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Fuzzy contra S-irresolute continuous mappings in $\hat{S}$ostak's fuzzy topological spaces

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Abstract

In this article, we have studied and analysed the idea of fuzzy contra S-irresolute continuous (resp. open and closed) mappings, fuzzy contra strongly semi continuous (resp. open and closed) mappings and fuzzy contra S-homeomorphism in Šostak's sense of fuzzy topological space. Some of their typical properties are evaluated. Further the new types of functions are illustrated and related counter examples are also defined.

Keywords

FcS-ICts, FcsS-Cts,FcS-IO, FcsSO, FcS-IC, FcsSC and FcS-H.

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Contents

1. Introduction and Preliminaries

After the introduction of fuzzy sets by Zadeh [18], Chang[1] was the one who initiated the idea of a fuzzy topology in a set X, in which a collection, T,of open sets of fuzzy subsets of X. He also defined new type of fuzzy topology. In [3, 4], the authors defined the idea of gradation of openness of fuzzy subsets of X. They also used the concept of fuzzy topology in the sense of Chang. In 1980 in [5],the basic idea of topology is itself fuzzy. As in 1985 by the customized work of Kubiak [8] and Šostak [13] in Lattice $L = I$ was initiated in 1995 by [6] and further by [9] in 1997, in that topology was used to explain the two membership latices namely L^X to M, where L and M are appropriate lattices. With all the basic ideas of topologies the basic lattices are fixed in [8, 13] and another significant generalizations of lattices L varies from space to space [7] is also represented. Using the developments of [5–9, 12–16], the notion of a fuzzy topology as a fuzzy subset tof powerset is independently consider with the base work of [5, 8, 13]. At present authors Chattopadhyay [3] et.al., Ramadan [11] and Ying [17] are also worked in this concept.

In this article, we defined a new concept of the fuzzy contra s-irresolute continuous (resp. open and closed) mapping, fuzzy contra strongly semi continuous (resp. open and closed) mapping and fuzzy contra s-homeomorphism as the base definition of Šostak and established defined some characteristic properties.

X, Y etc., denotes the non empty sets, $I = [0,\ 1]$ and $I_0 = (0,\ 1]$. In 1992 Ramadan [11] given the definition of smooth topological spaces (in short, sts) and the operators $C_\tau : I^X \times I_0 \to I^X$ as $C_\tau(\lambda,\ r) = \bigwedge\{\mu \in I^X : \lambda \leq \mu,\ \tau(\overline{1} - \mu) \geq r\}$. and $I_\tau : I^X \times I_0 \to I^X$ as $I_\tau(\lambda,\ r) = \bigvee\{\mu \in I^X : \lambda \geq \mu,\ \tau(\mu) \geq r\}$. Kim et. al.[10] discussed in a sts $(X,\ \tau)$, $\forall r \in I_0$. λ is called r-fuzzy strongly semiopen (resp. r-fuzzy semiopen & r-fuzzy preopen) (r-fsso (resp. r-fso & r-fpo) for short) iff $\lambda \leq I_\tau(C_\tau(I_\tau(\lambda,\ r),\ r),\ r)$ (resp. $\lambda \leq C_\tau(I_\tau(\lambda, r),\ r)$ & $\lambda \leq I_\tau(C_\tau(\lambda, r),\ r)$). The respective compliment sets are closed set. The operators r-fuzzy strongly semi-interior (resp. r-fuzzy strongly semi-closure) of λ, denoted by $SSI_\tau(\lambda,\ r)$ (resp. $SSC_\tau(\lambda,\ r)$) as $SSI_\tau(\lambda,\ r)=\bigvee\{\mu \in I^X : \mu \leq \lambda,\ \mu$ is r -fsso$\}$ (resp. $SSC_\tau(\lambda,\ r)=\bigwedge\{\mu \in I^X : \mu \geq \lambda,\ \mu$ is r -fssc$\}$). $f : (X,\ \tau) \to (Y,\ \eta)$ is fuzzy continuous (FCts for short) iff $\eta(\mu) \leq \tau(f^{-1}(\mu))$ for each $\mu \in I^Y$.

