Partial differentiability on graphs

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Abstract
Operations play a vital role in the field of Mathematics. There are many operations by which new graphs are obtained from the old ones. In this paper we try to form a new graph from the old one through a new operation, partial differentiability. Let $G$ be a graph of order $n$. Consider an arbitrary vertex $v$ in $G$. We remove all edges which are adjacent to the vertex $v$. The resultant graph is denoted by $G_v^{(1)}$. This is called the partial differentiation of $G$ with respect to the vertex $v$. Now we consider another vertex $u$ in $G_v^{(1)}$ and remove all edges which are adjacent to $u$. The resultant graph is denoted by $G_u^{(2)}$. This is the partial derivative of $G_v^{(1)}$ with respect to $u$. The minimum of $r$ such that $G^{(r)} \cong mK_1$, is called the order of partial differentiation, denoted by $r(G)$, where $m$ is a positive integer. In this paper we introduced the partial differentiability of graphs.

Keywords
Graph, Differentiation, Partial differentiation.

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1. Introduction

Partial derivatives can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid dynamics, elasticity, quantum mechanics etc. There has been a considerable effort to study differentiation on graphs. In \cite{1} introduced the differentiability of graphs. This motivated to study partial differentiation and its properties on graphs.

2. Main Results

Definition 2.1. Let $G$ be a graph of order $n$. Consider an arbitrary vertex $v$ in $G$. Remove all edges which are adjacent to the vertex $v$. The resultant graph is denoted by $G_v^{(1)}$. This is called the partial derivative of $G$ with respect to $v$. Now consider another vertex $u$ in $G_v^{(1)}$ and remove all edges which are adjacent to $u$. The resultant graph is denoted by $G_u^{(2)}$. This is the partial derivative of $G_v^{(1)}$ with respect to $u$.

The minimum of $r$ such that $G^{(r)} \cong mK_1$, is called the order of partial differentiation; where $m$ is a positive integer and it is denoted by $r(G)$.

Remark 2.2. 1. If $G$ is an empty graph, then $r(G) = 0$.

2. All graphs are partial differentiable.

Example 2.3.
Definition 2.4. The adjacency matrix of a graph $G$ with $n$ vertices and no parallel edges is an $n$ by $n$ symmetric binary matrix $X = [x_{ij}]$, defined over the ring of integers such that

$$x_{ij} = \begin{cases} 1, & \text{if there is an edge between } i^{th} \text{ and } j^{th} \text{ vertices} \\ 0, & \text{if there is no edges between them} \end{cases}$$

Finding order of partial differentiation by adjacency matrix

Algorithm:

1. Start
2. Read an adjacency matrix, $A$ and initialize $count = 0$
3. Find the row / column which has maximum number of entries 1.
4. If the maximum entries 1 in two or more column / row are same choose any one arbitrarily.
5. Remove that column and row
6. Set $count = count + 1$
7. Repeat steps 2, 3, 4 and 5 until we get matrix, $A$ with all entries zero.
8. Order of partial differentiation is $count$.
9. Stop.

Example 2.5.

Theorem 2.6. For a complete graph, $K_n, n \geq 2$, $G^{(n-1)} \cong K_1$.

Proof. We prove this theorem by mathematical induction.
First we prove the theorem is true for $n = 2$.
If $n = 2$, there are two vertices say $v_1$ and $v_2$.
Differentiate $G$ partially with respect to $v_1$ (or $v_2$). Then it remains $v_2$ (or $v_1$) only.

The theorem is true for $n = 2$.
By assumption, that the theorem is true for $n = m$.
Hence, for $K_m, G^{(m-1)} \cong K_1$
Now we prove the theorem is true for $n = m + 1$
Here $G$ contains $m + 1$ vertices.
Let $v_1, v_2, v_3, \ldots, v_{m+1}$ be the vertices.
Differentiate $G$ partially with respect to an arbitrary vertex $v_i$.
The resultant graph is $K_m$.
We know that, for $K_m, G^{(m-1)} \cong K_1$
Hence for $K_{m+1}, G^{(m+1-1)} \cong K_1$
i.e., $G^{(m)} \cong K_1$
i.e., $G^{(n-1)} \cong K_1$ (\because $n = m + 1$).

Proposition 2.7. For a connected graph $G$ with $n$ vertices, $1 \leq r(G) \leq n - 1$.

Proof. Let $G$ be a connected graph with $n$ vertices $v_1, v_2, v_3, \ldots, v_n$.
Let $v_i$ be an arbitrary vertex which is adjacent all other vertices and there exist no other edges between the remaining vertices.
If we differentiate $G$ partially with respect to $v_i$, we get $r(G) = 1$.
Again let each vertex of $G$ be adjacent to all other vertices in $G$. Then by theorem $r(G) = n - 1$.
Hence we get $1 \leq r(G) \leq n - 1$. 

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References


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