Stirling interpolation using triangular fuzzy number

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Abstract

In this paper the polynomial interpolation of triangular fuzzy number is discussed. First general form of the polynomial with fuzzy coefficients is proposed. The Stirling interpolation method is studied with triangular fuzzy number an example is provided to illustrate the algorithm.

Keywords

Interpolation; polynomial; triangular fuzzy numbers.

AMS Subject Classification

34K28, 65L06, 97N50.

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1. Introduction

In some problem, people have to deal with massive imprecise data. We usually represented them as triangular fuzzy numbers. Many researchers focus on fuzzy numbers, for the convenience in dealing with uncertainty people call this 'fuzzy mathematics'.

Interpolation is important concept in numerical analysis. It is necessary because in science and engineering we often need to deal with discrete experimental data. Interpolation is also used to simplify complicated functions by sampling data points can interpolating them using a simpler function. Interpolation method is the most popular method useful in fields of engineering, economy, business oriented problems. Chenyi Hu et. al.[7] have discussed interval polynomial interpolation problems in lagrange form. Hossein et. al.[10] have discussed interpolation of fuzzy data using cubic splines.

In this paper interpolation polynomial of triangular fuzzy number is formulated on comparing with stirling interpolation problem. the existence theorem and some of its properties are dealt with. An algorithm is proposed for stirling interpolating method using triangular fuzzy number illustrated by an example.

2. Preliminaries

Definition 2.1. A triangular fuzzy number is represented with three points as follows $A = (a^l, a^c, a^r)$. its membership function is interpreted as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x < a^l \\ \frac{x-a^l}{a^c-a^l} & , a^l \leq x \leq a^c \\ \frac{a^r-x}{a^r-a^c} & , a^c \leq x \leq a^r \\ 0 & , x > a^r \end{cases}$$

Definition 2.2. Let $\tilde{a} = (a^l, a^c, a^r)$, $\tilde{b} = (b^l, b^c, b^r)$ be two triangular fuzzy numbers. Then $\tilde{a} = \tilde{b}$ if and only if $a^l = b^l$, $a^c = b^c$, $a^r = b^r$.

Definition 2.3. The arithmetic operations of triangular fuzzy numbers are defined

$$(i). \tilde{a} + \tilde{b} = (a^l + b^l, a^c + b^c, a^r + b^r)$$

$$(ii). k\tilde{a} = \begin{cases} (ka^l, ka^c, ka^r), & \text{if } k \geq 0 \\ (ka^r, ka^c, ka^l), & \text{else.} \end{cases}$$

Definition 2.4. Let $\tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_m$ be triangular fuzzy number. A function $P_n(x, \tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_m)$ denoted by $\tilde{P}_n(x)$, is called the n -order polynomial with triangular fuzzy numbers coefficients, if it satisfies the following conditions.

- (1) $\tilde{P}_n(x)$ is an n -order polynomial about x ,
- (2) $\tilde{P}_n(x)$ is a 1-order polynomial about $\tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_m$.

Definition 2.5. Let $Q_0(x), Q_1(x), \dots, Q_m(x)$ be $m+1$ polynomials whose degree is no more than n and atleast one of them

is a polynomials of degree n . The fuzzy triangular polynomials $\tilde{P}_n(x)$ has the following them

$$\tilde{P}_n(x) = \tilde{a}_0 Q_0(x) + \tilde{a}_1 Q_1(x) + \dots + \tilde{a}_m Q_m(x)$$

The set of all triangular fuzzy polynomials of degree m ($m \leq n$) is denoted by $\tilde{P}_n$

3. Interpolation polynomials of fuzzy numbers

In this section the interpolation problem of triangular fuzzy numbers is formulated in detail by comparing with the stirling interpolation problem. The existence theorem of solutions investigated and some related properties are developed.

Definition 3.1. Let $x_0, x_1, \dots, x_n$ be $n+1$ distinct nodes.

