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A note on multi-objective integer linear programming problems using pentagonal fuzzy numbers

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Abstract
The focus of this paper is to find optimal solution of multi-objective integer linear programming problems (MOILPP) with pentagonal fuzzy numbers based on level-sum method. This approach is very easy to understand and useful. This is illustrated with relevant numerical example.

Keywords
Pentagonal fuzzy number, Multi objective integer linear programming problem.

AMS Subject Classification
90C10, 90C29, 90C70, 90C90.

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Contents

1. Introduction

The concept of linear programming problem is to find out the best solution to the real-world problems where the available information's are not exact or not precise. In that situation linear programming model can be applied. It plays a vital role in fuzzy modeling, which can formulate the uncertainty.

In [2] Herrera and Verdegay studied some models for dealing with fuzzy integer linear programming problems by using fuzzy constraints and fuzzy numbers. Allahviranloo et al.[1] have introduced a new method by reducing the fuzzy integer linear programming models to crisp integer linear programming models. Pandian [3] solved fuzzy linear programming problems using multi-objective technique. Sahaya sudha et al. [4] discussed multi-objective fuzzy linear programming problem with pentagonal fuzzy numbers. In [5] Stephen Dinagar and Mohamed Jeyavuthin discussed the concept of solving integer linear programming problems with pentagonal fuzzy numbers.

In this paper, section 2 contains some basic definitions needed for this work. In section 3, fuzzy integer linear programming with multi-objective functions are discussed. In section 4, A relevant numerical illustration is given. Finally, conclusion is included in section 5.

2. Preliminaries

In this section, we recall some definitions and basic results which will be used throughout the paper.

Definition 2.1. *[5] A fuzzy set $\tilde{A}$ is defined by*

$$\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}.$$

In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval $[0,1]$, called Membership function.

Definition 2.2. *[5] A fuzzy set $\tilde{A}$ is convex if*

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))), x_1, x_2 \in X$$

and $\lambda \in [0,1]$. Alternatively, a fuzzy set is convex, if all α-level sets are convex.

Definition 2.3. *[5] A fuzzy set $\tilde{A}$ is defined on the set of real numbers R is said to be fuzzy number if it has the following characteristics*
***(i)** $\tilde{A}$ is normal*
***(ii)** $\tilde{A}$ is convex set*
***(iii)** The support of $\tilde{A}$ is closed and bounded.*

Definition 2.4. *[5] A fuzzy number $\tilde{A}_P$ is Pentagonal fuzzy number denoted by $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$, where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership $\mu_{\tilde{A}_P}(x)$is given by*

$$\mu_{\tilde{A}_P}(x) = \begin{cases} 0 & ,x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & ,a_1 \leq x \leq a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & ,a_2 \leq x \leq a_3 \\ 1 & ,x = a_3 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{a_4-x}{a_4-a_3}\right) & ,a_3 \leq x \leq a_4 \\ \left(\frac{a_5-x}{a_5-a_4}\right) & ,a_4 \leq x \leq a_5 \\ 0 & ,x > a_5 \end{cases}$$

Definition 2.5. *A pentagonal fuzzy number can be defined as $\tilde{A}_P = (M_1(u), J_1(v), J_2(v), M_2(u))$ for $u \in [0, 0.5]$ and $v \in [0.5, 1]$*
(i)$M_1(u)$ *is strictly a increasing continuous function on [0,0.5]*
(ii)$J_1(v)$ *is strictly a increasing continuous function on [0.5,1]*
(iii)$J_2(v)$ *is strictly a decreasing continuous function on [1,0.5]*
(iv)$M_2(u)$ *is strictly a decreasing continuous function on [0.5,1].*

Remark 2.6. *The pentagonal fuzzy number $\tilde{A}_P$ becomes triangular fuzzy number if $a_3 - a_2 = a_4 - a_3$.*

3. Fuzzy Integer Linear Programming Problem

Consider the following fully fuzzy integer linear programming problems:
(P) Maximize (or) Minimize $\tilde{z} = \tilde{c}^T$ subject to
$\tilde{A} \times \tilde{x} \{\leq, =, \geq\} \tilde{b}, \tilde{x} \geq 0$ and are integers,

where the cost vectors
$\tilde{c}^T = (\tilde{c}_j)_{1\times n}, \tilde{A} = (\tilde{a}_{ij})_{m\times n}, \tilde{x} = (\tilde{x}_j)_{n\times 1}, \tilde{b} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{a}_{ij}, \tilde{x}_j, \tilde{b}_i, \tilde{c}_j \in F(R)$, for all $1 \leq j \leq n$ and $1 \leq i \leq m$.

