Intuitionistic fuzzy prime ideals of near-subtraction semigroups

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Abstract

In this paper, intuitionistic fuzzy strong ideal and intuitionistic fuzzy ideal of a near-subtraction semigroup N is characterized.

Keywords

Intuitionistic fuzzy ideal, Intuitionistic fuzzy strong ideals and Near subtraction semigroup.

AMS Subject Classification

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1. Introduction

Zadeh [17] introduced the concept of fuzzy sets in 1965. The notion of fuzzy subgroup was made by Rosenfeld [13] (1971). Liu [9], (1982), introduced the notion of fuzzy ideal of a ring. The notions of fuzzy sub near-ring, fuzzy ideal and fuzzy N -subgroup of a near-ring was introduced by Salah Abou-Zaid [1] (1991).

The concept of bi-ideals was applied to near-rings in [16]. Manikantan [12] introduced the notion of fuzzy bi-ideals of near-rings and discussed some of its properties. B.M Schein [15] considered a system $(\phi; \circ, \setminus)$ as a set of functions closed under the composition " $\circ$ " of function and the subtraction " $\setminus$ " as in set theory. He showed that each subtraction semigroup is isomorphic to a difference semigroup of a function which is invertible. The problem proposed by B.M Schein on the structure of multiplication in a subtraction semigroup was discussed by B.Zelinka [18]. Jun et.al, [8] initiated the work on ideals in subtraction algebra and characterized the ideals in it. Prime and fuzzy prime ideals of subtraction algebra and near-subtraction semigroup were introduced by Dheena

et.al,[5, 6].

The concept of Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [2] as a generalization of the notion of fuzzy set. Biswas [3] applied the concept of IFSs to the groups and studied intuitionistic Fuzzy (IF) subgroups of a group. The notion of an IF ideal of a near ring is given by Jun, Kim and Yon [7]. Also Cho and Jun [4] introduced the notion of IF R -subgroups in a near-rings and analyzed some properties. Kazanci, Yamak and Yilmaz [10] introduced intuitionistic Q -fuzzy R -subgroups of near-rings. Kim [11] introduced intuitionistic Q -fuzzy semiprime ideals of semigroups.

A nonempty set X with a binary operation $' - '$ satisfying the following (i). $x - (y - x) = x$, (ii). $x - (x - y) = y - (y - x)$ and (iii). $(x - y) - z = (x - z) - y$ for all $x, y, z \in X$ is called a subtraction algebra and the following are true in any subtraction algebra. 1. $x - 0 = x$ and $0 - x = 0$. 2. $(x - y) - x = 0$. 3. $(x - y) - y = x - y$. 4. $x - (x - y) \leq y$. 5. $(x - y) - (y - x) = x - y$. 6. $x - (x - (x - y)) = x - y$. 7. $(x - y) - (z - y) \leq x - z$. 8. $x \leq y$ iff $x = y - w$ for some $w \in X$. 9. $x \leq y$ implies $x - z \leq y - z$ and $z - y \leq z - x$ for all $z \in X$. 10. $x, y \leq z$ implies $x - y = x \wedge (z - y)$. In addition with $' - '$ operation, if X satisfies the following with $' - '$ operation 1. $(X; -)$ is a subtraction algebra. 2. $(X; \cdot)$ is a semigroup and 3. $x(y - z) = xy - xz$ and $(x - y)z = xz - yz$ for all $x, y, z \in X$ is called a subtraction semigroup. It is also clear that $0x = 0$ and $x0 = 0$ for all $x \in X$. For any two nonempty subset $A, B \in X$, $AB = \{ab | a \in A, b \in B\}$. A subtraction semigroup is called a near subtraction semigroup if $x(y - z) = xy - xz$ or $(x - y)z = xz - yz$ is true. In a near-subtraction semigroup X if $x0 = 0$ for all $x \in X$, then X is said to zero-symmetric. A nonempty subset I of a subtraction

algebra X is said to be a subalgebra of X , if $x - y \in I$ for all $x, y \in I$. A nonempty subset I of a near-subtraction semigroup is called *I1*. a left ideal of X if I is a subalgebra of $(X, -)$ and $xi - x(y - i) \in I$ for all $x, y \in X$ and $i \in I$. *I2* a right ideal if I is a subalgebra of $(X, -)$ and $IX \subseteq I$. and *I3* I is an ideal if I is both left and right ideal. If N is zero-symmetric then $xi - x(y - i) \in I$ for all $x, y \in X$ and $i \in I$. if and only if $XI \subseteq I$.

