Algorithm for finding location domination number of Corona product of graphs

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Abstract

A dominating set S which uniquely identify all vertices of the set $V(G) - S$ is a locating dominating set. This paper presents the procedure for finding the location domination number of Corona product of graphs. And location domination number of graph of forms $G \odot K_n$, $G \odot K_{m,n}$, $G \odot \bar{K}_n$, $G \odot S_n$, $G \odot P_n$, $G \odot C_n$, $G \odot W_n$ and $G \odot F_{1,n}$ are deduced.

Keywords

Dominating set, Locating set, Locating domination set, Corona product.

AMS Subject Classification

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1. Introduction

Oystein Ore [2] defined that the dominating set of a graph G is a subset S of the vertex set $V(G)$ such that all vertices in the set $V(G) - S$ is adjacent to atleast one vertex in S . Minimal cardinality of dominating set is known as domination number and it is denoted by $\gamma(G)$.

Slater [4, 5] defined the locating dominating set is a dominating set S with $S(v) \neq S(w)$, for any $v, w \in V(G) - S$, where

$S(v)$ is the set of vertices in S which are adjacent to v . A locating dominating set is denoted by LD -set. The minimum cardinality of an LD -set in G is called the location-domination number of G and it is denoted by $RD(G)$. An LD -set with $RD(G)$ elements is called as a referencing-dominating set or an RD -set.

In [1, 3] Sergio R. Canoy, Jr. et al. and Risan Nur Santi et al. have found the bound for location domination number of corona product of graphs. In this paper, we construct a systematic technique for finding location domination number of corona product of any graph and rectify some results in [3].

2. Location Domination Number of Corona Product of Graphs

In this section, we introduce a new definition “wholly located dominated graph” by which we determine location domination number of corona product of any graph.

Definition 2.1. Let G be any graph with RD -set S . If G has a vertex $v \in V(G) - S$, such that $S(v) = S$ then the vertex v is said to be wholly located dominated vertex with respect to the RD - set S .

Definition 2.2. Let $\mathcal{S}$ be the collection of all RD -set of G . Graph G is said to be wholly located dominated graph if for every RD -set $S \in \mathcal{S}$ there exist a wholly located dominated vertex.

Theorem 2.3. Let G_1 and G_2 be any two graphs with vertex set $V(G_1)$ and $V(G_2)$ respectively. For the graph $G_1 \odot G_2$, if G_2 wholly located dominated graph then

$$RD(G_1 \odot G_2) = RD(G_2)|V(G_1)| + \gamma(G_1)$$

otherwise,

$$RD(G_1 \odot G_2) = RD(G_2)|V(G_1)|$$

Proof.

Let $V(G_1) = \{u_1, u_2, \dots, u_m\}$ and $V(G_2) = \{v_1, v_2, \dots, v_n\}$, then $V(G_1 \odot G_2)$ is

$$\begin{aligned} & \{u_1, u_2, \dots, u_m\} \cup \{(u_1, v_1), (u_1, v_2), \dots, (u_1, v_n)\} \cup \\ & \{(u_2, v_1), (u_2, v_2), \dots, (u_2, v_n)\} \cup \dots \cup \\ & \{(u_m, v_1), (u_m, v_2), \dots, (u_m, v_n)\} \end{aligned}$$

By definition of corona product there are $|V(G_1)|$ copies of G_2 graph joined to each vertices of G_1 . To locate dominate each copy of G_2 , it requires $RD(G_2)$ number of vertices. Since there are $|V(G_1)|$ copies of G_2 , it requires $RD(G_2)|V(G_1)|$ number of vertices to locate and dominate. Therefore $RD(G_1 \odot G_2) \geq RD(G_2)|V(G_1)|$.

Let $S_1, S_2, \dots, S_m$ be the set to locate and dominate the m copies of the graph G_2 . That is S_i is the RD -set of the induced subgraph of $G_1 \odot G_2$ with vertex set $\{(u_i, v_1), (u_i, v_2), \dots, (u_i, v_n)\}$ where $i = 1, 2, \dots, m$. Clearly $S_i \cap S_j = \Phi$ if $i \neq j$. Now consider the set $S = S_1 \cup S_2 \cup \dots \cup S_m$. Case 1: Suppose G_2 is not wholly located dominated graph, then G_2 has atleast one RD -set S_{G_2} such that $S_{G_2}(v) \neq S_{G_2}$ for all $v \in V(G_2) - S_{G_2}$. Hence in the graph $G_1 \odot G_2$, $S((u_i, v_j)) = S_i(v_j) \neq S_i$ and $S(u_i) = S_i$.