Let the parameters $\tilde{z}_j, \tilde{a}_{ij}, \tilde{x}_j, \tilde{b}_i, \tilde{c}_j$ be the pentagonal fuzzy numbers

$$\begin{aligned} \tilde{z} &= (z_1, z_2, z_3, z_4, z_5), & \tilde{x}_j &= (x_j, y_j, t_j, u_j, v_j) \\ \tilde{a}_{ij} &= (a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}), & \tilde{b}_i &= (b_i, e_i, f_i, g_i, h_i) \\ \tilde{c}_j &= (p_j, q_j, r_j, s_j, t_j) \end{aligned}$$

respectively.
The mathematical formulation as follows:

$$\begin{aligned} (P)\ Maximize&(z_1, z_2, z_3, z_4, z_5) \\ &= \sum_{j=1}^{n} (p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \end{aligned}$$

subject to

$$\begin{aligned} \sum_{j=1}^{n} &(p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \\ &\{\leq, =, \geq\}\ (b_i, e_i, f_i, g_i, h_i); \\ &(x_j, y_j, t_j, u_j, v_j) \geq 0, (j = 1, 2, 3, ..., m) \end{aligned}$$

and are integers.
Now, we have to transform the fuzzy integer linear programming problem into multi-objective fuzzy integer linear programming problem by using level-sum method.

$$\begin{aligned} (M) Maximize\ z_1 &= \sum_{j=1}^{n} most\ lowest\ value\ of \\ &(p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \\ Maximize\ z_2 &= \sum_{j=1}^{n} lowest\ value\ of \\ &(p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \\ Maximize\ z_3 &= \sum_{j=1}^{n} middle\ value\ of \\ &(p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \\ Maximize\ z_4 &= \sum_{j=1}^{n} upper\ value\ of \\ &(p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \\ Maximize\ z_5 &= \sum_{j=1}^{n} most\ upper\ value\ of \\ &(p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j) \end{aligned}$$

subject to constraints

$$\begin{aligned} &\sum_{j=1}^{n} most\ lowest\ value\ of \\ &\quad (p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j)\ \{\leq, =, \geq\}\ b_i \\ &\sum_{j=1}^{n} lowest\ value\ of \\ &\quad (p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j)\ \{\leq, =, \geq\}\ e_i \\ &\sum_{j=1}^{n} middle\ value\ of \\ &\quad (p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j)\ \{\leq, =, \geq\}\ f_i \\ &\sum_{j=1}^{n} upper\ value\ of \\ &\quad (p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j)\ \{\leq, =, \geq\}\ g_i \\ &\sum_{j=1}^{n} most\ upper\ value\ of \\ &\quad (p_j, q_j, r_j, s_j, t_j) \times (x_j, y_j, t_j, u_j, v_j)\ \{\leq, =, \geq\}\ h_i \end{aligned}$$

for all $i = 1, 2, 3, \ldots, m$

$$\begin{aligned} &z_2 \geq z_1; z_3 \geq z_2; z_4 \geq z_3; z_5 \geq z_4; \\ &x_j \leq y_j, j = 1, 2, 3, \ldots, m, y_j \leq t_j, j = 1, 2, 3, \ldots, m, \\ &t_j \leq u_j, j = 1, 2, 3, \ldots, m, u_j \leq v_j, j = 1, 2, 3, \ldots, m, \\ &x_j \geq 0, j = 1, 2, 3, \ldots, m. \end{aligned}$$

Note 3.1. *Suppose a fuzzy integer linear programming problem having hexagonal fuzzy number is considered, we will* *have a multi-objective linear programming problem with six objectives. Here we establish the following theorem which gives relation between optimal solution of the fuzzy integer linear programming problem and an efficient solution to the corresponding multi-objective problem.*

Definition 3.2. *[4]*
Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be in $F(R)$ then $\tilde{A} > \tilde{B}$ if and only if $a_i \geq b_i, j = 1$ to 5 and $b_r > a_r$ for some $r \in \{1,2,3,4,5\}$.