$f:(X,\ \tau)\to(Y,\ \eta)$ is fuzzy semi continuous, (resp. fuzzy irresolute and fuzzy S-irresolute continuous) (FSC (resp. FI and FS-IC) for short) iff $f^{-1}(\mu)$ is r-fso (resp. r-fso and r-fsso) set of X for each $\eta(\mu)\geq r$, (resp. μ is r-fso and r-fsso)set of Y $r\in I_0$. $f:(X,\ \tau)\to(Y,\ \eta)$ is fuzzy open(resp. fuzzy closed) (FO (resp. FC) for short) iff $\tau(\lambda)\leq\eta(f(\lambda))$ (resp. $\tau(\overline{1}-\lambda)\leq\eta(\overline{1}-f(\lambda)))$ for each $\lambda\in I^X$. $f:(X,\ \tau)\to(Y,\ \eta)$ is fuzzy S-irresolute open (resp. fuzzy strongly semi-open and fuzzy strongly semi-closed) (FS-IO (resp. FsSO and FsSCC) for short) iff $f(\mu)$ is r-fsso (resp. r-fsso and r-fssc) set of Y for each r-fsso (resp.$\tau(\mu)\geq r$ &$tau(\overline{1}-\mu)\geq r$,) set $r\in I_0$. $f:(X,\ \tau)\to(Y,\ \eta)$ is fuzzy contra continuous (FcCts, for short) if for each $\mu\in I^Y$ we have $\tau(\overline{1}-f^{-1}(\mu))\geq\eta(\mu)$. $f:(X,\ \tau)\to(Y,\ \eta)$ is fuzzy contra open(resp. fuzzy contra closed) (FcO (resp. FcC) for short) if for each $\lambda\in I^X$ we have $\eta(\overline{1}-f(\lambda))\geq\tau(\lambda)$ (resp. $\eta(f(\lambda))\geq\tau(\overline{1}-\lambda)$). The pair $(X,\ \tau)$ is called a smooth topological space (or) Šostak Fuzzy topological space (or) fuzzy topological spaces (in short, sts, sfts, fts).

2. Fuzzy Contra S-irresolute Continuous Mapping

Definition 2.1. *Let $(X,\ \tau)$ and $(Y,\ \eta)$ be fts's and let $f:X\to Y$ be a mapping. Then f is called*

(1) fuzzy contra S-irresolute continuous (resp. fuzzy contra strongly semi-continuous) (FcS-ICts (resp. FcsSCts) for short) iff $f^{-1}(\mu)$ is r-fssc set of X for each r-fsso set $\mu\in I^Y$ (resp. $\eta(\mu)\geq r$), $r\in I_0$.

(2) fuzzy contra S-irresolute open (resp. fuzzy contra strongly semi-open fuzzy contra S-irresolute closed and fuzzy contra strongly semi-closed) (FcS-IO (resp. FcsSO FcS-IC and FcsSC) for short) iff $f(\mu)$ is r-fssc (resp. r-fssc,r-fsso and r-fsso) set of Y for each r-fsso set (resp. $\tau(\mu)\geq r$, r-fssc and $\tau(\overline{1}-\mu)\geq r$), set $\mu\in I^X$ $r\in I_0$.

(3) fuzzy contra S-homeomorphism (FcS-h, for short) iff f is bijective and both of f and f^{-1} are fuzzy contra S-irresolute continuous.

Theorem 2.2. *Let $f:(X,\ \tau_1)\to(Y,\ \tau_2)$ be a mapping. Then the below representaion are equivalent.*

(1) f is FcS-ICts.

(2) $\forall$ r-fssc $\mu\in I^Y$, $f^{-1}(\mu)$ is r-fsso.

