GSP closed sets in topological spaces

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Abstract

In this paper a new class of sets called #generalized semi pre closed sets introduced and its properties are studied. Further #gsp-open set and #gsp-neighbourhood are introduced.

Keywords

Pre closed set, #gs-open set, #gsp-closed set, #gsp-open set, #gsp-neighbourhood.

AMS Subject Classification

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1. Introduction

In 1996 H.Maki and T.Noiri[7] found the the new class of sets, the generalized pre closed sets. Later 2000 M.K.R.S Veera kumar[13] and [14] introduced *g-closed set and #gs-closed sets respectively. In this paper a new class of sets, #GSP closed sets in topological spaces are introduced and some of their fundamental properties are examined. Further #gsp-open set, #gsp-neighbourhood are introduced and their properties are discussed.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) represents topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of X the closure of A and interior of A denoted by $cl(A)$ and $int(A)$ [3]. The union of all preopen sets of X contained in A is called pre-interior of A and is denoted by $pint(A)$. The intersection of all preclosed sets of X containing A is called pre-closure of A and is denoted by $pcl(A)$. We recall the following definitions that are pre requisites for our present study.

Definition 2.1. A subset A of a topological space (X, τ) is called

- (i) a preo-pen set [8] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
- (ii) a semi-open set[4] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.
- (iii) a α -open set[10] if $A \subseteq int(cl(int(A)))$ and preclosed if $cl(int(cl(A))) \subseteq A$.

Definition 2.2. A subset A of a topological space (X, τ) is called

- (i) a generalized closed set(briefly g-closed)[5] if $cl(A) \subseteq U$ whenever $A \subseteq U$, U is open in (X, τ) .
- (ii) a weakly generalized closed set(briefly wg-closed) [9] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$, U is open in (X, τ) .
- (iii) a semi generalized closed set(briefly sg-closed) [2] if $scl(A) \subseteq U$, whenever $A \subseteq U$, U is semi-open in (X, τ) .
- (iv) a generalized semiclosed (briefly gs-closed) [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$, U is open in (X, τ) .
- (v) α -generalized closed (briefly α g-closed) [6] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$, and U is open in (X, τ) .
- (vi) a w-closed set [12] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- (vii) a *g-closed set [13] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open in (X, τ) .
- (viii) a #g-semi closed set(briefly #gs-closed) [14] if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is *g- open in (X, τ) .

3. Hash generalized semi pre closed sets

Definition 3.1 A subset A of topological space (X, τ) is called hash generalized semi preclosed set (briefly $\#gsp$ -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\#gs$ -open in X .

The family of all $\#gsp$ -closed sets in topological space X is denoted by $\#GSPC(X)$.

Theorem 3.2 Every closed set is $\#gsp$ -closed

Proof. The proof follows from the definition and the fact that every closed set is pre-closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.3 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a, c\}\}$. Let $A = \{a\}$, then A is $\#gsp$ -closed set but not closed.

Theorem 3.4 Every α -closed set is $\#gsp$ -closed.

Proof. The proof follows from the definitions and the fact that every α -closed set is $\#gsp$ -closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.5 Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a, b\}\}$. Let $A = \{a\}$, then A is $\#gsp$ -closed set but not α -closed.

Theorem 3.6 Every $\#gsp$ -closed set is gp closed.

Proof. Let A be any $\#gsp$ -closed set in X . Let U be open set containing A . Since every open set is $\#gs$ -open, we have $pcl(A) \subseteq U$. Hence A is gp -closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.7 Let $X = \{a, b, c, d, e\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{a, e\}$, then A is gp closed but not $\#gsp$ -closed.

Theorem 3.8 Every $\#gsp$ -closed set is weakly generalized closed.

Proof. Let A be any $\#gsp$ -closed set in X , Let U be any open set containing A , since every open set is $\#gs$ -closed and $cl(int(A)) \subseteq pcl(A)$, we have $cl(int(A)) \subseteq U$. Hence A is weakly generalized closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.9 Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{a, b, d\}$, then A is weakly generalized closed but not $\#gsp$ -closed set.

Remark: $\#gsp$ -closed sets are independent of g -closed, $*g$ -closed set and $\#gs$ -closed set, sg -closed, gs -closed and αg closed.

The following examples show that $\#gsp$ -closed sets are independent of g -closed($*g$ -closed)

Example 3.10 Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. The set $A = \{a, d\}$ is g -closed($*g$ -closed) but not $\#gsp$ -closed. The set $B = \{c\}$ is $\#gsp$ -closed but not g -closed($*g$ -closed).

