soShearEnergy of permutation of necklace

J. Chithra^{1*} and S.P. Jeyakokila²

Abstract

Pnueli, Lempel and Even introduced Permutation Graphs in 1971 [8]. If i, j belong to a permutation on p symbols $\{1, 2, \dots, p\}$ and if i is less than j then there is an edge between i and j in the permutation graph if i appears after j in the corresponding sequence. A subset D of A , whose closed neighborhood is A in π is a dominating set of π . Energy of a graph with respect to the minimal dominating set was introduced by *B.D.Acharya et al* [4]. soShearEnergy is introduced in [5]. In this paper necklace graph is realized as a permutation graph and soShearEnergy of permutation of necklace is calculated with respect to minimal dominating set.

Keywords

Permutation graphs, Necklace, Domination of first kind and second kind, soShearEnergy.

AMS Subject Classification

05C30, 05C50, 05C62, 05C69, 05C85.

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1. Introduction

Number of researches are carried out by the computer scientists all over the world since the concept is introduced and hence many notions of graphs in permutation graphs are modeled using polynomial time algorithms. Another way of defining the permutation graph is available in the literature. *Harary* and *Chartrand* defined a permutation graph as: for any permutation α in S_p , the α -permutation graph of a labeled graph G is the union of two disjoint copies G_1 and G_2 of G together with the lines joining point v_i of G_1 with $v_{\alpha(i)}$ of G_2 [1]. Both the kinds are unrelated. The study on the permutation graph introduced by *Pnueli, Lempel* and *Even* stimulated us to establish measures to find the domination numbers of the permutations corresponding to the permutation graphs with the perspective of graph theory. Our paper deals with the permutation graph defined by *Pnueli, Lempel* and *Even*. An

algorithm is developed by *J.Chithra* and *S.P.Subbiah* [2] to determine the minimal dominating sets for permutations and thus for permutation graphs. Energy of a graph was developed during 1970. Different types of energies were evolved from different matrices. Latter *B.D.Acharya et al.* introduced Energy of a graph of a dominating set. soShearEnergy of a graph with respect to the given minimal dominating set is introduced by *S.P.Jeyakokila* and *P.Sumathi* and an algorithm is also developed by them. In this article the graph named as necklace is realized by a permutation for which the soShearEnergy is determined. The results are found to be interesting since they integrate permutations, graphs and energy of graphs.

2. Preliminaries

Definition 2.1. Let π be a permutation on a finite set $V = \{a_1, a_2, a_3, \dots, a_p\}$ given by $\pi = \begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_p \\ a'_1 & a'_2 & a'_3 & \dots & a'_p \end{pmatrix}$ where $|a_{i+1} - a_i| = c > 0, 1 \leq i \leq p-1$. The sequence of π is given by $s(\pi) = \{a'_1, a'_2, a'_3, \dots, a'_p\}$.

Definition 2.2. Let $a_i, a_j \in V$. Then the residue of a_i and a_j in π is denoted by $\text{Res}(a_i, a_j)$ and is given by $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j))$.

Definition 2.3. When elements of V are ordered in L_1 and the sequence of π are represented in L_2 , then a line joining

a_i in L_1 and a_i in L_2 is represented as l_i . This is known as **line representation of a_i in π** [9]. Let l_i and l_j are the lines corresponding to the elements a_i and a_j respectively in π . Then l_i crosses l_j if $\text{Res}(a_i, a_j) < 0$.

Definition 2.4. Let π be a permutation on a finite set $V = \{a_1, a_2, a_3, \dots, a_p\}$ given by $\pi(a_i) = a'_i$, $1 \leq i \leq p$ where $|a_{i+1} - a_i| = c, c > 0$, $1 \leq i \leq p-1$. Then the **Permutation Graph** G_π is given by $G_\pi = (V_\pi, E_\pi)$ where $V_\pi = \{a_1, a_2, \dots, a_p\}$ and $a_i a_j \in E_\pi$, if $\text{Res}(a_i, a_j) < 0$. If l_i crosses l_j then $(a_i, a_j) \in E_\pi$.