Given $y_0 = f(x_0), y_1 = f(x_1), \dots, y_n = f(x_n)$

The problem is to find a polynomial $P_n(x) \in P_n$, called an interpolating polynomials, such that

$$P_n(x_i) = y_i \quad (i = 0, 1, 2, \dots, n)$$

Theorem 3.2. There exists a unique polynomial $P_n(x) \in P_n$ such that $P_n(x_i) = y_i$ for $i = 0, 1, \dots, n$

The stirling interpolation of $P_n(x)$ is defined by

$$\begin{aligned} P_n(x) = & f(x_0) + \frac{u}{2} [\Delta_1 \phi] + \phi \frac{u^2}{2!} [\Delta_2 \phi] \\ & + \frac{u(u^2 - 1^2)}{3!} [\Delta_3 \phi] + \frac{u^2(u^2 - 1^2)}{4!} [\Delta_4 \phi] \\ & + \frac{u(u^2 - 1^2)(u^2 - 2^2)}{5!} [\Delta_5 \phi] \\ & + \frac{u^2(u^2 - 1^2)(u^2 - 2^2)}{6!} [\Delta_6 \phi] + \dots \end{aligned}$$

where

$$\begin{aligned} \Delta_1 \phi & \text{ for } i = 1 & f_i - f_{-i} = f_1 - f_{-1} \\ \Delta_2 \phi & \text{ for } i = 2 & f_{i-1} - 2f_{i-2} + f_{-i+1} \\ & & = f_1 - 2f_0 + f_{-1} \\ \Delta_3 \phi & \text{ for } i = 3 & f_{i-1} - 2f_{i-2} + 2f_{-i+1} - f_{-i+1} \\ & & = f_2 - 2f_1 + 2f_{-1} - f_{-2} \\ \Delta_4 \phi & \text{ for } i = 4 & f_{i-2} - 4f_{i-3} + 6f_{i-4} + 4f_{-i+3} + f_{-i+2} \\ & & = f_2 - 4f_1 + 6f_0 + 4f_{-1} + f_{-2} \\ \Delta_5 \phi & \text{ for } i = 5 & f_{i-2} - 5f_{i-3} + 5f_{i-4} - 5f_{-i+4} + 5f_{-i+3} - f_{-i+2} \\ & & = f_3 - 5f_2 + 5f_1 - 5f_{-1} + 5f_{-2} - f_{-3} \\ \Delta_6 \phi & \text{ for } i = 6 & f_{i-3} - 6f_{i-4} + 15f_{i-5} - 20f_{i-6} + 15f_{-i+5} - 6f_{-i+4} + f_{-i+3} \\ & & = f_3 - 6f_2 + 15f_1 - 20f_0 + 15f_{-1} - 6f_{-2} + f_{-3} \end{aligned}$$

for the $n+1$ pairs (x_i, y_i) in which (y_i) are triangular fuzzy numbers.

Theorem 3.3. Let $x_0, x_1, \dots, x_n$ be $n+1$ distinct nodes and let $\tilde{y}_i = (y_i^l, y_i^c, y_i^r)$ $i = 0, 1, 2, \dots, n$ be $n+1$ triangular fuzzy numbers. There exists atleast one fuzzy polynomial such that $\tilde{P}_n(x_i) = \tilde{y}_i$ for $i = 0, 1, 2, \dots, n$

Proof. To prove existence let us use a constructive approach providing an expression for $\tilde{P}_n(x)$. for arbitrary $y_i, i = 0, 1, 2, \dots, n$. Suppose that $P_n(x, y_0, y_1, \dots, y_n)$ is an interpolating polynomial such that

$$P_n(x_i, y_0, y_1, \dots, y_{n1}) = y_i, \quad (i = 0, 1, 2, \dots, n)$$

Define

$$\tilde{P}_n(x) = (P_n^l(x), P_n^c(x), P_n^r(x))$$

$$\text{where, } P_n^l(x) = \inf_{\forall y_i \in \tilde{y}_i, i=0,1,\dots,n} P_n(x, y_0, y_1, \dots, y_n)$$

$$P_n^r(x) = \sup_{\forall y_i \in \tilde{y}_i, i=0,1,\dots,n} P_n(x, y_0, y_1, \dots, y_n)$$

$$\begin{aligned} P_n^c(x) = & f(x_0) + \frac{u}{2} [\Delta_1 \phi] + \phi \frac{u^2}{2!} [\Delta_2 \phi] \\ & + \frac{u(u^2 - 1^2)}{3!} [\Delta_3 \phi] + \frac{u^2(u^2 - 1^2)}{4!} [\Delta_4 \phi] \\ & + \frac{u(u^2 - 1^2)(u^2 - 2^2)}{5!} [\Delta_5 \phi] \\ & + \frac{u^2(u^2 - 1^2)(u^2 - 2^2)}{6!} [\Delta_6 \phi] + \dots \end{aligned}$$

