Fuzzy contra S-irresolute continuous mappings in Šostak’s fuzzy topological spaces

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Abstract

In this article, we have studied and analysed the idea of fuzzy contra S-irresolute continuous (resp. open and closed) mappings, fuzzy contra strongly semi continuous (resp. open and closed) mappings and fuzzy contra S-homeomorphism in Šostak’s sense of fuzzy topological space. Some of their typical properties are evaluated. Further the new types of functions are illustrated and related counter examples are also defined.

Keywords


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1. Introduction and Preliminaries

After the introduction of fuzzy sets by Zadeh [18], Chang[1] was the one who initiated the idea of a fuzzy topology in a set \(X\), in which a collection, \(T\), of open sets of fuzzy subsets of \(X\). He also defined new type of fuzzy topology. In [3, 4], the authors defined the idea of gradation of openness of fuzzy subsets of \(X\). They also used the concept of fuzzy topology in the sense of Chang. In 1980 in [5], the basic idea of topology is itself fuzzy. As in 1985 by the customized work of Kubiak [8] and Šostak [13] in Lattice \(L = I\) was initiated in 1995 by [6] and further by [9] in 1997, in that topology was used to explain the two membership lattices namely \(L^\alpha\) to \(M\), where \(L\) and \(M\) are appropriate lattices. With all the basic ideas of topologies the basic lattices are fixed in [8, 13] and another significant generalizations of lattices \(L\) varies from space to space [7] is also represented. Using the developments of [5–9, 12–16], the notion of a fuzzy topology as a fuzzy subset of a power set is independently consider with the base work of [5, 8, 13]. At present authors Chattopadhyay [3] et al., Ramadan [11] and Ying [17] are also worked in this concept.

In this article, we defined a new concept of the fuzzy contra S-irresolute continuous (resp. open and closed) mapping, fuzzy contra strongly semi continuous (resp. open and closed) mapping and fuzzy contra S-homeomorphism as the base definition of Šostak and established defined some characteristic properties.

\(X, Y\) etc., denotes the non empty sets, \(I = [0, 1]\) and \(I_0 = (0, 1]\). In 1992 Ramadan [11] given the definition of smooth topological spaces (in short, st s) and the operators \(C_I : I^X \times I_0 \rightarrow I^X\) as \(C_I(\lambda, r) = \bigwedge\{\mu \in I^X : \lambda \leq \mu, \tau(1 - \mu) \geq r\}\). and \(I_I : I^X \times I_0 \rightarrow I^X\) as \(I_I(\lambda, r) = \bigvee\{\mu \in I^X : \lambda \geq \mu, \tau(\mu) \geq r\}\). Kim et. al.[10] discussed in a st s \((X, \tau, \forall r \in I_0\), \(\lambda\) is called r-fuzzy strongly semiopen (resp. r-fuzzy semiopen & r-fuzzy preopen) (r-fsso (resp. r-fso & r-fpso) for short) iff \(\lambda \leq I_I(C_I(\lambda, r), r), r\) (resp. \(\lambda \leq C_I(I_I(\lambda, r), r) \& \lambda \leq I_I(C_I(\lambda, r), r))\). The respective compliment sets are closed set. The operators r-fuzzy strongly semi-interior (resp. r-fuzzy strongly semi-closure) of \(\lambda\), denoted by \(SSC_I(\lambda, r)\) (resp. \(SSC_I(\lambda, r)\)) as \(SSC_I(\lambda, r) = \bigwedge\{\mu \in I^X : \mu \leq \lambda, \mu \in r - fsso\}\) (resp. \(SSC_I(\lambda, r) = \bigvee\{\mu \in I^X : \mu \geq \lambda, \mu \in r - fssc\}\). f : \((X, \tau) \rightarrow (Y, \eta)\) is fuzzy continuous (FCts for short) iff \(\eta(\mu) \leq \tau(f^{-1}(\mu))\) for each \(\mu \in I^Y\).
2. Fuzzy Contra S-irresolute Continuous Mapping

Definition 2.1. Let \( (X, \tau) \) and \( (Y, \eta) \) be tfs’s and let \( f : X \rightarrow Y \) be a mapping. Then \( f \) is called

1. fuzzy contra S-irresolute continuous (resp. fuzzy contra strongly semi-continuous) (FcsS-ICs resp. FcsSCts) for short) iff \( f^{-1}(\mu) \) is r-fsso for each \( \mu \in I^Y \).