Example 2.5. Let π be a permutation on a finite set $V = \{1, 2, 3, 4, 5\}$ given by $\pi(1) = 3$, $\pi(2) = 5$, $\pi(3) = 1$, $\pi(4) = 4$ and $\pi(5) = 2$. (i.e) $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$.

Then the line l_1 crosses l_3 and l_5 ; l_2 crosses l_3 , l_4 , and l_5 ; l_3 crosses l_1 and l_2 ; l_4 crosses l_2 and l_5 ; l_5 crosses l_1 , l_2 and l_4 .

Definition 2.6. A graph G is a permutation graph if there exists π such that $G_\pi \cong G$. (i.e) a graph is a **permutation graph** if it is realizable by a permutation π . Otherwise it is not a permutation graph.

Example 2.7. Let $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$. Then $G_\pi = (V_\pi, E_\pi)$ where $V_\pi = \{1, 2, 3, 4, 5\}$ and $E_\pi = \{(1, 3), (1, 5), (2, 3), (2, 4), (2, 5), (4, 5)\}$.

Definition 2.8. Let G be a graph and S be a subset of $V(G)$. Let $v \in V - S$. The **idegree** of v with respect to S is the number of neighbours of v in $V - S$ and it is denoted by $\text{id}_S(v)$.

Definition 2.9. Let G be a graph and S be a subset of $V(G)$. Let $v \in V - S$. The **odegree** of v with respect to S is the number of neighbours of v in $V - S$, in the set S and is denoted as $\text{od}_S(v)$.

Definition 2.10. Let G be graph and S be a subset of $V(G)$. Let $v \in V - S$. The **oiddegree** of v with respect to S is $\text{od}_S(v) - \text{id}_S(v)$ if $\text{od}_S > \text{id}_S$ and it is denoted by $\text{oid}_S(v)$.

Definition 2.11. Let G be a graph and D be a minimal dominating set. **oShearEnergy** of a graph with respect to D denoted by $\text{osE}_D(G)$ is the sum of all oid if $\text{od}_S > \text{id}_S$, zero otherwise.

Definition 2.12. Let G be a graph and D be a minimal dominating set, then **ShearEnergy curve** is the curve obtained by joining the oShearEnergies of G with respect to D_i for $0 \leq i \leq n-1$, marked off by taking the number of vertices of D_i along the X-axis and the oShearEnergies along the Y-axis

Definition 2.13. Let G be a graph and D be a minimal dominating set. **soShearEnergy** of a graph with respect to D is $\sum_{i=0}^{|V-D|} \text{osE}_{D_i}(G)$ where $D_{i+1} = D_i \cup V_{i+1}$, and V_{i+1} is a singleton set with minimum oidegree of $V - D_i$ and D_0 is a minimal dominating set where $0 \leq i \leq |V - D|$. It is denoted by $\text{sosE}_D(G)$.

3. Permutation realising a Necklace

Definition 3.1. An n -point contact of K_2 is NK_n (i. e) $NK_n = K_2 + nK_1$ along with an edge $K_2 = uv$ is called a necklace with rope uv and is denoted by NK_n with rope uv . Here n denotes number of beads, where a bead is an element of degree 2 in NK_n . [Ref:[3]]

Theorem 3.2. Let π be a permutation on p symbols $V = \{a_1, a_2, \dots, a_p\}$ given by $\pi(a_i) = a'_i$, $1 \leq i \leq p$, where $|a_{i+1} - a_i| = c, c > 0$. Then a transposition $(a_i a_j)$, $i < j$, realizes a permutation graph which is a subgraph of G_π isomorphic to a Necklace with the edge $a_i a_j$ as a rope and with $n = a_i - a_j - 1$ number of beads, as a component and other elements are isolates.

Proof.

Let $\pi =$

$$\begin{pmatrix} a_1 & a_2 & \dots & a_i & a_{i+1} & \dots & a_j & a_{j+1} & \dots & a_p \\ a_1 & a_2 & \dots & a_j & a_{i+1} & \dots & a_i & a_{j+1} & \dots & a_p \end{pmatrix}.$$

The transposition $(a_i a_j)$ is given by

$$(a_i a_j) = \begin{pmatrix} a_i & a_{i+1} & a_{i+2} & \dots & a_j \\ a_j & a_{i+1} & a_{i+2} & \dots & a_i \end{pmatrix}.$$

Here l_i crosses $l_{i+1}, l_{i+2}, l_{i+3}, \dots, l_j$ and l_j crosses $l_{j-1}, l_{j-2}, l_{j-3}, \dots, l_i$.