Clearly S_i locate and dominate all vertices of the set $\{(u_i, v_1), (u_i, v_2), \dots, (u_i, v_n)\} - S_i$ uniquely and it also uniquely locate and dominate the vertex u_i . So all the vertices of $(G_1 \odot G_2) - S$ are uniquely located and dominated by the set S and it is minimal. Hence,

$$\begin{aligned} RD(G_1 \odot G_2) &= |S| = |S_1 \cup S_2 \cup \dots \cup S_m| \\ &= |S_1| + |S_2| + \dots + |S_m|, \\ &\quad \text{as } S_i \cap S_j = \Phi \text{ if } i \neq j \\ &= m RD(G_2) \\ &= RD(G_2) |V(G_1)|. \end{aligned}$$

Case 2: Suppose assume that G_2 is wholly located dominated graph, then for RD -set S_{G_2} of G_2 there exist every some vertex $v_j \in V(G_2) - S_{G_2}$ such that $S_{G_2}(v_j) = S_{G_2}$.

Now for the graph $G_1 \odot G_2$, if we look upon the set $S = S_1 \cup S_2 \cup \dots \cup S_m$, it clear that S_i will be the RD -set of i^{th} copy of G_2 in the graph $G_1 \odot G_2$. By our assumption there exist a vertex (u_i, v_j) , where $i = 1, 2, \dots, m$ such that $S((u_i, v_j)) = S_i(v_j) = S_i$ and $S(u_i) = S_i$. Hence S could not be the LD -set of $G_1 \odot G_2$. But S dominates all the vertices of $G_1 \odot G_2$ and locate all the vertices uniquely except $u_1, u_2, \dots, u_m$.

As $S(u_i) = S_i$, it is enough to dominate the vertices $u_1, u_2, \dots, u_m$ with the help of the vertices in the set $\{u_1, u_2, \dots, u_m\}$, so that $S(u_i) \supset S_i$ and it would be the LD -set of $G_1 \odot G_2$ and could not be further reduced.

Therefore RD -set of $G_1 \odot G_2$ is the set S union dominating set of $\{u_1, u_2, \dots, u_m\}$. That is, S union dominating set of G_1 is the RD -set of $G_1 \odot G_2$. Hence,

$$\begin{aligned} RD(G_1 \odot G_2) &= |S| + \gamma(G_1) \\ &= RD(G_2)|V(G_1)| + \gamma(G_1). \end{aligned} \quad \square$$

Remark 2.4. Corona product of two graphs is not commutative. Therefore $RD(G_1 \odot G_2)$ need not be equal to $RD(G_2 \odot G_1)$.

Remark 2.5. All disconnected graphs will not be a wholly located dominated graph. In the graph $G_1 \odot G_2$, if G_2 is disconnected then it will not a wholly located dominated graph. Thus $RD(G_1 \odot G_2) = RD(G_2)|V(G_1)|$.

3. Illustration

3.1 $G \odot K_1$

Corollary 3.1. As K_n is not wholly located dominated graph when $n = 1$, $RD(G \odot K_1) = |V(G)|$. For $n > 1$, K_n are all wholly located dominated graph so

$$\begin{aligned} RD(G \odot K_n) &= RD(K_n)|V(G)| + \gamma(G) \\ &= (n-1)|V(G)| + \gamma(G). \end{aligned}$$

3.2 $G \odot K_{m,n}$

Corollary 3.2. For $m, n \geq 2$, $K_{m,n}$ are all not wholly located dominated graph. Hence

$$\begin{aligned} RD(G \odot K_{m,n}) &= RD(K_{m,n})|V(G)| \\ &= (m+n-2)|V(G)|. \end{aligned}$$

3.3 $G \odot \bar{K}_n$

Corollary 3.3. $\bar{K}_n$ is not wholly located dominated graph for all $n \geq 1$. By Theorem 2.3 we obtain

$$\begin{aligned} RD(G \odot \bar{K}_n) &= RD(\bar{K}_n)|V(G)| \\ &= n|V(G)|. \end{aligned}$$

3.4 $G \odot S_n$

Result 3.1. Consider the star graph $S_n = K_{1,n}$ as shown in Fig. 1. It's vertex set be $\{u, v_1, v_2, \dots, v_n\}$, where $\deg(u) = n$ and $\deg(v_i) = 1$ for $i = 1, 2, \dots, n$.