Then $\tilde{P}_n(x)$ satisfies the interpolation condition

$$\text{Since, } P_n^l(x) = \inf_{\forall y_i \in \tilde{y}_i, i=0,1,\dots,n} P_n(x, y_0, y_1, \dots, y_n)$$

$$= \inf_{\forall y_i \in \tilde{y}_i, i=0,1,\dots,n} y_i = y_i^l$$

$$P_n^r(x) = \sup_{\forall y_i \in \tilde{y}_i, i=0,1,\dots,n} P_n(x, y_0, y_1, \dots, y_n)$$

$$= \sup_{\forall y_i \in \tilde{y}_i, i=0,1,\dots,n} y_i = y_i^r$$

Therefore

$$\begin{aligned} \tilde{P}_n(x_i) = & (P_n^l(x), P_n^c(x), P_n^r(x)) \\ = & (y_i^l, y_i^c, y_i^r) \\ = & \tilde{y}_i \end{aligned}$$

As a consequence the interpolating polynomial of triangular fuzzy number exists. $\square$

Theorem 3.4. The stirling interpolation form of the interpolating polynomial of fuzzy numbers is

$$\begin{aligned} P_n^c(x) = & f(x_0) + \frac{u}{2} [\Delta_1 \phi] + \phi \frac{u^2}{2!} [\Delta_2 \phi] \\ & + \frac{u(u^2 - 1^2)}{3!} [\Delta_3 \phi] + \frac{u^2(u^2 - 1^2)}{4!} [\Delta_4 \phi] \\ & + \frac{u(u^2 - 1^2)(u^2 - 2^2)}{5!} [\Delta_5 \phi] \\ & + \frac{u^2(u^2 - 1^2)(u^2 - 2^2)}{6!} [\Delta_6 \phi] + \dots \end{aligned}$$

where,

$$\begin{aligned} \Delta_1 \phi & \text{ for } i = 1 & f_i - f_{-i} = f_1 - f_{-1} \\ \Delta_2 \phi & \text{ for } i = 2 & f_{i-1} - 2f_{i-2} + f_{-i+1} \\ & & = f_1 - 2f_0 + f_{-1} \\ \Delta_3 \phi & \text{ for } i = 3 & f_{i-1} - 2f_{i-2} + 2f_{-i+1} - f_{-i+1} \end{aligned}$$

$$\begin{aligned}
&= f_2 - 2f_1 + 2f_{-1} - f_{-2} \\
\Delta_4 \varphi \quad \text{for } i=4 \quad & f_{i-2} - 4f_{i-3} + 6f_{i-4} + 4f_{-i+3} + f_{-i+2} \\
&= f_2 - 4f_1 + 6f_0 + 4f_{-1} + f_{-2} \\
\Delta_5 \varphi \quad \text{for } i=5 \quad & f_{i-2} - 5f_{i-3} + 5f_{i-4} - 5f_{-i+4} + 5f_{-i+3} - f_{-i+2} \\
&= f_3 - 5f_2 + 5f_1 - 5f_{-1} + 5f_{-2} - f_{-3} \\
\Delta_6 \varphi \quad \text{for } i=6 \quad & f_{i-3} - 6f_{i-4} + 15f_{i-5} - 20f_{i-6} + 15f_{-i+5} - 6f_{-i+4} + f_{-i+3} \\
&= f_3 - 6f_2 + 15f_1 - 20f_0 + 15f_{-1} - 6f_{-2} + f_{-3}
\end{aligned}$$