In this paper, we introduce the notion of intuitionistic Q -fuzzy strong bi-ideal of a near-ring and obtain the characterization of a strong bi-ideal in terms of a intuitionistic Q -fuzzy strong bi-ideal of a near-ring.

2. Preliminaries

In what follows, let N denote a near-subtraction algebra.

Definition 2.1. A strong left ideal P of N is called a prime left ideal if $IJ \subseteq P$ implies $I \subseteq P$ or $J \subseteq P$ for all left ideals I, J .

Definition 2.2. A strong left ideal P of N is called a weakly prime left ideal if $IJ \subseteq P$ implies $I = P$ or $J = P$ for all left ideals I, J .

Definition 2.3. Let X be an non-empty fixed set. An intuitionistic fuzzy set (briefly, IFS) A is object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions $\mu_A : N \rightarrow [0, 1]$ and $\nu_A : N \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in N$ to the set A , respectively, and $\mu_A(x) + \nu_A \leq 1$ for all $x \in N$. For the sake of simplicity, we use the symbol $A = \langle \mu_A, \lambda_A \rangle$ for the IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in N \}$.

Definition 2.4. An IFS A in N is called an IF ideal of N if the following conditions are satisfied: (IF1) $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in N$, (IF2) $\mu_A(ax - a(b - x)) \geq \mu_A(x)$ and $\nu_A(ax - a(b - x)) \leq \nu_A(x)$ for all $a, b, x \in N$ and (IF3) $\mu_A(xy) \geq \mu_A(x)$ and $\nu_A(xy) \leq \nu_A(x)$ for all $x, y \in N$. If (IF1) and (IF2) are satisfied then A is an IF left ideal of N and if (IF1) and (IF3) are satisfied then A is called an IF right ideal of N .

3. Intuitionistic Fuzzy Strong ideal of Near-ring

Definition 3.1. A non-empty subset I of a near-subtraction semigroup N is called a strong ideal if

(IFS1) $x - y \in I$ for all $x \in I$ and for all $y \in N$.

(IFS2) $xi - x(y - i) \in I$ for all $x, y \in N$ and for all $i \in I$.

(IFS3) $IX \subseteq I$.

If (IFS1) and (IFS2) are true, then I is a strong left ideal of N , and (IFS1) and (IFS2) are true then I is a strong right ideal of N .

Remark 3.2. Every strong ideal of N is an ideal of N , but the converse need not be true.

Remark 3.3. A prime left ideal is always a weakly prime left ideal and the converse is not true. It is illustrated in the following example.

Example 3.4. Let $N = \{0, x, y, z\}$ be a near-subtraction semigroup with two binary operations $' - '$ and $' . '$ is defined as follows.

-	0	x	y	z
0	0	0	0	0
x	x	0	z	y
y	y	0	0	y
z	z	0	z	0

.	0	x	y	z
0	0	0	0	0
x	0	x	y	z
y	0	0	0	0
z	0	x	y	z

Here the strong left ideal $\{0, z\}$ is weakly prime left ideal but it is not a prime left ideal, because $\{0, y\} \not\subseteq \{0, z\}$ is a strong left ideal such that $\{0, y\} \cdot \{0, y\} \subseteq \{0, z\}$.

Theorem 3.5. Let A be an IFS of a near-subtraction semigroup N . Then A is an IF ideal iff the level subset $A_{(t,t')}$ is an ideal of N for all $(t, t') \in [0, 1]^2$ with $t + t' \leq 1$ whenever nonempty.