2. fuzzy contra S-irresolute open (resp. fuzzy contra strongly semi-open fuzzy contra S-irresolute closed and fuzzy contra strongly semi-closed) (FcsSO FcsS-ICs and FcsSCs) for short) iff \( f \) is r-fsso for each \( \mu \in I^Y \).

3. fuzzy contra S-homeomorphism (Fcs-h, for short) iff \( f \) is bijective and both of \( f \) and \( f^{-1} \) are fuzzy contra S-irresolute continuous.

Theorem 2.2. Let \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) be a mapping. Then the below representation are equivalent.

1. \( f \) is FcsS-ICs.
2. \( \forall r \)-fssc \( \mu \in I^X, f^{-1}(\mu) \) is r-fsso.
3. \( f(SS\tau_{\tau_1}(\mu, r)) \geq SSC\tau_{\tau_2}(f(\lambda), r) \), \( \forall \lambda \in I^X \).
4. \( SS\tau_{\tau_1}(f^{-1}(\mu), r) \geq f^{-1}(SS\tau_{\tau_2}(\mu, r)) \), \( \forall \mu \in I^X \).
5. \( f^{-1}(SS\tau_{\tau_2}(\mu, r)) \geq (SS\tau_{\tau_1}(f^{-1}(\mu), r)) \), \( \forall \mu \in I^X \).

Proof. (1)\(\Rightarrow\)(2): For a r-fssc set \( \mu \in Y, f^{-1}(\tau - \mu) \) is \( f^{-1}(\tau - f(\lambda)) \). By (1) \( f^{-1}(\tau - f(\lambda)) \) is r-fsso in \( X \). \( \Rightarrow \) \( f^{-1}(\tau - \mu) \) is r-fsso set in \( X \). Thus proved (2).

(2)\(\Rightarrow\)(3): Assume \( f(SS\tau_{\tau_1}(\lambda, r)) \geq SSC\tau_{\tau_2}(f(\lambda), r) \) for a \( \lambda \in I^X \) and \( r \in I_0 \). Then \( f^{-1}(\tau - f(\lambda)) \subseteq \tau - \mu \). Hence \( f^{-1}(\tau - \mu) \) is r-fsso set in \( X \). Thus proved (4).

(3)\(\Rightarrow\)(4): \( \forall \mu \in I^X \), \( r \in I_0 \). Put \( \lambda = f^{-1}(\mu) \).

Thus \( f(SS\tau_{\tau_1}(f^{-1}(\mu), r)) \geq SSC\tau_{\tau_2}(f^{-1}(\mu), r) \).

Hence, we have \( T - SSC\tau_{\tau_1}(f^{-1}(\mu), r) \geq f^{-1}(T - SS\tau_{\tau_1}(f^{-1}(\mu), r)) \).

Thus proved (5). (Let \( \mu \) be r-fsso set of \( Y \). From Theorem ?(3), \( \mu \geq SS\tau_{\tau_2}(\mu, r) \). By (5), \( f^{-1}(SS\tau_{\tau_2}(\mu, r)) \geq SSC\tau_{\tau_1}(f^{-1}(\mu), r) \).

Thus proved (1). □

Theorem 2.3. For a \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \), the below representation are equivalent.

1. \( A \) map \( f \) is FcsSCts.
2. For each \( r \)-fc set \( \mu \in I^Y \), \( f^{-1}(\mu) \) is r-fsso in \( X \).
3. \( f(SS\tau_{\tau_1}(\lambda, r)) \geq C\tau_{\tau_2}(f(\lambda), r), \forall \lambda \in I^X \) and \( r \in I_0 \).
Theorem 2.4. For a bijective \( f : (X, \tau) \rightarrow (Y, \eta) \) the below representation are equivalent.

\( f \) is FcS-ICts.

