-A))$. Also $C_{\mu_i}i_{\mu_j}(X-A) = X - i_{\mu_i}C_{\mu_j}(A)$, $X-A \subset X - i_{\mu_i}C_{\mu_j}(A)$ and so $i_{\mu_i}C_{\mu_j}(A) \subset A$.
$(b) \Rightarrow (c)$.
If $i_{\mu_i}C_{\mu_j}(A) \subset A$, then $i_{\mu_i}C_{\mu_j}(A) \subset i_{\mu_i}(A)$ and so $i_{\mu_i}(C_{\mu_j}(A)) = i_{\mu_i}(A)$.
$(c) \Rightarrow (d)$.
If $F = C_{\mu_j}(A)$, then F is a μ_j-closed set (resp. μ_i). Now $i_{\mu_i}(F) = i_{\mu_i}(C_{\mu_j}(A)) = i_{\mu_i}(A) \subset A \subset F$ which proves (d).
$(d) \Rightarrow (a)$.
Suppose there exist a μ_j-closed set F (resp.μ_i) such that $i_{\mu_j}(F) \subset A \subset F$. Then $X-F \subset X-A \subset X - i_{\mu_i}(F) = C_{\mu_i}(X-F)$. Since $X-F$ is μ_{ij}-semi open, $X-A$ is μ_{ij}-semi open and A is μ_{ij}-semi closed. □

Definition 3.12. We say that A is C_{μ_j} dense if $C_{\mu_i}(A) = X$.

Theorem 3.13. Let (X,μ_1,μ_2) be a bigeneralized topological space. Let A be a subset of X. Then the following are equivalent.

(a). $C_{\mu i}(A) = X$

(b). $C_{\sigma_{ij}}(A) = X$

(c) If B is any μ_{ij}-semi closed subset of X such that $A \subset B$, then $B = X$

(d). Every non empty μ_{ij}-semi open set has a non empty intersection with A

(e). $i_{\sigma_{ij}}(X-A) = \phi$.

Proof. $(a) \Rightarrow (b)$.
Suppose $x \notin C_{\sigma_{ij}}(A)$. Then by Theorem 3.5$(c)$, there exist a μ_{ij}-semi open set G containing X such that $G \cap A = \phi$. Since G is a nonempty μ_{ij}-semi open set, there exist μ_i open set H such that $H \subset G \subset C_{\mu_j}H$ and $H \cap A = \phi$ which implies $C_{\mu_i}(A) \neq X$, a contradiction. Hence $C_{\sigma_{ij}}(A) = X$
$(b) \Rightarrow (c)$.
If B is any μ_{ij}-semi closed subset of X such that $A \subset B$, then

$X = C_{\sigma_{ij}}(A) \subset C_{\sigma_{ij}}(B) = B$ and so $X = B$
$(c) \Rightarrow (d)$.
If G is any nonempty μ_{ij}-semi open set such that $G \cap A = \phi$. Then $A \subset X - G$ and $X - G$ is μ_{ij}-semi closed which implies $X - G = X$ and so $G = \phi$, a contradiction. Therefore $G \cap A \neq \phi$.
$(d) \Rightarrow (e)$.
If $i_{\sigma_{ij}}(X - A) \neq \phi$. Then $i_{\sigma_{ij}}(X - A)$ is a nonempty μ_{ij}-semi open set such that $i_{\sigma_{ij}}(X - A) \cap A \neq \phi$, a contradiction. Therefore $i_{\sigma_{ij}}(X - A) = \phi$
$(e) \Rightarrow (a)$.
Suppose $i_{\sigma_{ij}}(X - A) = \phi$. Then $X - i_{\sigma_{ij}}(X - A) = X$ and so $C_{\sigma_{ij}}(A) = X$. Also $C_{\sigma_{ij}}(A) \subset C_{\mu_i}(A)$ (resp. μ_j) for any subset A of X and hence $C_{\sigma_{ij}}(A) = X$ implies $C_{\mu_i}(A) = X$. □

References

[1] C. Boonpok, Weakly open functions on bigeneralized topological spaces, *Int. J. of. Math. Analysis*, 15(5)(2010), 891–897.

[2] A. Csaszar, Generalized topology, generalized continuity, *Acta. Math. Hungar.*, 96(2002), 351–357.

[3] A. Csaszar, Extremally disconnected generalized topologies, *Annales. Univ. Sci. Budapest*, 47(2004), 91–96.

[4] A. Csaszar, Generalized open sets in generalized topologies, *Acta. Math. Hungar.*, 106(2005), 53–66.

[5] A. Csaszar, Modification of generalized topologies via hereditary classes, *Acta. Math. Hungar.*, 115(2007), 29–36.

[6] W.K. Min, Almost continuity on generalised topological spaces, *Acta. Math. Hungar.*, 125(2009), 121–125.

[7] P. Sivagami and D. Sivaraj, On γ-semi open sets, *Tamkang. J. Mathematics*, 39(4)(2008), 303–308.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