Proof. Let A be an IF ideal of a near-subtraction semigroup N . Let $(t, t') \in [0, 1]^2$ with $t + t' \leq 1$ and $x, y \in A_{(t,t')}$. Then $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t$ and $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq t'$. Thus $x - y \in A_{(t,t')}$ for all $x, y \in A_{(t,t')}$. If $x \in A_{(t,t')}$, then $\mu_A(zx - z(y - x)) \geq \mu_A(x) \geq t$ and $\nu_A(zx - z(y - x)) \leq \nu_A(x) \leq t'$. Further for $x, y \in A_{(t,t')}$, $\mu_A(xy) \geq \mu_A(x) \geq t$ and $\nu_A(xy) \leq \nu_A(x) \leq t'$, for all $x \in A_{(t,t')}$ and $y, z \in A_{(t,t')}$. Thus $A_{(t,t')}$ is an ideal for all $(t, t') \in [0, 1]^2$ with $t + t' \leq 1$ whenever nonempty. Conversely, let $x, y, z \in N$. For $t = \min\{\mu_A(x), \mu_A(y)\}$ and $t' = \max\{\nu_A(x), \nu_A(y)\}$, $x, y \in A_{(t,t')}$. As $x - y \in A_{(t,t')}$, $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\} = t$ and $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\} = t'$. Let $(t, t') = A(x)$, then $x \in A_{(t,t')}$ and $zx - z(y - x), xy \in A(x) = (\mu_A(x), \nu_A(x))$ for all $y, z \in N$. Therefore $\mu_A(zx - z(y - x)) \geq \mu_A(x) = t$, $\nu_A(zx - z(y - x)) \leq \nu_A(x) = t'$ and $\mu_A(xy) \geq \mu_A(x) = t$, $\nu_A(xy) \leq \nu_A(x) = t'$. Thus A is an IF ideal of N . $\square$

Theorem 3.6. Let A be an IFS of a near-subtraction semigroup N . Then A is an IF strong ideal iff the level subset $A_{(t,t')}$ is an ideal of N for all $(t, t') \in [0, 1]^2$ with $t + t' \leq 1$ whenever nonempty.

Proof. Let A be an IF strong ideal of a near-subtraction semigroup N . Let $(t, t') \in [0, 1]^2$ with $t + t' \leq 1$. If $x \in A_{(t,t')}$, then $\mu_A(x - y) \geq \mu_A(x) \geq t$, $\nu_A(x - y) \leq \nu_A(x) \leq t'$, $\mu_A(zx - z(y - x)) \geq \mu_A(x) \geq t$, $\nu_A(zx - z(y - x)) \leq \nu_A(x) \leq t'$, $\mu_A(xy) \geq \mu_A(x) \geq t$, $\nu_A(xy) \leq \nu_A(x) \leq t'$. Thus $x - y, zx - z(y - x), xy \in A_{(t,t')}$ for all $x \in A_{(t,t')}$ and for all $y, z \in N$. Therefore $A_{(t,t')}$ is a strong ideal of N .

Conversely, let $x, y, z \in N$. Let $(t, t') = (\mu_A(x), \nu_A(x))$. Now $x \in A_{(t, t')}$. Therefore $x - y, zx - z(y - x), xy \in A_{(t, t')}$ for all $y, z \in N$. Thus $\mu_A(x - y) \geq \mu_A(x) = t, \mu_A(zx - z(y - x)) \geq \mu_A(x) = t$ and $\mu_A(xy) \geq \mu_A(x) = t, \nu_A(x - y) \leq \nu_A(x) = t', \nu_A(zx - z(y - x)) \leq \nu_A(x) = t'$ and $\nu_A(xy) \leq \nu_A(x) = t'$. Therefore A is a IF strong ideal of N . $\square$

Corollary 3.7. Every IF strong ideal is IF ideal.

However the converse of the above corollary is not true in general as shown in the following example.

Example 3.8. Let $N = \{0, x, y, z\}$ be a near-subtraction semigroup with two binary operations $' - '$ and $' \cdot '$ is defined as follows.

-	0	x	y	z
0	0	0	0	0
x	x	0	z	y
y	y	0	0	y
z	z	0	z	0

·	0	x	y	z
0	0	0	0	0
x	0	x	x	x
y	x	0	z	y
z	0	x	y	z

$$\mu_A(a) = \begin{cases} 0.8 & \text{if } a \in \{0, x\} \\ 0.3 & \text{otherwise} \end{cases}$$

$$\nu_A(a) = \begin{cases} 0.1 & \text{if } a \in \{0, x\} \\ 0.6 & \text{otherwise} \end{cases}$$

Clearly $A = (\mu_A(x), \nu_A(x))$ is a IF right ideal of N , but not a IF right strong ideal of N , since $\mu_A(x - y = z) = 0.3 < 0.9 = \mu_A(x)$.

Definition 3.9. The product of two IFSs $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ of N is defined as

$$(A.B)(x) = \begin{cases} (\sup_{x=yz} \min\{\mu_A(y), \mu_B(z)\}, \\ \inf_{x=yz} \max\{\nu_A(y), \nu_B(z)\}) & \text{if } x = yz \\ (0, 1) & \text{otherwise} \end{cases}$$

Definition 3.10. An IF strong ideal A of a near -subtraction semigroup N is called IF prime if $C.D \subseteq A$ implies $C \subseteq A$ or $D \subseteq A$ for all IF strong ideals C, D of N .