Clearly $S_1 = K_{1,1} = K_2$ is a wholly located dominated graph. Any set with cardinality n will be the RD -set of S_n . Therefore $S = \{u, v_1, v_2, \dots, v_{n-1}\}$ will be the RD -set of S_n . For $n > 1$, S_n has the RD -set S which has no wholly located dominated vertex. Hence $S_n, n > 1$ are all not wholly located dominated graph.

 Figure 1. Star Graph S_n

 Figure 2. Wheel Graph W_n

Corollary 3.4. For the graph S_n , by Theorem 2.3 and Result 3.1 we get

$$RD(G \odot S_1) = RD(S_1) |V(G)| + \gamma(G) = |V(G)| + \gamma(G)$$

and for $n > 1$,

$$RD(G \odot S_n) = RD(S_n) |V(G)| = n |V(G)|.$$

3.5 $G \odot P_n$

Result 3.2. Path graph P_2, P_3 are wholly located dominated graph while $P_n, n \neq 2, 5$ are all not wholly located dominated graph.

Corollary 3.5. For the graph P_n , combining with the fact of Result 3.2 and Theorem 2.3 we get

$$RD(G \odot P_n) = \begin{cases} RD(P_n) |V(G)| + \gamma(G), & \text{if } n = 2, 5 \\ RD(P_n) |V(G)|, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \lceil \frac{2n}{5} \rceil |V(G)| + \gamma(G), & \text{if } n = 2, 5 \\ \lceil \frac{2n}{5} \rceil |V(G)|, & \text{otherwise} \end{cases}$$

3.6 $G \odot C_n$

Result 3.3. For the cycle graph $C_n, n \geq 3$ it is obvious that $C_3 = K_3$ and C_5 are wholly located dominated graph and $C_n, n \neq 3, 5$ they are all not wholly located dominated graph.

Corollary 3.6. For the cycle $C_n, n \geq 3$

$$RD(G \odot C_n) = \begin{cases} RD(C_n) |V(G)| + \gamma(G), & \text{if } n = 3, 5 \\ RD(C_n) |V(G)|, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \lceil \frac{2n}{5} \rceil |V(G)| + \gamma(G), & \text{if } n = 3, 5 \\ \lceil \frac{2n}{5} \rceil |V(G)|, & \text{otherwise} \end{cases}$$

3.7 $G \odot W_n$

Result 3.4. $W_n, n \geq 3$ is the wheel graph whose vertex set is $\{u, v_1, v_2, \dots, v_n\}$ with $\deg(u) = n$ and $\deg(v_i) = 3$ is shown in the Fig. 2.

Graph W_3 and W_4 are obviously wholly located dominated graph. While W_5 has a RD -set $\{u, v_1, v_2\}$ which does not has a wholly located dominated vertex. Hence W_5 is not a wholly located dominated graph.

If W_6 has a RD -set containing $\{u\}$ then it must also contain two vertices from the set $\{v_1, v_2, v_3, v_4, v_5\}$ in such a way that it locate and dominates 3 vertices, so there exist a wholly located dominated vertex. If $\{u\}$ does not belongs to the RD -set, then u will be the wholly located dominated vertex. Hence in all possible RD -set there exist a wholly located dominated vertex. So W_6 is a wholly located dominated graph.

For $n > 6$, if $n \equiv 1 \pmod{5}$ that is $n = 5k + 1$ where $k \geq 2$, W_n has a RD -set $\{u, v_2, v_4, v_7, v_9, \dots, v_{5k-3}, v_{5k-1}\}$ without a wholly located dominated vertex. So W_n is not a wholly located dominated graph when $n > 6$ is $n \equiv 1 \pmod{5}$.

Similarly when $n > 6$ is $n \equiv 3 \pmod{5}$, then W_n is not a wholly located dominated graph with RD -set $\{u, v_2, v_4, v_7, v_9, \dots, v_{5k-3}, v_{5k-1}, v_{5k+1}\}$ which has no wholly located dominated vertex.

But when n is of form $n = 5(k + 1)$ or $n = 5k + 2$ or $n = 5k + 4$ where $k \geq 1$, none of the RD -set contains $\{u\}$. But as u is adjacent to all the vertices, it will definitely be the wholly located dominated vertex with respect to any RD -set. So for all $n > 6$, W_n will be the wholly located dominated graph when $n \equiv 0, 2, 4 \pmod{5}$.