The following examples show that $\#gsp$ -closed sets are independent of $\#gs$ closed.

Example 3.11 Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. Let $A = \{b\}$, then A is $\#gs$ -closed but not $\#gsp$ -closed.

Example 3.12 Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a, b\}\}$. Let $A = \{b, c\}$, then A is $\#gsp$ -closed but not $\#gs$ -closed.

The following examples show that $\#gsp$ -closed sets are independent of sg -closed(sg -closed)

Example 3.13 Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}\}$. Then the set $A = \{a\}$ $\#gsp$ -closed but not sg -closed(sg -closed).

Example 3.14 Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $A = \{b, c\}$ is sg -closed(sg -closed) but not $\#gsp$ -closed.

The following examples show that $\#gsp$ -closed sets are independent of αg -closed.

Example 3.15 Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then the set $A = \{a, d\}$ is αg -closed but not $\#gsp$ -closed.

Example 3.16 Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}\}$. The set $A = \{a\}$ is $\#gsp$ -closed but not αg -closed.

4. Characteristics of $\#GSP$ -closed and $\#GSP$ -open sets

Theorem 4.1 Let A be a $\#gsp$ -closed set in (X, τ) . Then $pcl(A) - A$ does not contain any non empty $\#gs$ -closed.

Proof. Let A be a $\#gsp$ -closed set and such that F be a $\#gs$ -closed set such that $F \subseteq pcl(A) - A$ then $F \subseteq X - A$ implies $A \subseteq F$. A is $\#gsp$ -closed and $X - F$ is $\#gs$ -open. Therefore $pcl(A) \subseteq X - F$. That is $F \subseteq X - pcl(A)$. Hence $F \subseteq pcl(A) \cap (X - pcl(A)) = \phi$. Hence $F = \phi$. $\square$

Theorem 4.2 Let $A \subseteq X$ be a $\#gsp$ -closed set. Then A is pre-closed if and only if $pcl(A) - A$ is $\#gs$ -closed.

Proof. Necessity: Let A be pre-closed. Then $pcl(A) = A$ and so $pcl(A) - A = \phi$ which is $\#gs$ -closed.

Sufficiency: Suppose $pcl(A) - A$ is $\#gs$ -closed. Now A is $\#gsp$ -closed and since $pcl(A) - A$ is $\#gs$ -closed subset of itself by theorem 4.1 $pcl(A) - A = \phi$. This implies that $pcl(A) = A$ and so A is pre closed. $\square$

Theorem 4.3 If $A \subseteq X$ is both $\#gs$ -open and $\#gsp$ -closed then A is pre-closed in X .

Proof. Let A is a $\#gs$ -open and $\#gsp$ -closed set in X , then $pcl(A) \subseteq A$. But $A \subseteq pcl(A)$ is always true. Therefore $pcl(A) = A$ and hence A is pre-closed. $\square$

Theorem 4.4 If $A \subseteq X$ is a $\#gsp$ -closed set in X and $A \subseteq B \subseteq pcl(A)$, then B is $\#gsp$ -closed in X .

Proof. Let G be a $\#gs$ -open set in X such that $B \subseteq G$, then $A \subseteq G$. since A is $\#gsp$ -closed then $pcl(A) \subseteq G$. Now $pcl(B) \subseteq pcl(pcl(A)) = pcl(A) \subseteq G$. So $pcl(B) \subseteq G$. $\square$

Remark: Union of ${}^{\#}\text{gsp}$ -closed sets need not be ${}^{\#}\text{gsp}$ -closed as seen from the following example.

Example 4.5 Consider $X = \{a, b, c, d, e\}$ with the topology $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Let $A = \{c\}$ and $B = \{d\}$. Here A and B are ${}^{\#}\text{gsp}$ -closed but $A \cup B = \{c, d\}$ is not ${}^{\#}\text{gsp}$ -closed.

Definition 4.6 A set $A \subseteq X$ is called hash generalized semi pre open if and only if its complement is hash generalized semi closed.

Remark 4.7 Let (X, τ) be a topological space. Then for any $A \subseteq X$, $\text{pcl}(X-A) = X - \text{pint}A$

Proof. It follows from the result 3.3[11], $\text{pint}A = X - \text{pcl}(X-A)$. $\square$

Theorem 4.8 A set $A \subseteq X$ is ${}^{\#}\text{gsp}$ -open set if and only if $G \subseteq \text{pint}A$ whenever G is ${}^{\#}\text{gs}$ -closed and $G \subseteq A$.