Definition 3.11. An IF strong ideal A of a near -subtraction semigroup N is called IF weakly prime if $C.D \subseteq A$ implies $C = A$ or $D = A$ for all IF strong ideals C, D which contains N .

Any IF prime left ideals is an if weakly prime ideal. However the converse is not true, it is shown in the following example.

Example 3.12. Let us consider the N in Example 3.4 and define IFS $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B)$ as

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in \{0, z\} \\ 0.2 & \text{otherwise} \end{cases} \quad \nu_A(x) = \begin{cases} 0 & \text{if } x \in \{0, z\} \\ 0.7 & \text{otherwise} \end{cases}$$

$$\mu_B(x) = \begin{cases} 0.8 & \text{if } x \in \{0, y\} \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_B(x) = \begin{cases} 0.1 & \text{if } x \in \{0, y\} \\ 1 & \text{otherwise} \end{cases} \quad \mu_{(B.B)}(x) = \begin{cases} 1 & \text{if } x \in \{0\} \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{B.B}(x) = \begin{cases} 0 & \text{if } x \in \{0\} \\ 1 & \text{otherwise} \end{cases}$$

Here the IF strong left ideal A is IF weakly prime left ideal but not IF prime left ideal, because $B.B \subseteq A$ but $B \not\subseteq A$.

Lemma 3.13. Let A be a non constant IF weakly prime left ideal of N , then $ImA = \{(1, 0), (t, t')\}$ with $t + t' \leq 1$, where $(1, 0) > (t, t') \geq (0, 1)$.

Proof. If $ImA = \{(t_1, t'_1), (t_2, t'_2), (t_3, t'_3)\}$ for $(1, 0) > (t_1, t'_1) > (t_2, t'_2) > (t_3, t'_3) \geq (0, 1)$, then there exist $a, b, c \in N$ such that $(\mu_A(a), \nu_A(a)) = (t_1, t'_1), (\mu_A(b), \nu_A(b)) = (t_2, t'_2), (\mu_A(c), \nu_A(c)) = (t_3, t'_3)$. Choose $(s_1, s'_1), (s_2, s'_2)$ such that $(1, 0) > (s_1, s'_1) > (t_1, t'_1) > (s_2, s'_2) > (t_2, t'_2)$. Define $C(x) = (\mu_C(x), \nu_C(x))$ and $D(x) = (\mu_D(x), \nu_D(x))$ as

$$C(x) = \begin{cases} (s_1, s'_1) & \text{if } x \in A_{(t_1, t'_1)} \\ (t_2, t'_2) & \text{otherwise} \end{cases}$$

$$D(x) = \begin{cases} (t_1, t'_1) & \text{if } x \in A_{(t_1, t'_1)} \\ (s_2, s'_2) & \text{if } x \in A_{(t_2, t'_2)} - A_{(t_1, t'_1)} \\ (t_3, t'_3) & \text{otherwise} \end{cases}$$

Clearly $A \subseteq C, A \subseteq D$.

$$(D.C)(x) = \begin{cases} (t_1, t'_1) & \text{if } x = yz \quad y, z \in A_{(t_1, t'_1)} \\ (s_2, s'_2) & \text{if } x = yz, \quad y \in A_{(t_2, t'_2)} - A_{(t_1, t'_1)}, z \in A_{(t_1, t'_1)} \\ (t_2, t'_2) & \text{if } x = yz, \quad y, z \in A_{(t_2, t'_2)} - A_{(t_1, t'_1)} \\ (t_3, t'_3) & \text{if } x = yz, \quad y \in A_{(t_3, t'_3)} - A_{(t_2, t'_2)} \\ (0, 1) & \text{otherwise} \end{cases}$$

Therefore $D.C \subseteq A$, but $(\mu_C(a), \nu_C(a)) = (s_1, s'_1) > (t_1, t'_1) = (\mu_A(a), \nu_A(a))$ and $(\mu_D(a), \nu_D(a)) = (s_2, s'_2) > (t_2, t'_2) = (\mu_A(b), \nu_A(b))$. Thus $C \neq A, D \neq A$ a contradiction.