Definition 3.3. *[3]*
A feasible point x^0 is said to be efficient solution if there exists no other feasible point x in P such that $f_i(x) \leq f_i(x^0), i = 1,2,\ldots,k$ and $f_r(x) < f_r(x^0)$ for some $r \in \{1,2,\ldots,k\}$.

Theorem 3.4. *Let $X^0 = \{x_j^0, y_j^0, t_j^0, u_j^0, v_j^0; j = 1,2,\ldots,m\}$ be an efficient solution to the problem (M).Then,*
$\tilde{X}^0 = \{(x_j^0, y_j^0, t_j^0, u_j^0, v_j^0); j = 1,2,\ldots,m\}$ is an optimal solution to the problem (P).

Proof. Let $X^0 = \{x_j^0, y_j^0, t_j^0, u_j^0, v_j^0; j = 1,2,\ldots,m\}$ be an efficient solution to the problem (M).Then,
$\tilde{X}^0 = \{(x_j^0, y_j^0, t_j^0, u_j^0, v_j^0); j = 1,2,\ldots,m\}$ is a feasible solution to the problem (P).

Assume that $\tilde{X}^0 = \{(x_j^0, y_j^0, t_j^0, u_j^0, v_j^0); j = 1,2,\ldots,m\}$ is not an optimal solution to the problem (P). Then, there exists a feasible solution, $\tilde{X}^0 = \{(x_j^0, y_j^0, t_j^0, u_j^0, v_j^0); j = 1,2,\ldots,m\}$to the problem (P) such that $Z(\tilde{X}) > Z(\tilde{X})^0$,that is,
$z_i(x,y,t,u,v) \geq z_i(x^0, y^0, t^0, u^0, v^0), i = 1,2,3,4,5$ and
$z_r(x,y,t,u,v) > z_r(x^0, y^0, t^0, u^0, v^0)$,for some $r \in \{1,2,3,4,5\}$
where,

$$\begin{aligned}
x^0 &= \{x_j^0; j = 1,2,3,\ldots,m\}, & y^0 &= \{y_j^0; j = 1,2,3,\ldots,m\},\\
t^0 &= \{t_j^0; j = 1,2,3,\ldots,m\}, & u^0 &= \{u_j^0; j = 1,2,3,\ldots,m\},\\
v^0 &= \{v_j^0; j = 1,2,3,\ldots,m\}, & x &= \{x_j; j = 1,2,3,\ldots,m\},\\
y &= \{y_j; j = 1,2,3,\ldots,m\}, & t &= \{t_j; j = 1,2,3,\ldots,m\},\\
u &= \{u_j; j = 1,2,3,\ldots,m\}, & v &= \{v_j; j = 1,2,3,\ldots,m\}.
\end{aligned}$$

This means that $X^0 = \{x_j^0, y_j^0, t_j^0, u_j^0, v_j^0; j = 1,2,\ldots,m\}$ is not an efficient solution to the problem (M). This is a contradiction, which proves the theorem.

□

Algorithm 3.5. *The computational procedure for finding optimal solution to the fuzzy integer linear programming problem with the aid of level-sum method is given as:*

Step 1 *construct the multi-objective integer linear programming problem from the given fuzzy integer linear programming problem.*

Step 2 *Find an efficient solution to the multi-objective integer linear programming problem obtained in step 1 using the sum of objectives method [3].*

Step 3 *The efficient solution obtained in step 2 to the multi-objective integer linear programming problem gives an optimal fuzzy solution to the fuzzy integer linear programming problem by the above theorem.*

4. Numerical Example

Example 4.1. *Consider the following fuzzy integer linear programming problem*

$$\text{Maximize } \tilde{z} = (1,2,3,4,5)\tilde{x}_1 + (6,7,8,9,10)\tilde{x}_2$$

Subject to constraints

$$\begin{aligned}
(2,4,6,8,10)\tilde{x}_1 + (3,6,9,12,15)\tilde{x}_2 &\leq (5,10,15,20,25);\\
(1,2,3,4,5)\tilde{x}_1 + (2,4,6,8,10)\tilde{x}_2 &\leq (10,20,30,40,50);
\end{aligned}$$

