Solving transportation problem with Monalisha's approximation method under fuzzy environment

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Abstract

This work finds initial basic feasible solution and optimal solution to the fuzzy transportation problem by using Monalisha's Approximation Method (MAM). In this work the supply and demand are represented by the Interval-Valued Triangular Fuzzy numbers (IVTFNs). The solution procedure is illustrated with numerical example.

Keywords

Transportation Problem, Interval-Valued Triangular Fuzzy Number (IVTFN), Monalisha's Approximation Method.

AMS Subject Classification

90C08, 90C90.

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1. Introduction

Transportation Problem is a modern class of Linear Programming Problem in which supply and demand of the commodities transported from several sources to the different destinations. Nagoor Gani and Stephen Dinagar[4] studied a special note on solving linear programming in fuzzy environment. In [3] Monalisha Pattnaik introduced the Monalisha's approximation method for solving optimization problems. S. Vimala and S. Krishna Prabha[2] proposed a modified method for solving balanced fuzzy transportation problem. S. Vimala and S. Krishna Prabha[1] solved Fuzzy Transportation Problem through Monalisha's Approximation Method. D. Stephen Dinagar and R. Keerthivasan[5] studied the fuzzy transportation problems with the aid of interval-valued triangular fuzzy numbers. In this paper, the fuzzy transportation problems using Interval-Valued Triangular Fuzzy Numbers (IVTFNs) with Monalisha's approximation method have been studied.

The organization of this paper is structured as follows. In section 2, we introduce some basic concepts related to Interval-Valued Triangular Fuzzy Numbers (IVTFNs) and some arithmetic operation of the above said numbers. IVTFN-transportation problem is introduced in section 3. In section 4, the algorithm of the Monalisha's approximation method is given. A relevant numerical illustration is presented in Section 5. Finally the conclusion part is included in section 6.

2. Basic Concepts

Definition 2.1. An interval-valued fuzzy number $\tilde{A}$ on R is given by $\tilde{A} = \{(x, [\mu^{\tilde{A}^L}(x), \mu^{\tilde{A}^U}(x)])\}$, for all $x \in R$, $0 \leq \mu^{\tilde{A}^L}(x) \leq \mu^{\tilde{A}^U}(x) \leq 1$ and $\mu^{\tilde{A}^L}(x), \mu^{\tilde{A}^U}(x) \in [0, 1]$ and denoted by $\mu^{\tilde{A}}(x) = [\mu^{\tilde{A}^L}(x), \mu^{\tilde{A}^U}(x)]$, $x \in R$ or $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$.

Definition 2.2. The interval-valued triangular fuzzy number $\tilde{A}$ on R represented by the two fuzzy numbers $[\tilde{A}^L, \tilde{A}^U]$, where $\tilde{A}^L = (a_1^L, a_2^L, a_3^L; w_A^L)$ and $\tilde{A}^U = (a_1^U, a_2^U, a_3^U; w_A^U)$:

$$\begin{aligned}\tilde{A} &= [\tilde{A}^L, \tilde{A}^U] \\ &= [(a_1^L, a_2^L, a_3^L; w_A^L), (a_1^U, a_2^U, a_3^U; w_A^U)]\end{aligned}$$

satisfying that $a_1^U \leq a_1^L$, $a_3^L \leq a_3^U$ and $w_A^L \leq w_A^U$ where w_A^U and w_A^L are the weights of $\tilde{A}^U$ and $\tilde{A}^L$ respectively.

Definition 2.3. Arithmetic Operations

Let $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a_1, b_1, c_1; w_{\tilde{A}}), (a_2, b_2, c_2; w_{\tilde{A}})]$, $\tilde{B} = [\tilde{B}^L, \tilde{B}^U] = [(p_1, q_1, r_1; w_{\tilde{B}}), (p_2, q_2, r_2; w_{\tilde{B}})]$ be the IVTFNs,

Figure 1. Interval-Valued Triangular Fuzzy Number.