If $ImA = \{(t_1, t'_1), (t_2, t'_2)\}$ for $(1, 0) > (t_1, t'_1) > (t_2, t'_2) \geq (0, 1)$, then there exist $d, e \in N$ such that $(\mu_A(d), \nu_A(d)) = (t_1, t'_1), (\mu_A(e), \nu_A(e)) = (t_2, t'_2)$. Choose s_1, s_2 in such a way that $(1, 0) > (s_1, s'_1) > (t_1, t'_1) > (s_2, s'_2) > (t_2, t'_2)$. Define $C(x) = (\mu_C(x), \nu_C(x))$ and $D(x) = (\mu_D(x), \nu_D(x))$ as

$$C(x) = \begin{cases} (s_1, s'_1) & \text{if } x \in A_{(t_1, t'_1)} \\ (t_2, t'_2) & \text{otherwise} \end{cases}$$

$$D(x) = \begin{cases} (t_1, t'_1) & \text{if } x \in A_{(t_1, t'_1)} \\ (s_2, s'_2) & \text{otherwise} \end{cases}$$

Clearly $A \subseteq C, A \subseteq D$.

$$(D.C)(x) = \begin{cases} (t_1, t'_1) & \text{if } x = yz, \quad y, z \in A_{(t_1, t'_1)} \\ (s_2, s'_2) & \text{if } x = yz, \quad y \in A_{(t_2, t'_2)} - A_{(t_1, t'_1)}, \\ z \in A_{(t_1, t'_1)} \\ (t_2, t'_2) & \text{if } x = yz, \quad y, z \notin A_{(t_1, t'_1)} \\ (0, 1) & \text{otherwise} \end{cases}$$

Thus $D.C \subseteq A$, but $C(d) = (s_1, s'_1) > (t_1, t'_1) = A(a)$, $D(e) = (s_2, s'_2) > (t_2, t'_2) = A(b)$. Therefore $C \neq A$, $D \neq A$ a contradiction. Therefore $ImA = \{(1, 0), (t, t')\}$ with $t + t' \leq 1$, where $(1, 0) > (t, t') \geq (0, 1)$. $\square$

Lemma 3.14. *If A is a non constant IF weakly prime ideals of N , then $A_{(1,0)}$ is a prime left ideal on N .*

Proof. Let A be a non constant IF weakly prime ideal of N . Let C, D be left ideals of N such that $C.D \subseteq A_{(1,0)}, A_{(1,0)} \subseteq C$ and $A_{(1,0)} \subseteq D$. Define IFSs I, J as

$$I(x) = \begin{cases} (1, 0) & \text{if } x \in C \\ (t, t') & \text{otherwise} \end{cases} \quad J(x) = \begin{cases} (1, 0) & \text{if } x \in D \\ (t, t') & \text{otherwise} \end{cases}$$

Clearly I, J are IF strong left ideals of $N, A \subseteq I, A \subseteq J$.

$$(I.J)(x) = \begin{cases} (1, 0) & \text{if } x \in C.D \\ (t, t') & \text{if } x = yz, y \notin Corz \notin D \\ (0, 1) & \text{otherwise} \end{cases}$$

Therefore $I.J \subseteq A$ implies $I = A$ or $J = A$. Thus $C = I$ or $D = J$. Hence $A_{(1,0)}$ is a weakly prime left ideal of N . $\square$

Theorem 3.15. *Let A be a non constant IF left ideal of N . Then A is an IF weakly prime left ideal of N if and only if*
(i) $ImA = \{(1, 0), (t, t')\}$, where $(1, 0) > (t, t') \geq (0, 1)$.
(ii) $A_{(1,0)}$ is a weakly prime left ideal of N .

Proof. If A is an IF weakly prime left ideal of N , then (i), (ii) is true from Lemma 3.13 and Lemma 3.14.