(2) \( SSI_\eta(f(\lambda), r) \geq f(SSC_\tau(\lambda, r)), \forall \lambda \in \mathbb{I}^X \& r \in I_0. \)

Proof. (1)\( \Leftrightarrow \) (2): Let \( f \) be a FcS-ICts mapping and \( \lambda \in \mathbb{I}^X \) and \( r \in I_0. \) Then \( f^{-1}(SSI_\eta(f(\lambda), r)) \) is r-fssc in \( X. \) Since \( f \) is 1-1, using Theorem 2.2 we have,

\[ f^{-1}(SSI_\eta(\mu, r)) \geq SSC_\tau(f^{-1}(\mu), r). \]

Let \( \mu = f(\lambda). \) Then, \( f^{-1}(SSI_\eta(f(\lambda), r)) \geq SSC_\tau(f^{-1}(f(\lambda)), r) = SSC_\tau(\lambda, r). \) Again since \( f \) is onto, we have \( SSI_\eta(f(\lambda), r) = ff^{-1}(SSI_\eta(f(\lambda), r)) \geq f(SSC_\tau(\lambda, r)). \) Thus (2) is proved.

(2) \( \Rightarrow \) (1): Let \( \mu \) be r-fsso set of \( Y. \) Then by Theorem 2.15 in [10] (3), \( \mu = SSI_\eta(\mu, r). \) By (2),

\[ f(SSC_\tau(f^{-1}(\mu), r) \leq SSI_\eta(f(f^{-1}(\mu), r)) = SSI_\eta(\mu, r) = \mu \]

and \( SSC_\tau(\mu, r) = f^{-1}(f(SSC_\tau(f^{-1}(\mu), r)) \leq f^{-1}(\mu). \) Also, by Theorem 2.15 in [10](5), \( SSC_\tau(\mu, r) \geq f^{-1}(\mu) \). Thus, \( f^{-1}(\mu) = SSC_\tau(\mu, r) \). (i.e) \( f^{-1}(\mu) \) is r-fssc.

Thus (1). \( \Box \)

Theorem 2.5. Let \( (X, \tau) \) and \( (Y, \eta) \) be fts's and let \( f : X \rightarrow Y \) be a mapping. Then the below representation are equivalent.

\( f \) is called FcS-IO.

(2) \( f(SSI_\tau(\lambda, r)) \leq SSC_\eta(f(\lambda), r), \forall \lambda \in \mathbb{I}^X \& r \in I_0. \)

(3) \( SSI_\tau(f^{-1}(\mu), r) \leq f^{-1}(SSC_\eta(\mu, r)), \forall \mu \in \mathbb{I}^Y \& r \in I_0. \)

(4) For any \( \mu \in \mathbb{I}^Y \) and any r-fssc with \( f^{-1}(\mu) \leq \lambda, \exists a \text{ r-fsso } \rho \in \mathbb{I}^Y \text{ with } \mu \leq \rho \Rightarrow f^{-1}(\rho) \leq \lambda. \)

Proof. (1)\( \Rightarrow \) (2): For each \( \lambda \in \mathbb{I}^X \), since, \( SSI_\tau(\lambda, r) \leq \lambda \) from Theorem 2.5(5), we have

\[ f(SSI_\tau(\lambda, r)) \leq f(\lambda). \]

From (1), \( f(SSI_\tau(\lambda, r)) \) is r-fssc. Hence

\[ f(SSI_\tau(\lambda, r)) \leq SSC_\eta(f(\lambda), r). \]

Thus proved (2).

(2) \( \Rightarrow \) (3): For all \( \mu \in \mathbb{I}^Y \), put \( \lambda = f^{-1}(\mu) \) from (2). Then, \( f(SSI_\tau(f^{-1}(\mu), r)) \leq SSC_\eta(f(f^{-1}(\mu)), r) \leq SSC_\eta(\mu, r) \). It implies

\[ SSI_\tau(f^{-1}(\mu), r) \leq f^{-1}(SSC_\eta(\mu, r)). \]

