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Partial differentiability on graphs

M. Regees[1] and T. Sajitha Kumari[2]*

Abstract

Operations play a vital role in the field of Mathematics. There are many operations by which new graphs are obtained from the old ones. In this paper we try to form a new graph from the old one through a new operation, partial differentiability. Let G be a graph of order n. Consider an arbitrary vertex v in G. We remove all edges which are adjacent to the vertex v. The resultant graph is denoted by $G_v^{(1)}$. This is called the partial differentiation of G with respect to the vertex v. Now we consider another vertex u in $G_v^{(1)}$ and remove all edges which are adjacent to u. The resultant graph is denoted by $G_u^{(2)}$. This is the partial derivative of $G_v^{(1)}$ with respect to u. The minimum of r such that $G^{(r)} \cong mK_1$, is called the order of partial differentiation, denoted by $r(G)$, where m is a positive integer. In this paper we introduced the partial differentiability of graphs.

Keywords

Graph, Differentiation, Partial differentiation.

AMS Subject Classification

05C78.

[1] *Department of Mathematics, Malankara Catholic College, Kaliakavilai-629153, Tamil Nadu, India.*

[2] *Research Scholar, Department of Mathematics, Scott Christian College, Nagercoil- 629001, Tamil Nadu, India.*

*__Corresponding author__: [1] regeesregees@gmail.com; [2] sajithasabu09@gmail.com

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Contents

1. Introduction

Partial derivatives can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid dynamics, elasticity, quantum mechanics etc. There has been a considerable effort to study differentiation on graphs. In [1] introduced the differentiability of graphs. This motivated to study partial differentiation and its properties on graphs.

2. Main Results

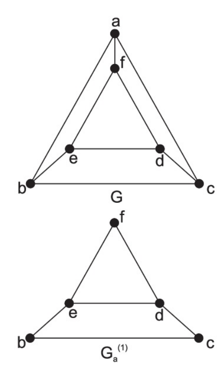
Definition 2.1. *Let G be a graph of order n. Consider an arbitrary vertex v in G. Remove all edges which are adjacent to the vertex v. The resultant graph is denoted by $G_v^{(1)}$. This is called the partial derivative of G with respect to v. Now consider another vertex u in $G_v^{(1)}$ and remove all edges which are adjacent to u. The resultant graph is denoted by $G_v^{(2)}$. This is called the partial derivative of $G_v^{(1)}$ with respect to u.*

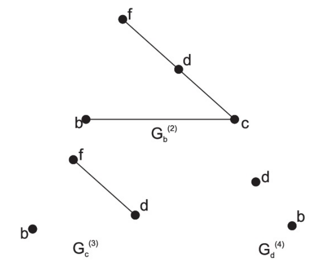
The minimum of r such that $G^{(r)} \cong mK_1$, is called the order of the partial differentiation; where m is a positive integer and it is denoted by $r(G)$.

Remark 2.2.

1. *If G is an empty graph, then $r(G) = 0$.*
2. *All graphs are partial differentiable.*

Example 2.3.





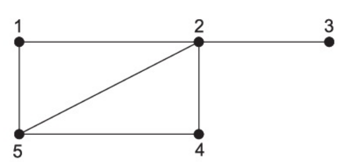
Definition 2.4. *The adjacency matrix of a graph G with n vertices and no parallel edges is an n by n symmetric binary matrix $X = [x_{ij}]$, defined over the ring of integers such that*

$$x_{ij} = \begin{cases} 1, \text{if there is an edge between } i^{th} \text{ and } j^{th} \text{ vertices} \\ 0, \quad \text{if there is no edges between them.} \end{cases}$$

Finding order of partial differentiation by adjacency matrix
Algorithm:

1. Start
2. Read an adjacency matrix , A and initialize count = 0
3. Find the row / column which has maximum number of entries 1.
4. If the maximum entries 1 in two or more column / row are same choose any one arbitrarily.
5. Remove that column and row
6. Set count = count +1
7. Repeat steps 2, 3, 4 and 5 until we get matrix, A with all entries zero.
8. Order of partial differentiation is count.
9. Stop.

Example 2.5.



Adjacency matrix, $[a_{ij}] =$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Step 1

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

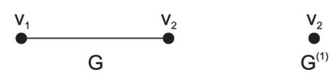
Step 2

$$\begin{array}{c} \\ 1 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Here $r(G) = 2$

Theorem 2.6. *For a complete graph, $K_n; n \geq$, $G^{(n-1)} \cong K_1$.*

Proof. We prove this theorem by mathematical induction.
First we prove the theorem is true for $n = 2$.
If $n = 2$, there are two vertices say v_1 and v_2.
Differentiate G partially with respect to v_1 (or v_2). Then it remains v_2 (or v_1) only.



The theorem is true for $n = 2$.
By assumption, that the theorem is true for $n = m$.
Hence, for K_m, $G^{(m-1)} \cong K_1$
Now we prove the theorem is true for $n = m+1$
Here G contains $m+1$ vertices.
Let $v_1, v_2, v_3, \ldots, v_{m+1}$ be the vertices.
Differentiate G partially with respect to an arbitrary vertex v_i.
The resultant graph is K_m.
We know that, for K_m, $G^{(m-1)} \cong K_1$
Hence for K_{m+1}, $G^{(m-1+1)} \cong K_1$
i.e., $G^{(m)} \cong K_1$
i.e., $G^{(n-1)} \cong K_1 \quad (\because n = m+1)$. □

Proposition 2.7. *For a connected graph G with n vertices, $1 \leq r(G) \leq n-1$.*

Proof. Let G be a connected graph with n vertices $v_1, v_2, v_3, \ldots, v_n$.
Let v_i be an arbitrary vertex which is adjacent all other vertices and there exist no other edges between the remaining vertices. If we differentiate G partially with respect to v_i, we get $r(G) = 1$. Again let each vertex of G be adjacent to all other vertices in G. Then by theorem $r(G) = n-1$.
Hence we get $1 \leq r(G) \leq n-1$.

□

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