Then

(i) **Addition:**

$$\tilde{A}(+) \tilde{B} = [(a_1 + p_1, b_1 + q_1, c_1 + r_1; \min\{w_{\tilde{A}}, w_{\tilde{B}}\}), (a_2 + p_2, b_2 + q_2, c_2 + r_2; \min\{w_{\tilde{A}}, w_{\tilde{B}}\})]$$

(ii) **Subtraction:**

$$\tilde{A}(-) \tilde{B} = [(a_1 - r_1, b_1 - q_1, c_1 - p_1; \min\{w_{\tilde{A}}, w_{\tilde{B}}\}), (a_2 - r_2, b_2 - q_2, c_2 - p_2; \min\{w_{\tilde{A}}, w_{\tilde{B}}\})]$$

(iii) **Scalar Multiplication:**

$$k\tilde{A} = [(ka_1, kb_1, kc_1; w_{\tilde{A}}), (ka_2, kb_2, kc_2; w_{\tilde{A}})] \text{ if } k \geq 0$$

$$k\tilde{A} = [(kc_1, kb_1, ka_1; w_{\tilde{A}}), (kc_2, kb_2, ka_2; w_{\tilde{A}})] \text{ if } k < 0$$

(iv) **Multiplication:**

$$\tilde{A}(\times) \tilde{B} = [(a_1.R(\tilde{B}), b_1.R(\tilde{B}), c_1.R(\tilde{B}); \min\{w_{\tilde{A}}, w_{\tilde{B}}\}), (a_2.R(\tilde{B}), b_2.R(\tilde{B}), c_2.R(\tilde{B}); \min\{w_{\tilde{A}}, w_{\tilde{B}}\})]. \text{ if } R(\tilde{B}) \geq 0$$

$$R(\tilde{B}) \geq 0$$

$$\tilde{A}(\times) \tilde{B} = [(a_3.R(\tilde{B}), a_2.R(\tilde{B}), a_1.R(\tilde{B}); \min\{w_{\tilde{A}}, w_{\tilde{B}}\}), (a_3.R(\tilde{B}), a_2.R(\tilde{B}), a_1.R(\tilde{B}); \min\{w_{\tilde{A}}, w_{\tilde{B}}\})]. \text{ if } R(\tilde{B}) < 0$$

$$\text{Where, } R(\tilde{B}) = (p_1^L + q_1^L + r_1^L + p_2^U + q_2^U + r_2^U)/6.$$

3. IVTFN- Transportation Problem

Consider the transportation with m origins(rows) and n destination(columns).

Let C_{ij} be a fuzzy cost of transporting one unit of the product from i^{th} origin to j^{th} destination.

Let $\tilde{A}_i = [(a_i^L, b_i^L, c_i^L; w_{\tilde{A}_i}), (a_i^U, b_i^U, c_i^U; w_{\tilde{A}_i})]$ be the fuzzy quantity of commodity available at origin i,

$\tilde{B}_j = [(p_j^L, q_j^L, r_j^L; w_{\tilde{B}_j}), (p_j^U, q_j^U, r_j^U; w_{\tilde{B}_j})]$ be the fuzzy quantity of commodity requirement at destination j,

x_{ij} is quantity transported from i^{th} origin to j^{th} destination. The fuzzy transportation problem can be represented in the following table.

S.No	D_1	D_2	D_3	...	D_n	Fuzzy available
O_1	x_{11}	x_{12}	x_{13}	...	x_{1n}	$\tilde{A}_1$
O_2	x_{21}	x_{22}	x_{23}	...	x_{2n}	$\tilde{A}_2$
O_3	x_{31}	x_{32}	x_{33}	...	x_{3n}	$\tilde{A}_3$
...	...	...	...	...	...	...
O_m	x_{m1}	x_{m2}	x_{m3}	...	x_{mn}	$\tilde{A}_m$
Fuzzy requirement	$\tilde{B}_1$	$\tilde{B}_2$	$\tilde{B}_3$	...	$\tilde{B}_n$	$\sum_{i=1}^m \tilde{A}_i = \sum_{j=1}^n \tilde{B}_j$

Table 3.1. IVTFN transportation table

The above IVTFN transportation problem can be represented in the form of fuzzy LPP as:

$$\min z = \sum_{i=1}^m \sum_{j=1}^n x_{ij} [(d_{ij}^L, e_{ij}^L, f_{ij}^L; w_{c_{ij}}), (d_{ij}^U, e_{ij}^U, f_{ij}^U; w_{c_{ij}})]$$

Subject to Constrains

$$\sum_{j=1}^n x_{ij} = [(a_i^L, b_i^L, c_i^L; w_i), (a_i^U, b_i^U, c_i^U; w_i)] \text{ for } i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = [(p_j^L, q_j^L, r_j^L; w_j), (p_j^U, q_j^U, r_j^U; w_j)] \text{ for } j = 1, 2, \dots, n$$

for all $x_{ij} \geq 0$.