Therefore a_i and a_j are of degree $a_j - a_i$.

All $l_{i+1}, l_{i+2}, \dots, l_{j-1}$ cross l_i and l_j .

Hence $a_{i+1}, a_{i+2}, \dots, a_{j-1}$ are of degree 2.

Hence they are bead vertices and they are $a_j - a_i - 1$ in number.

Therefore by Definition 3.1 $(a_i a_j)$ realizes a necklace NK_n with $a_i a_j$ as a rope and with $n = a_j - a_i - 1$ number of beads.

As other elements are their own images they are isolates.

Hence the proof. $\square$

Example 3.3. Let π be a permutation on p symbols $V = \{a_1, a_2, \dots, a_p\}$ given by $\pi(a_i) = a'_i$, $1 \leq i \leq p$, where $|a_{i+1} - a_i| = c, c > 0$ and

$$\pi = \begin{pmatrix} a_i & a_{i+1} & a_{i+2} & a_{i+3} & a_{i+4} & a_{i+5} \\ a_{i+5} & a_{i+1} & a_{i+2} & a_{i+3} & a_{i+4} & a_i \end{pmatrix} = (a_i a_{i+5}).$$

domination of first kind and domination of second kind as follows.

- (i) Domination of first kind denoted by ${}^1D$ is the minimal dominating set with one of the vertices in the rope.
- (ii) Domination of second kind denoted by ${}^2D$ is the minimal dominating set containing vertices of the beads.

Lemma 4.2. Let NK_m be the Necklace, then

(i) (a) $\text{soSE}_{NK_m}({}^1D) = n^2 + n + 1$ where $n = \frac{m}{2}$ when m is even .

(b) $\text{soSE}_{NK_m}({}^1D) = (m + 1) + \text{SE}_{NK_{m-2}}({}^1D)$ when m is odd, and $\text{soSE}_{NK_1}({}^1D) = 1$.

(ii) $\text{soSE}_{NK_m}({}^2D^c) = 3m - 1$.

Proof. Let NK_m be the given necklace graph.

(i) Let the dominating set ${}^1D$ be of first kind. Then the dominating set ${}^1D$ is the singleton set with vertex which is adjacent to all other vertices of the necklace. Then the set $V - {}^1D$ contains one vertex of the rope and m number of beads.

Case(i) Let m be even.

At the initial stage, odegree of all the vertices is 1 and idegree of all the bead vertices is 1 and the vertex of rope is $m - 2$. Hence $\text{osE}_{D_1}(NK_m)$ is zero for the $\frac{m}{2} - 1$ number of stages. At the $\frac{m}{2}$ nd stage, odegree of the vertex of the rope is $\frac{m}{2}$ since $\frac{m}{2}$ vertices are shifted to the dominating set.

Hence oid of that vertex is 1.

At the $\frac{m}{2} + 1$ th stage, the vertex of the rope is shifted to the dominating set.

The remaining vertices of $V - {}^1D$ are independent and hence the idegree of all the vertices are zero and odegree of the vertices are 2.

Hence at this stage $\text{osE}_{D_{\frac{m}{2}+1}}(NK_m) = 2 \frac{m}{2} = m$.

Since m is even ,let us consider $m=2n$, then $\text{osE}_{D_{\frac{m}{2}+1}}(NK_m) = 2n$.

The remaining vertices are shifted one by one till cardinality of $V - {}^1D$ is zero, so that the $\text{osE}_D(NK_m)({}^1D)$ decreases 2 by 2 till the cardinality reaches zero. Therefore $\text{soSE}_{D_1}(NK_m) = 1 + 2n + 2(n - 1) + \dots + 2 + 0$ On simplification, $\text{soSE}_{NK_m}({}^1D) = n^2 + n + 1$ where $n = \frac{m}{2}$.

Case(ii)

Let m be odd.