(3) \( \Rightarrow \) (4): Let \( \lambda \) be r-fsso set of \( X \) \( \exists f^{-1}(\mu) \leq \lambda \). Since \( \overline{\lambda} = \overline{f^{-1}(\overline{\mu})} \) and \( SSI_\tau(\overline{\lambda}, r) = f^{-1}(\overline{\mu}), r = \overline{\lambda} \). \( SSI_\tau(\overline{\lambda}, r) = \overline{\lambda} \leq SSC_\eta(f^{-1}(\overline{\mu}, r)). \) From (3), \( \overline{\lambda} = SSC_\eta(f^{-1}(\overline{\mu}, r), \leq f^{-1}(SSC_\eta(\overline{\mu}, r)). \) It implies \( \lambda \geq f^{-1}(SSC_\eta(\overline{\mu}, r)) = f^{-1}(\overline{\lambda}) \).
Then, \( f^{-1}(SSI_{\eta}(\mu, r)) \) Hence \( \exists \) a r-fsso \( SSI_{\eta}(\mu, r) \in I^Y \) with \( \mu \leq SSI_{\eta}(\mu, r) \)
\[
 f^{-1}(SSI_{\eta}(\mu, r)) \leq \lambda.
\]

Thus (4) is proved.

(4) \( \Rightarrow \) (1): Let \( \omega \) be r-fsso set of \( X \). Now, we have to prove that \( f(\omega) \) is r-fss set of \( Y \). Put \( \mu = T - f(\omega) \) and \( \lambda = T - \omega \)
\[
\Rightarrow \lambda = r-fss. \quad \text{We obtain} \quad f^{-1}(\mu) = f^{-1}(T - f(\omega)) = T - f^{-1}(f(\omega)) = T - f^{-1}(f(\omega) \leq T - \omega = \lambda. \quad \text{From (4), \( \exists \) a r-fsso set} \rho \text{ with} \mu \leq \rho \ \exists
\[
 f^{-1}(\rho) \leq \lambda = T - \omega.
\]

It implies \( \omega \leq T - f^{-1}(\rho) = f^{-1}(T - \rho) \). Thus, \f(\omega) \leq f(f^{-1}(T - \rho)) = T - \rho \quad \text{(2.5)}
\[
\text{Also, since} \mu \leq \rho,\quad f(\omega) = T - \mu \geq T - \rho \quad \text{(2.6)}
\]

Hence from equations (2.5) and (2.6), we get, \( f(\omega) = T - \rho \).

(i.e) \( f(\omega) \) is r-fssc. Thus (1) is proved.

**Theorem 2.6.** Let \( (X, \tau) \) and \( (Y, \eta) \) be fts’s and let \( f : X \to Y \) be a mapping. The below representation are equivalent.

(1) \( f \) is called F c S-I closed.

(2) \( f(SSI_c(\lambda, r)) \geq SSI_{\eta}(f(\lambda), r) \), \( \forall \lambda \in I^X \) and \( r \in I_0 \).

(3) \( SSC_c(f^{-1}(\mu), r) \geq f^{-1}(SSI_{\eta}(\mu, r)) \), \( \forall \mu \in I^X \) and \( r \in I_0 \).

(4) For any \( \mu \in I^X \) and r-fsso \( \lambda \in I^X \) with \( f^{-1}(\mu) \leq \lambda \), \( \exists \) a r-fssc \( \rho \in I^Y \) with \( \mu \geq \rho \Rightarrow f^{-1}(\rho) \geq \lambda \).

**Proof.** (1) \( \Rightarrow \) (2): Let \( f \) be F c S-I closed. For each \( \lambda \in I^X \) and \( r \in I_0 \), since \( SSC_c(\lambda, r) \geq \lambda \) from Theorem 2.15 in [10](5), we have
\[
f(SSI_c(\lambda, r)) \geq f(\lambda).
\]

From (1), \( f(SSI_c(\lambda, r)) \) is r-fsso. Hence \( f(SSI_c(\lambda, r)) \geq SSI_{\eta}(f(\lambda), r) \).

Thus (2) is proved.

(2) \( \Rightarrow \) (3): For all \( \mu \in I^Y \), \( r \in I_0 \) put \( \lambda = f^{-1}(\mu) \) from (2).

Then,
\[
f(SSC_c(f^{-1}(\mu), r)) \geq SSI_{\eta}(f(f^{-1}(\mu)), r) \geq SSI_{\eta}(\mu, r).
\]

It implies \( SSC_c(f^{-1}(\mu), r) \geq f^{-1}(SSI_{\eta}(\mu, r)) \). Thus proved (3).