Corollary 3.7. For wheel graph $W_n, n \geq 3$ by Theorem 2.3 and Result 3.4 we get

$$RD(G \odot W_n) = \begin{cases} RD(W_n) |V(G)| + \gamma(G), & \text{if } n = 3, 4, 6 \text{ or} \\ & \text{if } n > 6, \text{ and } n \equiv 0, 2, 4 \pmod{5} \\ RD(W_n) |V(G)|, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 3 |V(G)| + \gamma(G), & \text{if } n = 3, 6 \\ 2 |V(G)| + \gamma(G), & \text{if } n = 4 \\ 3 |V(G)|, & \text{if } n = 5 \\ \lceil \frac{2n}{5} \rceil |V(G)| + \gamma(G), & \text{if } n > 6 \text{ and } n \equiv 0, 2, 4 \pmod{5} \\ \lceil \frac{2n}{5} \rceil |V(G)|, & \text{if } n > 6 \text{ and } n \equiv 1, 3 \pmod{5}. \end{cases}$$

3.8 $G \odot F_{1,n}$

Result 3.5. Fan graph $F_{1,n} = K_1 + P_n$ with vertex set $\{u, v_1, v_2, \dots, v_n\}$ is shown in the Fig. 3.

Clearly $F_{1,1}, F_{1,2}, F_{1,3}$ and $F_{1,4}$ are all wholly located dominated graph. $F_{1,5}$ has an RD -set $\{u, v_3, v_4\}$ with no wholly located dominated vertex. Hence $F_{1,5}$ is not a wholly located dominated graph.

 Figure 3. Fan Graph $F_{1,n}$

For the graph $F_{1,6}$ all possible RD -set has a wholly located dominated vertex. So $F_{1,6}$ should be a wholly located dominated graph.

Similar to W_n , we can show that $F_{1,n}$ is wholly located dominated graph when $n > 6$ and $n \equiv 0, 2, 4 \pmod{5}$.

For $n > 6$, if $n \equiv 1 \pmod{5}$, that is $n = 5k + 1$ where integer $k \geq 2$, then we can find a RD -set $\{u, v_2, v_4, v_7, v_9, \dots, v_{5k-3}, v_{5k-1}\}$ with no wholly located dominated vertex. Similarly for $n > 6$, if $n \equiv 3 \pmod{5}$ then $F_{1,n}$ has an RD -set $\{u, v_2, v_4, v_7, v_9, \dots, v_{5k-3}, v_{5k-1}, v_{5k+1}\}$ with no wholly located dominated vertex. Hence $F_{1,n}$, $n > 6$ with $n \equiv 1, 3 \pmod{5}$ are all not wholly located dominated graph.

Corollary 3.8. Location domination number of the graph $G \odot F_{1,n}$ is given by

$$RD(G \odot F_{1,n}) = \begin{cases} RD(F_{1,n})|V(G)| + \gamma(G), & \text{if } n = 1, 2, 3, 4, 6 \text{ or} \\ & \text{if } n > 6 \text{ and } n \equiv 0, 2, 4 \pmod{5} \\ RD(F_{1,n})|V(G)|, & \text{otherwise} \end{cases}$$

$$= \begin{cases} |V(G)| + \gamma(G), & \text{if } n = 1 \\ 2|V(G)| + \gamma(G), & \text{if } n = 2, 3, 4 \\ 3|V(G)|, & \text{if } n = 5 \\ 3|V(G)| + \gamma(G), & \text{if } n = 6 \\ \lceil \frac{2n}{5} \rceil |V(G)| + \gamma(G), & \text{if } n > 6 \text{ and } n \equiv 0, 2, 4 \pmod{5} \\ \lceil \frac{2n}{5} \rceil |V(G)|, & \text{if } n > 6 \text{ and } n \equiv 1, 3 \pmod{5}. \end{cases}$$

Remark 3.9. By replacing graph G in the Corollary 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, with known graph we could directly find the location domination number of graph of form $G \odot K_n$, $G \odot K_{m,n}$, $G \odot \bar{K}_n$, $G \odot S_n$, $G \odot P_n$, $G \odot C_n$, $G \odot W_n$ and $G \odot F_{1,n}$.

4. Conclusion

In this paper by introducing wholly located dominated graph we have found location domination number of corona product of graphs. And we have deduced the location domination number of some general graphs like $G \odot K_n$, $G \odot K_{m,n}$, $G \odot \bar{K}_n$, $G \odot S_n$, $G \odot P_n$, $G \odot C_n$, $G \odot W_n$ and $G \odot F_{1,n}$.

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