Also it is noted that fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^m \tilde{A}_i = \sum_{j=1}^n \tilde{B}_j,$$

if the total fuzzy available is equal to the total fuzzy requirement.

4. Monalisha's approximation method

Step 1 Determine the cost table from the given problem. Examine whether total demand equals total supply then, go to step 2. Unless we introduce a dummy row/column having all its cost elements as zero and supply demand is the positive difference of supply and demand.

Step 2 Find the smallest cost in each row of the given cost matrix and then subtract the same from each cost of that row.

Step 3 In the reduced matrix obtained in step 2, locate the smallest cost of each column and then subtract the same from each cost of that column.

Step 4 For each row of the transportation table identify the smallest and the next - to - smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

Step 5 Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference

correspond to i^{th} row and let 0 be in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the $(i, j)^{th}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner.

Step 6 Again compute the column and row differences for the reduced transportation table and go to step 5. Repeat the procedure until all the rim requirements are satisfied.

5. Numerical Illustrations

Let us consider the numerical example of IVTFN transportation problem as:
Finding The Optimal Cost for The Problem of Assigning Three Sources to Four Destinations.

S.NO	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	1	10	12	11	[(16,24,30;0.3), (12,22,34;0.8)]
S ₂	15	21	22	9	[(26,42,54;0.5), (24,40,60;0.9)]
S ₃	22	16	32	15	[(34,44,58;0.2), (30,52,70;0.8)]
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(4,13,26;0.3), (2,23,28;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]	

Solution:

S.NO	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	1	10	12	11	[(16,24,30;0.3), (12,22,34;0.8)]
S ₂	15	21	22	9	[(26,42,54;0.5), (24,40,60;0.9)]
S ₃	22	16	32	15	[(34,44,58;0.2), (30,52,70;0.8)]
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(4,13,26;0.3), (2,23,28;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]	[(76,110,142;0.2), (66,114,164;0.8)]

Table 5.1. Examine whether total demand equals total supply.

S.NO	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	0	8	0	10	[(16,24,30;0.3), (12,22,34;0.8)]
S ₂	6	11	2	0	[(26,42,54;0.5), (24,40,60;0.9)]
S ₃	7	0	6	0	[(34,44,58;0.2), (30,52,70;0.8)]
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(4,13,26;0.3), (2,23,28;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]	[(76,110,142;0.2), (66,114,164;0.8)]

Table 5.2. Reduced Matrix.

S.NO	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	0	8	0	10	[(16,24,30;0.3), (12,22,34;0.8)]	0
S ₂	6	11	22	0	[(26,42,54;0.5), (24,40,60;0.9)]	2
S ₃	7	0	6	0	[(34,44,58;0.2), (30,52,70;0.8)]	0
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(4,13,26;0.3), (2,23,28;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]	[(76,110,142;0.2), (66,114,164;0.8)]	
Penalty	6	8	2	0		

Table 5.3. Finding The Penalty.

S.NO	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	0	8	0	10	[(16,24,30;0.3), (12,22,34;0.8)]	0
S ₂	6	11	22	0	[(26,42,54;0.5), (24,40,60;0.9)]	2
S ₃	7	0	6	0	[(34,44,58;0.2), (30,52,70;0.8)]	0
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(4,13,26;0.3), (2,23,28;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]	[(76,110,142;0.2), (66,114,164;0.8)]	
Penalty	6	8	2	0		

Table 5.4. Choose maximum penalty, The allocation cell (S_3, D_2). Leave D_2 .