Conversely, if there exist IF left ideals I, J of N containing A such that $I.J \subseteq A$ with $I \neq A$ and $J \neq A$. Therefore there exist $a, b \in N$ such that $I(a) = (s_1, s'_1) > (t, t') = A(a)$ and $J(b) = (s_2, s'_2) > (t, t') = A(b)$. Thus $a \in I_{(s_1, s'_1)}$ but $a \notin A_{(1,0)}$ and $b \in J_{(s_2, s'_2)}$ but $b \notin A_{(1,0)}$. Clearly $I_{(s_1, s'_1)}, J_{(s_2, s'_2)}$ are strong left ideals. Let $x \in A_{(1,0)}$. As $A \subseteq I, A(x) = (1, 0)$, then $I(x) = (1, 0)$. Thus $I(x) \geq (s_1, s'_1)$. Therefore $A_{(1,0)} \subseteq I_{(s_1, s'_1)}$. Similarly, $A_{(1,0)} \subseteq J_{(s_2, s'_2)}$. If $I_{(s_1, s'_1)}, J_{(s_2, s'_2)} \subseteq A_{(1,0)}$, then $I_{(s_1, s'_1)} = A_{(1,0)}$ or $A_{(1,0)} = J_{(s_2, s'_2)}$ which is a contradiction. Thus $I_{(s_1, s'_1)}, J_{(s_2, s'_2)} \not\subseteq A_{(1,0)}$. Then $cd \notin A_{(1,0)}$ for some $c \in I_{(s_1, s'_1)}$

and for some $d \in J_{(s_2, s'_2)}$. Thus $A(cd) = (t, t')$.

$(I.J)(cd) \geq I(c) \wedge J(d) = (s_1, s'_1) \wedge (s_2, s'_2) > (t, t') = A(cd)$. Which is a contradiction to $I.J \subseteq A$. Therefore A is an IF weakly prime left ideals of N . $\square$

Theorem 3.16. *Let A be a non-constant IF strong left ideal of N . Then A is an IF prime left ideal of N if and only if*
(i) $IMA = \{(1, 0), (t, t')\}$
(ii) $A_{(1,0)}$ is a prime left ideal of N .

Proof. Let A be an IF prime left ideal of N . Then A is an IF weakly prime left ideal of N . From Lemma 3.13 $IMA = \{(1, 0), (t, t')\}$, where $(1, 0) > (t, t') \geq (0, 1)$. Let C, D be strong left ideals of N such that $C.D \subseteq A_{(1,0)}$. Then χ_C, χ_D are IF strong left ideals of N .

$$(\chi_C \cdot \chi_D)(x) = \begin{cases} (1, 0) & \text{if } x \in C.D \\ (0, 1) & \text{otherwise} \end{cases}$$

Therefore $\chi_C \cdot \chi_D \subseteq A$ and so $\chi_C \subseteq A$ or $\chi_D \subseteq A$. Hence $C \subseteq A$ or $D \subseteq A$. Hence $A_{(1,0)}$ is a prime left ideals of N .

Conversely, if there exist IF left ideals I, J of N containing A such that $I.J \subseteq A$ with $I \not\subseteq A$ and $J \not\subseteq A$. Therefore there exist $a, b \in N$ such that $I(a) = (s_1, s'_1) > (t, t') = A(a)$ and $J(b) = (s_2, s'_2) > (t, t') = A(b)$. Thus $a \in I_{(s_1, s'_1)}$ but $a \notin A_{(1,0)}$ and $b \in J_{(s_2, s'_2)}$ but $b \notin A_{(1,0)}$. Clearly $I_{(s_1, s'_1)}, J_{(s_2, s'_2)}$ are strong left ideals. Let $x \in A_{(1,0)}$. As $A \subseteq I, A(x) = (1, 0)$, then $I(x) = (1, 0)$. Thus $I(x) \geq (s_1, s'_1)$. Therefore $A_{(1,0)} \subseteq I_{(s_1, s'_1)}$. Similarly, $A_{(1,0)} \subseteq J_{(s_2, s'_2)}$. If $I_{(s_1, s'_1)}, J_{(s_2, s'_2)} \subseteq A_{(1,0)}$, then $I_{(s_1, s'_1)} \subseteq A_{(1,0)}$ or $A_{(1,0)} \subseteq J_{(s_2, s'_2)}$ which is a contradiction. Thus $I_{(s_1, s'_1)}, J_{(s_2, s'_2)} \not\subseteq A_{(1,0)}$. Then $cd \notin A_{(1,0)}$ for some $c \in I_{(s_1, s'_1)}$ and for some $d \in J_{(s_2, s'_2)}$. Thus $A(cd) = (t, t')$.

$(I.J)(cd) \geq I(c) \wedge J(d) = (s_1, s'_1) \wedge (s_2, s'_2) > (t, t') = A(cd)$. Which is a contradiction to $I.J \subseteq A$. Therefore A is an IF prime left ideals of N . $\square$

4. Conclusion

In this paper the notion of Intuitionistic fuzzy ideals of a near-subtraction semigroup are presented and properties of these ideals are discussed.

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