Proof. Let A be a ${}^{\#}\text{gsp}$ -open set and suppose $G \subseteq A$ where G is ${}^{\#}\text{gs}$ -closed. Then $X-A$ is a ${}^{\#}\text{gsp}$ -closed set contained in the ${}^{\#}\text{gs}$ -open set $X-G$, $\text{pcl}(X-A) \subseteq X-G$. Since $\text{pcl}(X-A) = X - \text{pint}A$ by the remark 4.7. Then $X - \text{pint}(A) \subseteq X-G$, that is $G \subseteq \text{pint}(A)$.

Conversely $G \subseteq \text{pint}(A)$ be whenever $G \subseteq A$ and G is ${}^{\#}\text{gs}$ -closed then $X - \text{pint}(A) \subseteq X-G$. That is $\text{pcl}(X-A) \subseteq X-G$. This implies $X-A$ is ${}^{\#}\text{gsp}$ -closed and A is ${}^{\#}\text{gs}$ -open in X . $\square$

5. ${}^{\#}\text{GSP}$ neighbourhoods

Definition 5.1 A subset M of a topological space X is called ${}^{\#}\text{gsp}$ -neighbourhood (briefly ${}^{\#}\text{gsp}$ -nbhd) of a point $x \in X$, if there exists a ${}^{\#}\text{gsp}$ -open set U such that $x \in U \subseteq M$.

The collection of all ${}^{\#}\text{gsp}$ -nbhds of $x \in X$ is called ${}^{\#}\text{gsp}$ -nbhd system of x and is denoted by ${}^{\#}\text{gsp}N(x)$.

Theorem 5.2 Every neighbourhood of a point $x \in X$ is ${}^{\#}\text{gsp}$ -nbhd of that point.

Proof. Follows from the definition and fact that every open set is ${}^{\#}\text{gsp}$ -open set. $\square$

Converse of the above theorem need not be true as seen from the following example.

Example 5.3 Consider $X = \{a, b, c, d, e\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. The subset $\{a, b, c, e\}$ is ${}^{\#}\text{gsp}$ -nbhd of $e \in X$ but it is not a nbhd of e as there is no open set G such that $e \in G \subseteq \{a, b, c, e\}$.

Theorem 5.4 If A is ${}^{\#}\text{gsp}$ -open then it is ${}^{\#}\text{gsp}$ -nbhd of each of its points.

Proof. If A is ${}^{\#}\text{gsp}$ -open set then for each $x \in A$, $x \in A \subseteq A$. Therefore A is ${}^{\#}\text{gsp}$ -nbhd of each of its points. $\square$

Theorem 5.5 If $A \subseteq X$ is a ${}^{\#}\text{gsp}$ -closed set and $x \in A^c$ then there exists a ${}^{\#}\text{gsp}$ -nbhd F of x such that $F \cap A = \emptyset$.

Proof. $A \subseteq X$ be the ${}^{\#}\text{gsp}$ -closed set and then A^c is ${}^{\#}\text{gsp}$ -open therefore by theorem 5.4, A^c is ${}^{\#}\text{gsp}$ -nbhd of each of its points. Let $x \in A^c$ then there exists a ${}^{\#}\text{gsp}$ -open set F such that $x \in F \subseteq A^c$. That is $F \cap A = \emptyset$. $\square$

Theorem 5.6 Let x be a point in a space X then

- (i) ${}^{\#}\text{gsp}N(x) \neq \emptyset$.
- (ii) If $A \in {}^{\#}\text{gsp}N(x)$ and then $x \in A$.
- (iii) If $A \in {}^{\#}\text{gsp}N(x)$ and $B \supseteq A$ then $B \in {}^{\#}\text{gsp}N(x)$.

Proof. (i) Since $x \in {}^{\#}\text{gsp}N(x)$, ${}^{\#}\text{gsp}N(x) \neq \emptyset$.

(ii) Let $A \in {}^{\#}\text{gsp}N(x)$, then there exist a ${}^{\#}\text{gsp}$ -open set G such that $x \in G \subseteq A$ this implies $x \in A$.

(iii) Let $x \in {}^{\#}\text{gsp}N(x)$, then there exists a ${}^{\#}\text{gsp}$ -open set G such that $x \in G \subseteq A$, since $A \subseteq B$, then $x \in G \subseteq B$ this implies $B \in {}^{\#}\text{gsp}N(x)$. $\square$

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