When $m = 1$, the dominating set contains a vertex of degree 2 which is one of the vertices of the rope. Vertices in the set $V - {}^2D$ are vertices of rope of degree 2 and a bead vertex is also of degree 2.

Hence odegree and idegree of both the vertices are 1.

Therefore at the first stage oidegree of both the vertices are zero and $\text{osE}_{D_1}(NK_3)({}^1D) = 0$.

At the second stage the vertex of the rope is shifted to the dominating set and hence idegree of the remaining vertex is zero and odegree is 2.

Hence $\text{osE}_{D_2}(NK_3) = 2$.

$\text{osE}_D(NK_m)$ attains integer value only at $\lceil \frac{m}{2} \rceil$ nd stage.

NK_4 with rope $a_i a_{i+5}$

4. soShearEnergy of permutation of Necklace

Definition 4.1. Two minimal Dominating sets are possible for NK_m . Let us name the two minimal dominating sets as

Hence $os\epsilon_{1D_{\lceil \frac{m}{2} \rceil}}(NK_m)$ is $(m+1)$. And it is the repetition of necklace of size $m-1$ after this stage.

Hence $os\epsilon_{NK_m}(^1D) = (m+1) + \epsilon_{NK_{m-2}}(^1D)$ when m is odd, $os\epsilon_{NK_1}(^1D) = 1$.

(ii) Let the dominating set be of second kind, that is, the dominating set contains the bead vertices. Hence the $V - ^2D$ set contains the vertices of ropes for all values of m .

As they are adjacent to each other the idegree of both the vertex is 1.

They are also adjacent to all the bead vertex and hence odegree of both the vertex is m . At the first stage oidegree is $m-1$ for both the vertices.

Therefore $os\epsilon_{2D_1}(NK_m) = 2(m-1)$.

At the second stage, any one of the vertices is shifted to the dominating set, idegree of the other vertex is 0 and odegree is $m+1$, and oidegree is $m+1$.

Therefore $os\epsilon_{2D}(NK_m) = 2(m-1) + (m+1) = 3m-1$. $\square$

5. Energy curves of Necklace

From the graphs given below it is evident that the flow of oShearEnergies of the with respect to first type is a continuous flow but with second kind of domination it starts with a very high value.

6. soShearEnergy of permutation of a necklace

Theorem 6.1. Let π be a permutation on p symbols $V = \{a_1, a_2, \dots, a_p\}$ given by a transposition $(a_1 a_p)$, where $|a_{i+1} - a_i| = c, c > 0$. Then

(i) (a) $os\epsilon_{NK_m}(^1D) = n^2 + n + 1$ where $n = \frac{m}{2}$ when m is even

(b) $os\epsilon_{NK_m}(^1D) = (m+1) + os\epsilon_{NK_{m-2}}(^1D)$ when m is odd and $os\epsilon_{NK_1}(^1D) = 2$

(ii) $os\epsilon_{NK_m}(^2D^c) = 3m-1$.

Proof. Let π be a permutation on p symbols $V = \{a_1, a_2, \dots, a_p\}$ given by a transposition $(a_1 a_p)$, where $|a_{i+1} - a_i| = c, c > 0$. Then by the theorem 3.2 π realizes a necklace NK_m with $m = a_p - a_1 - 1 = p - 2$ number of beads and $a_1 a_p$ as rope. For a permutation graph a minimal dominating set can be established using the algorithm given in [2].

The possible minimal dominating sets are $D = \{a_1\}$ or $\{a_p\}$ and $D = \{a_2, a_3, \dots, a_{p-1}\}$.

If $D = \{a_1\}$ or $\{a_p\}$ then D contains the vertices of the rope and hence it is the domination of first kind by Definition 4.1. Therefore by Lemma 4.2 $os\epsilon_{NK_m}(^1D) = n^2 + n + 1$ where $n = \frac{m}{2}$ when m is even and $os\epsilon_{NK_m}(^1D) = (m+1) + os\epsilon_{NK_{m-2}}(^1D)$ when m is odd and $os\epsilon_{NK_1}(^1D) = 2$.

If $D = \{a_2, a_3, \dots, a_{p-1}\}$ then D is the domination of second kind. Hence $os\epsilon_{NK_m}(^2D^c) = 3m-1$.

Hence the proof. $\square$

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