3.4 Numerical Example

The following table gives the values of e^x for certain equidistant values of x . Find the value of e^x when $x = 0.644$

x	$y = e^x$
0.61	(1.830320, 1.840431, 1.851542)
0.62	(1.847818, 1.858928, 1.869019)
0.63	(1.866500, 1.877610, 1.888721)
0.64	(1.885370, 1.896481, 1.907592)
0.65	(1.904430, 1.915541, 1.926651)
0.66	(1.923681, 1.934792, 1.945803)
0.67	(1.943126, 1.954237, 1.965348)

Solution

$$\begin{aligned}
P_n(x) &= f(x_0) + \frac{u}{2}[f_1 - f_{-1}] + \frac{u^2}{2!}[f_1 - 2f_0 + f_{-1}] \\
&+ \frac{u(u^2 - 1^2)}{3!}[f_2 - 2f_1 + 2f_{-1} - f_{-2}] \\
&+ \frac{u^2(u^2 - 1^2)}{4!}[f_2 - 4f_1 + 6f_0 + 4f_{-1} + f_{-2}] \\
&+ \frac{u(u^2 - 1^2)(u^2 - 2^2)}{5!}[f_3 - 5f_2 + 5f_1 - 5f_{-1} \\
&+ 5f_{-2} - f_{-3}] + \frac{u^2(u^2 - 1^2)(u^2 - 2^2)}{6!}[f_3 - 6f_2 \\
&+ 15f_1 - 20f_0 + 15f_{-1} - 6f_{-2} + f_{-3}] + \dots
\end{aligned}$$

$$\begin{aligned}
u &= \frac{x - x_0}{h} \\
&= \frac{0.644 - 0.64}{0.1} \\
u &= 0.4
\end{aligned}$$

$$\begin{aligned}
P_n(x) &= (1.885370, 1.896481, 1.907502) \\
&+ (0.2)[1.015709, 0.037931, 0.060151] \\
&+ (0.08) \begin{bmatrix} (1.904430, 1.915541, 1.926651) \\ -(3.77074, 3.792962, 3.815124) \\ +(1.866500, 1.877600, 1.888721) \end{bmatrix} \\
&- 0.056 \begin{bmatrix} (1.923681, 1.934792, 1.945803) \\ -(3.80886, 3.831082, 3.853302) \\ +(3.733000, 3.755220, 3.777442) \\ -(1.847818, 1.858928, 1.869019) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
&-0.0056 \begin{bmatrix} (1.923681, 1.934792, 1.945803) \\ -(7.617720, 7.662164, 7.706604) \\ +(11.312222, 11.378886, 11.445372) \\ -(7.466000, 7.510440, 7.554884) \\ +(1.847818, 1.858928, 1.869019) \end{bmatrix} \\
&-0.010752 \begin{bmatrix} (1.943126, 1.954237, 1.965348) \\ -(9.618405, 9.673960, 9.729015) \\ +(9.522150, 9.577705, 9.633255) \\ -(9.332500, 9.388050, 9.443605) \\ -(1.830320, 1.840431, 1.851542) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
P_n(x) &= (1.885370, 1.896481, 1.907502) \\
&+ (0.003141, 0.007586, 0.012003) \\
&+ (0.08) \begin{bmatrix} (3.77093, 3.793151, 3.815372) \\ -(3.77074, 3.792962, 3.815124) \end{bmatrix} \\
&- (0.056) \begin{bmatrix} -(1.929621, 1.89629, 1.863057) \\ +(1.863981, 1.896292, 1.929624) \end{bmatrix} \\
&- (0.0056) \begin{bmatrix} -(5.782923, 5.727372, 5.671917) \\ +(3.757338, 3.868446, 3.846222) \\ +(1.847818, 1.859828, 1.869019) \end{bmatrix} \\
&+ (0.010752) \begin{bmatrix} (20.704366, 20.826582, 20.943698) \\ -(20.781225, 20.9024441, 21.024162) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
P_n(x) &= (1.885370, 1.896481, 1.907502) \\
&+ (0.003141, 0.007586, 0.012003) \\
&+ (-0.003535, 0.000015, 0.003570) \\
&- (0.000000, 0.000000, 0.000015) \\
&+ (0.000379, 0.000000, 0.000373) \\
&+ (-0.003438, -0.000816, 0.001747).
\end{aligned}$$

Thus, we have

$$P_n(x) = (1.881866, 1.903266, 1.925255).$$

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