S.NO	D ₁	D ₃	D ₄	Supply	Penalty
S ₁	0	0	10	[(16,24,30;0.3), (12,22,34;0.8)]	0
S ₂	6	2	0	[(26,42,54;0.5), (24,40,60;0.9)]	2
S ₃	7	6	0	[(8,31,54;0.2), (2,29,68;0.8)]	6
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]		
Penalty	6	2	0		

Table 5.5. Choose maximum penalty, The allocation cell (S_1, D_1). Leave S_1 .

S.NO	D ₁	D ₃	D ₄	Supply	Penalty
S ₂	0	0	0	[(26,42,54;0.5), (24,40,60;0.9)]	0
S ₃	1	4	0	[(8,31,54;0.2), (2,29,68;0.8)]	1
Demand	[(12,22,36;0.3), (2,24,42;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]		
Penalty	1	4	0		

Table 5.6. Choose maximum penalty, The allocation cell (S_2, D_3). Leave D_3 .

S.NO	D ₁	D ₄	Supply	Penalty
S ₂	0	0	[(12,8,32;0.2), (-22,15,45;0.8)]	0
S ₃	1	0	[(8,31,54;0.2), (2,29,68;0.8)]	1
Demand	[(12,22,36;0.3), (2,24,42;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]		
Penalty	1	0		

Table 5.7. Choose maximum penalty, The allocation cell (S_2, D_1). Leave D_1 .

S.NO	D ₄	Supply	Penalty
S ₂	0	[(32,10,44;0.2), (-48,11,69;0.8)]	-
S ₃	0	[(8,31,54;0.2), (2,29,68;0.8)]	-
Demand	[(25,40,55;0.4), (15,46,65;0.9)]		
Penalty	0		

Table 5.8. Choose any cell and allocate it.

The allocation table

S.NO	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	1	10	12	11	[(16,24,30;0.3), (12,22,34;0.8)]
S ₂	6	21	22	9	[(26,42,54;0.5), (24,40,60;0.9)]
S ₃	7	0	6	0	[(34,44,58;0.2), (30,52,70;0.8)]
Demand	[(18,22,36;0.3), (10,26,38;0.9)]	[(4,13,26;0.3), (2,23,28;0.8)]	[(22,34,38;0.2), (15,25,46;0.8)]	[(25,40,55;0.4), (15,46,65;0.9)]	[(76,110,142;0.2), (66,114,164;0.8)]

Optimum Cost

$$\begin{aligned}
 &= 1 \times [(16, 24, 30; 0.3), (12, 22, 34; 0.8)] + \\
 &15 \times [(-12, -2, 20; 0.3), (-24, 4, 26; 0.8)] + \\
 &22 \times [(22, 34, 38; 0.2), (15, 25, 46; 0.8)] + \\
 &9 \times [(-32, 10, 44; 0.2), (-48, 11, 69; 0.8)] + \\
 &16 \times [(4, 13, 26; 0.3), (2, 23, 28; 0.8)] + \\
 &15 \times [(8, 31, 54; 0.2), (2, 29, 68; 0.8)] \\
 &= [(16, 24, 30; 0.3), (12, 22, 34; 0.8)] + \\
 &[(-180, -30, 300; 0.3), (-360, 60, 390; 0.8)] + \\
 &[(484, 748, 836; 0.2), (330, 550, 1012; 0.8)] + \\
 &[(-288, 90, 396; 0.2), (-432, 99, 621; 0.8)] + \\
 &[(64, 208, 416; 0.3), (32, 368, 448; 0.8)] +
 \end{aligned}$$

$$\begin{aligned}
& [(120, 465, 810; 0.2), (30, 435, 1020; 0.8)]. \\
& = [(216, 1505, 2788; 0.2), (-388, 1534, 3525; 0.8)]. \\
& = 1530.
\end{aligned}$$

6. Conclusion

In this paper, We have derived the optimal transportation cost by using the proposed fuzzy arithmetic operations. This method is very useful for the decision makers to take a decision during uncertainty. So this technique can be applied for any type of fuzzy transportation problems occurring in real life situations. The same procedure can be extended for solving problems in balanced and unbalanced transportation problems, transshipment problems, project scheduling problems, and network flow problems also.

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