(3) \( \Rightarrow \) (4): Let \( \lambda \) be r-fsso set of \( X \) \( \exists f^{-1}(\mu) \geq \lambda \). Since \( T - \lambda \geq f^{-1}(T - \mu) \) and \( SSC_c(T - \lambda, r) = T - \lambda \), \( SSC_c(T - \lambda, r) \geq SSC_c(f^{-1}(T - \mu, r)) \). From (3), \( T - \lambda \geq SSC_c(f^{-1}(T - \mu, r)) \geq f^{-1}(SSI_{\eta}(T - \mu, r)) \) it implies \( \lambda \leq T - f^{-1}(SSI_{\eta}(T - \mu, r)) = f^{-1}(T - SSI_{\eta}(T - \mu, r)) \) \( \leq f^{-1}(SSI_{\eta}(\mu, r)) \). Hence \( \exists \) a r-fsso set \( SSS_{\eta}(\mu, r) \in I^X \) with \( \mu \geq SSS_{\eta}(\mu, r) \) \( \exists f^{-1}(SSI_{\eta}(\mu, r)) \geq \lambda \).

(4) \( \Rightarrow \) (1): Let \( \omega \) be r-fssc set of \( X \). Put \( \mu = T - f(\omega) \) and \( \lambda = T - \omega \geq \lambda \) is r-fsso. We obtain \( f^{-1}(\mu) = f^{-1}(T - f(\omega)) = T - f^{-1}(f(\omega)) = T - f^{-1}(f(\omega) \leq T - \omega = \lambda. \quad \text{From (4), \( \exists \) a r-fssc set} \rho \text{ with} \mu \geq \rho \ \exists
\[
f^{-1}(\rho) \geq \lambda = T - \omega.
\]

It implies \( \omega \geq T - f^{-1}(\rho) = f^{-1}(T - \rho) \). Thus, \( f(\omega) \geq f(f^{-1}(T - \rho)) = T - \rho \). \quad \text{(2.7)}

Also, since \( \mu \leq \rho,\quad f(\omega) = T - \mu \geq T - \rho \quad \text{(2.8)}
\]

Hence from (2.7) and (2.8), we have \( f(\omega) = T - \rho \). (i.e) \( f(\omega) \) is r-fsso. Thus (1).

**Theorem 2.7.** Let \( (X, \tau) \) and \( (Y, \eta) \) be fts’s and let \( f : X \to Y \) be a bijection mapping. Then the below representation hold:

(1) \( f \) is a F c S-I closed iff \( f^{-1}(SSI_{\eta}(\mu, r)) \leq SSC_{\tau}(f^{-1}(\mu), f^{-1}(r), \forall \mu \in I^X \) and \( r \in I_0 \).

(2) \( f \) is F c S-I closed iff \( f \) if F c S-I open.

**Proof.** (1) \( \Rightarrow \) (2): Let \( f \) be F c S-I closed. For each \( \lambda \in I^X \) and \( r \in I_0 \), since \( SSC_{\tau}(\lambda, r) \geq \lambda \) from Theorem 2.15 in [10](5), we have
\[
f(SSC_{\tau}(\lambda, r)) \geq f(\lambda).
\]

From (1), \( f(SSC_{\tau}(\lambda, r)) \) is r-fsso. Hence \( f(SSC_{\tau}(\lambda, r)) \geq SSI_{\eta}(f(\lambda), r) \).

Thus (2) is proved.

(2) \( \Rightarrow \) (1): For all \( \mu \in I^Y \), \( r \in I_0 \) put \( \lambda = f^{-1}(\mu) \) from (2).

Then,
\[
f(SSC_{\tau}(f^{-1}(\mu), r)) \geq SSI_{\eta}(f(f^{-1}(\mu)), r) \geq SSI_{\eta}(\lambda, r).
\]

It implies \( SSC_{\tau}(f^{-1}(\mu), r) \geq f^{-1}(SSI_{\eta}(\lambda, r)) \).