$\tilde{x}_1, \tilde{x}_2 \geq 0$ *and are integers.*
Let

$$\tilde{x}_1 = (x_1, y_1, t_1, u_1, v_1), \tilde{x}_2 = (x_2, y_2, t_2, u_2, v_2) \text{ and}$$
$$\tilde{z} = (z_1, z_2, z_3, z_4, z_5)$$

Now, using the step 1, the multi-objective problem related to the given problem is,

$$\begin{aligned}
(M)\text{Maximize } z_1 &= x_1 + 6x_2, & \text{Maximize } z_2 &= 2y_1 + 7y_2\\
\text{Maximize } z_3 &= 3t_1 + 8t_2, & \text{Maximize } z_4 &= 4u_1 + 9u_2,\\
\text{Maximize } z_5 &= 5v_1 + 10v_2
\end{aligned}$$

Subject to constraints

$$\begin{aligned}
2x_1 + 3x_2 &\leq 5; & 4y_1 + 6y_2 &\leq 10; & 6t_1 + 9t_2 &\leq 15;\\
8u_1 + 12u_2 &\leq 20; & 10v_1 + 15v_2 &\leq 25; & 1x_1 + 2x_2 &\leq 10;\\
2y_1 + 4y_2 &\leq 20; & 3t_1 + 6t_2 &\leq 30; & 4u_1 + 8u_2 &\leq 40;\\
5v_1 + 10v_2 &\leq 50;
\end{aligned}$$

$$\begin{aligned}
&z_2 \geq z_1; z_3 \geq z_2; z_4 \geq z_3; \quad z_5 \geq z_4; y_1 \geq x_1; t_1 \geq y_1;\\
&u_1 \geq t_1; v_1 \geq u_1; y_2 \geq x_2; \quad t_2 \geq y_2; u_2 \geq t_2; v_2 \geq u_2
\end{aligned}$$

Now, by the step 2, we consider the following linear programming problem (S) related to the above multi-objective integer linear programming problem,
(S) Maximize $\tilde{z}$
$= x_1 + 6x_2 + 2y_1 + 7y_2 + 3t_1 + 8t_2 + 4u_1 + 9u_2 + 5v_1 + 10v_2$
subject to

$$\begin{aligned}
2x_1 + 3x_2 &\leq 5; & 4y_1 + 6y_2 &\leq 10; & 6t_1 + 9t_2 &\leq 15;\\
8u_1 + 12u_2 &\leq 20; & 10v_1 + 15v_2 &\leq 25; & 1x_1 + 2x_2 &\leq 10;\\
2y_1 + 4y_2 &\leq 20; & 3t_1 + 6t_2 &\leq 30; & 4u_1 + 8u_2 &\leq 40;\\
5v_1 + 10v_2 &\leq 50;
\end{aligned}$$

$$\begin{aligned}
2y_1 + 7y_2 - x_1 - 6x_2 &\leq 0; & 3t_1 + 8t_2 - 2y_1 - 7y_2 &\leq 0;\\
4u_1 + 9u_2 - 3t_1 - 8t_2 &\leq 0; & 5v_1 + 10v_2 - 4u_1 - 9u_2 &\geq 0;
\end{aligned}$$

$$\begin{aligned}
&y_1 \geq x_1; t_1 \geq y_1; \quad u_1 \geq t_1; \quad v_1 \geq u_1;\\
&y_2 \geq x_2; t_2 \geq y_2; \quad u_2 \geq t_2; \quad v_2 \geq u_2.
\end{aligned}$$

and solve it by Gomory's fractional cut method. The optimal solution to the problem (S) is
$x_1 = 1, x_2 = 1, y_1 = 1, y_2 = 1, t_1 = 1, t_2 = 1, u_1 = 1,$
$u_2 = 1, v_1 = 1, v_2 = 1$ *with* $Z = 55$,
which is the efficient solution to the problem (M).
Now, by using step 3, the optimal fuzzy solution to the given problem is
$\tilde{x}_1 = (1,1,1,1,1), \tilde{x}_2 = (1,1,1,1,1)$ *and* $\tilde{z} = (7,9,11,13,15)$.

5. Conclusion

In this work, the notion called level-sum method to find an optimal fuzzy solution to a fuzzy integer linear programming problem using pentagonal fuzzy numbers have been proposed. The main advantage of this method is easy to solve the optimal solution for the given fuzzy integer linear programming problem and it provides an appropriate best solution to different types of linear programming models. This notion can be extended to some other optimization problems in future.

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