\( \Rightarrow f \) is F c S-I closed. (2) It is evident from : \( f^{-1}(SSI_{\eta}(\mu, r)) \leq SSC_{\tau}(f^{-1}(\mu), r) \Leftrightarrow f^{-1}(T - SSC_{\eta}(T - \mu, r)) \leq T - SSC_{\tau}(T - f^{-1}(\mu), r) \Leftrightarrow T - f^{-1}(SSI_{\eta}(T - \mu, r)) \leq T - SSC_{\tau}(T - \mu, r) \Leftrightarrow f^{-1}(SSI_{\eta}(T - \mu, r)) \geq SSC_{\tau}(f^{-1}(T - \mu), r) \Leftrightarrow f^{-1}(SSI_{\eta}(T - \mu, r)) \geq SSI_{\eta}(T - \mu, r). \)

Hence \( f \) is F c S-I open.

**Theorem 2.8.** Let \( f : (X, \tau) \to (Y, \eta) \) be a bijection mapping from \( \text{an fts} (X, \tau) \) into \( \text{an fts} (Y, \eta) \). Then the below representation are equivalent.

(1) \( f \) is F c S-I homeomorphism.

(2) \( f \) is F c S-I Cts and F c S-I closed.

(3) \( f \) is F c S-I Cts and F c S-I open.
(4) \( f(SSC_\tau(\lambda, r)) = SSC_\eta(f(\lambda), r), \forall \lambda \in I^X \) and \( r \in I_0 \).
(5) \( f(SSC_\tau(\lambda, r)) = SSC_\eta(f(\lambda), r), \forall \lambda \in I^X \) and \( r \in I_0 \).
(6) \( SSC_\tau(f^{-1}(\mu), r) = f^{-1}(SSC_\eta(\mu, r)), \forall \mu \in I^Y \) and \( r \in I_0 \).
(7) \( SSC_\tau(f^{-1}(\mu), r) = f^{-1}(SSC_\eta(\mu, r)), \forall \mu \in I^Y \) and \( r \in I_0 \).

**Proof.** (1)\( \Rightarrow \) (2): Since \( f \) is c S-I homeomorphism, \( f \) and \( f^{-1} \) are c S-I Cts. Let \( \mu \) be any r-fsso set in \( Y \). As \( f \) is a c S-I Cts, \( f^{-1}(\mu) \) is r-fsso set of \( X \). Also \( f^{-1} \) is a c S-I Cts implies that \( f(f^{-1}(\mu)) = \mu \) is r-fsso set in \( Y \). This implies that \( f \) is c S-I closed map.

(2)\( \Rightarrow \) (1): Let \( \mu \) be any r-fsso set of \( Y \). Since \( f \) is F S-I Cts, \( f^{-1}(\mu) \) is r-fsso set of \( Y \). Since \( f \) is F S-I Cts, \( f^{-1}(\mu) = \mu \) is r-fsso in \( X \). Thus \( f^{-1} \) is F S-I Cts.

(3)\( \Rightarrow \) (4): For each \( \lambda \in I^X \) and \( r \in I_0 \), since \( f \) is c S-I closed, using Theorem 2.6(2), we have,
\[
\bigl( f(\SSC_\tau(\lambda, r)) \bigr) \leq \SSC_\eta(f(\lambda), r).
\]

Also \( f \) is F S-I Cts, so by Theorem 2.4,
\[
\SSC_\eta(f(\lambda), r) \geq f(\SSC_\tau(\lambda, r)),
\]
\( \forall \lambda \in I^X \) and \( r \in I_0 \). Hence, \( f(\SSC_\tau(\lambda, r)) \geq SSC_\eta(f(\lambda), r) \).

(3)\( \Rightarrow \) (6): For each \( \mu \in I^Y \) and \( r \in I_0 \), since \( f \) is c S-I open, using Theorem 2.5(2), we have,
\[
\bigl( f(\SSC_\tau(\lambda, r)) \bigr) \geq \SSC_\eta(f(\lambda), r).
\]

Also \( f \) is F S-I Cts, so by Theorem 2.2, \( f(\SSC_\tau(\lambda, r)) \geq SSC_\eta(f(\lambda), r) \). Hence, \( f(\SSC_\tau(\lambda, r)) = f(\SSC_\eta(\lambda, r), \forall \lambda \in I^X \) and \( r \in I_0 \). Then from Theorem 2.4,
\[
\SSC_\eta(f(\lambda), r) \geq f(\SSC_\tau(\lambda, r)).
\]

Since \( f \) is c S-I closed, by Theorem 2.6(3),
\[
SSC_\tau(f^{-1}(\mu), r) \geq f^{-1}(SSC_\eta(\mu, r)),
\]
\( \forall \mu \in I^Y \) and \( r \in I_0 \). By putting \( \lambda = f^{-1}(\mu), \forall \mu \in I^Y \) and \( r \in I_0 \). Thus proves (7).

(3)\( \Rightarrow \) (6): For each \( \lambda \in I^X \) and \( r \in I_0 \). Then from Theorem 2.2,
\[
f(\SSC_\tau(\lambda, r)) \leq \SSC_\eta(f(\lambda), r).
\]

Since \( f \) is c S-I open, by Theorem 2.5,
\[
SSC_\tau(f^{-1}(\mu), r) \leq f^{-1}(SSC_\eta(\mu, r)),
\]
\( \forall \mu \in I^Y \) and \( r \in I_0 \). By putting \( \lambda = f^{-1}(\mu), \forall \mu \in I^Y \) and \( r \in I_0 \). Thus proves (6).

**Remark 2.9.** For a mapping \( f : X \to Y \), the below representation are valid:

(1) \( f \) is FcS-ICts \( \Rightarrow \) \( f \) is FcSSCts.
(2) \( f \) is FcS-ICts \( \Rightarrow \) \( f \) is FcCs.
(3) \( f \) is FcS-IO (resp. FcS-IC) \( \Rightarrow \) \( f \) is FcSO (resp. FcSSC).
(4) \( f \) is FcS-IO (resp. FcS-IC) \( \Rightarrow \) \( f \) is FcO (resp. FcC).

But the converses need not be true as shown by the following examples.

### 3. Example

**Example 3.1.** Consider the fs's \((X, \tau) \& (Y, \eta)\) with \( X = \{x, y, z\} \) and \( Y = \{x, y, z\} \) and
\[
\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \emptyset \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \lambda = v_1 \\ 0, & \text{otherwise} \end{cases},
\]
\( \eta(\nu) = \begin{cases} 1, & \text{if } \nu = \emptyset \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \nu = v_2 \\ 0, & \text{otherwise} \end{cases} \)
where \( v_1(x) = 0.3, v_1(y) = 0.3, v_1(z) = 0.5; v_2(x) = 0.7, v_2(y) = 0.7, v_2(z) = 0.5; v_3(x) = 0.8, v_3(y) = 0.8, v_3(z) = 0.5. \)
Then the identity function \( f : X \to Y \) is

(i) FcSSCts but not FcS-ICts because \( v_3 = \frac{1}{2} \) fsso in \((Y, \eta)\) \( \Rightarrow \) \( f^{-1}(v_3) = v_3 \) is not \( \frac{1}{2} \) fssc in \((X, \tau)\).

(ii) FcCs but not FcS-ICts because \( v_3 = \frac{1}{2} \) fsso in \((Y, \eta)\) \( \Rightarrow \) \( f^{-1}(v_3) = v_3 \) is not \( \frac{1}{2} \) fssc in \((X, \tau)\).

**Example 3.2.** Consider the fs's \((X, \tau) \& (Y, \eta)\) with \( X = \{x, y, z\} \) and \( Y = \{x, y, z\} \) and
\[
\tau(\nu) = \begin{cases} 1, & \text{if } \nu = \emptyset \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \nu = v_1 \\ 0, & \text{otherwise} \end{cases},
\]
\( \eta(\mu) = \begin{cases} 1, & \text{if } \mu = \emptyset \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \mu = v_2 \\ 0, & \text{otherwise} \end{cases} \)
where \( v_1(x) = 0.7, v_1(y) = 0.7, v_1(z) = 0.5; v_2(x) = 0.3, v_2(y) = 0.3, v_2(z) = 0.5; v_3(x) = 0.8, v_3(y) = 0.8, v_3(z) = 0.5. \)
Then the identity function \( f : X \to Y \) is FcSO but not FcS-IO because \( v_3 = \frac{1}{2} \) fsso in \((X, \tau)\) \( \Rightarrow \) \( f^{-1}(v_3) = v_3 \) is not \( \frac{1}{2} \) fssc in \((Y, \eta)\).

**Theorem 3.3.** If \( f : X \to Y \) is FcS-IO and \( g : Y \to Z \) is FcS-IC, then \( g \circ f : X \to Z \) is FcSO map.

**Proof.** Let \( \lambda \) be r-fsso set in \( X \). As \( f \) is FcSSO, \( f(\lambda) \) is r-fsso set of \( Y \). Also since \( g : Y \to Z \) is FcS-IC, \( g(f(\lambda)) \) is r-fsso in \( Z \). \( \{e\} \) \( (g \circ f)(\lambda) = g(f(\lambda)) \) is r-fsso in \( Z \). Thus \( g \circ f \) is FcSO map. \( \square \)
Theorem 3.4. If $f : X \to Y$ is $Fcs$-$IO$ and $g : Y \to Z$ is $Fcs$-$IC$, then $g \circ f : X \to Z$ is $FS$-$IO$ map.

Proof. Let $\lambda$ be $r$-$fsso$ set in $X$. As $f$ is $Fcs$-$IO$, $f(\lambda)$ is $r$-$fsso$ set of $Y$. Also since $g : Y \to Z$ is $Fcs$-$I$, $g(f(\lambda))$ is $r$-$fsso$ in $Z$. (i.e) 

$$ (g \circ f)(\lambda) = g(f(\lambda)) $$

is $r$-$fsso$ in $Z$. Thus $g \circ f$ is $FS$-$IO$ map. \hfill $\square$

Theorem 3.5. If $f : X \to Y$ is $Fc$-$C$ and $g : Y \to Z$ is $Fcs$-$SC$, then $g \circ f : X \to Z$ is $Fs$-$SC$ map.

Proof. Let $\mu$ be $r$-$fc$ set in $X$. As $f$ is $Fc$-$C$, $f(\mu)$ is $r$-$fc$ set of $Y$. Also since $g : Y \to Z$ is $Fcs$-$SC$, $g(f(\mu))$ is $r$-$fsso$ in $Z$. (i.e) $(g \circ f)(\mu) = g(f(\mu))$ is $r$-$fsso$ in $Z$. Thus $g \circ f$ is $Fs$-$SC$ map. \hfill $\square$

Theorem 3.6. If $f : X \to Y$ is $Fc$-$C$ and $g : Y \to Z$ is $Fcs$-$SO$, then $g \circ f : X \to Z$ is $Fs$-$SO$ map.

Proof. Let $\mu$ be $r$-$fc$ set in $X$. As $f$ is $Fc$-$C$, $f(\mu)$ is $r$-$fc$ set of $Y$. Also since $g : Y \to Z$ is $Fcs$-$SO$, $g(f(\mu))$ is $r$-$fsso$ in $Z$. (i.e) $(g \circ f)(\mu) = g(f(\mu))$ is $r$-$fsso$ in $Z$. Thus $g \circ f$ is $Fs$-$SO$ map. \hfill $\square$

Theorem 3.7. If $f : X \to Y$ is $Fc$-$IC$s and $g : Y \to Z$ is $FS$-$IC$s, then $g \circ f : X \to Z$ is $Fc$-$IC$s.

Proof. Let $\mu$ be $r$-$fsso$ set in $Z$. As $g$ is $FS$-$IC$s, $g^{-1}(\mu)$ is $r$-$fsso$ set of $Y$. Also since $f : Y \to Z$ is $Fc$-$IC$s, $f^{-1}(g^{-1}(\mu))$ is $r$-$fsso$ in $X$. (i.e) $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ is $r$-$fsso$ set in $X$. Thus $g \circ f$ is $Fc$-$I$.

4. Conclusion

Šostak's fuzzy topology has been recently of major interest among fuzzy topologies. The concepts of $Fc$-$IC$s, $Fcs$-$IC$s, $Fcs$-$IO$, $Fcs$-$SC$, $Fcs$-$SO$, $Fcs$-$IC$, $Fcs$-$SC$, and $Fcs$-$H$ in fuzzy topological space in Šostak's sense are introduced and studied. Some of their characteristic properties are considered. Also a comparison between these new types of functions are established and counter examples are also given. These results will help to extend the some generalized continuous mappings, compactness and hence it will help to improve smooth topological and bi-topological spaces.

References