(3) $f(SSI_{\tau_1}(\lambda,\ r))\geq SSC_{\tau_2}(f(\lambda),\ r)$, $\forall\ \lambda\in I^X$ & $r\in I_0$.

(4) $SSI_{\tau_1}(f^{-1}(\mu),\ r)\geq f^{-1}(SSC_{\tau_2}(\mu,\ r))$, $\forall\ \mu\in I^Y$ & $r\in I_0$.

(5) $f^{-1}(SSI_{\tau_2}(\mu,\ r))\geq(SSC_{\tau_1}(f^{-1}(\mu),\ r)$, $\forall\ \mu\in I^Y$ & $r\in I_0$.

Proof. (1)⇒(2): For a r-fssc set in $\mu\in Y$, $f^{-1}(\overline{1}-\mu)=\overline{1}-f^{-1}(\mu)$. By (1) $f^{-1}(\overline{1}-\mu)$ is r-fssc in X. $\Rightarrow$ $\overline{1}-f^{-1}(\mu)$ is r-fssc set in X. $\Rightarrow$ $f^{-1}(\mu)$ is r-fsso set in X. Thus proved (2).

(2)⇒(3): Assume $f(SSI_{\tau_1}(\lambda,\ r))\ngeq SSC_{\tau_2}(f(\lambda),\ r)$ for a $\lambda\in I^X$ & $r\in I_0$, then $y\in Y$ and $t\in(0,1)\ni f(SSI_{\tau_1}(\lambda,\ r))(y)<t<SSC_{\tau_2}(f(\lambda),\ r)(y)$. If $f^{-1}(\{y\})=\phi$, because $f(SSI_{\tau_1}(\lambda,\ r))(y)=0$. it is a contradiction. Also $f^{-1}(\{y\})\neq\phi$ $\exists$ $x\in f^{-1}(\{y\})\ni$

$$\begin{aligned} f(SSI_{\tau_1}(\lambda,\ r))(y) &\leq (SSI_{\tau_1}(\lambda,\ r)(x) \\ &< t \\ &< SSC_{\tau_2}(f(\lambda),\ r)(f(x)) \end{aligned} \tag{2.1}$$

Since $SSC_{\tau_2}(f(\lambda),\ r)(f(x))>t$, $\exists$ r-fssc $\mu\in I^Y$ with $f(\lambda)\geq\mu\ni$

$$SSC_{\tau_2}(f(\lambda),\ r)(f(x))>\mu(f(x))>t.$$

However, $f(\lambda)\geq\mu\Rightarrow\lambda\geq f^{-1}(\mu)$. From (2), $f^{-1}(\mu)$ is r-fsso. Thus
$SSI_{\tau_1}(\lambda,\ r)(x)\geq f^{-1}(\mu)(x)=\mu(f(x))>t$. It contradicts equation 2.1. Hence $f(SSI_{\tau_1}(\lambda,r))\geq SSC_{\tau_2}(f(\lambda),r)$.

(3)⇒(4): $\forall\ \mu\in I^Y$, $r\in I_0$, put $\lambda=f^{-1}(\mu)$.
From (3), we have

$$\begin{aligned} f(SSI_{\tau_1}(f^{-1}(\mu),\ r)) &\geq SSC_{\tau_2}(f(f^{-1}(\mu)),\ r) \\ &\geq SSC_{\tau_2}(\mu,\ r). \end{aligned} \tag{2.2}$$

$$\begin{aligned} \Rightarrow SSI_{\tau_1}(f^{-1}(\mu),\ r) &\geq f^{-1}(f(SSI_{\tau_1}(f^{-1}(\mu),\ r))) \\ &\geq f^{-1}(SSC_{\tau_2}(\mu,\ r)). \end{aligned}$$

Thus $SSI_{\tau_1}(f^{-1}(\mu),\ r)\geq f^{-1}(SSC_{\tau_2}(\mu,\ r))$. Thus (4) is proved.
(4)⇒ (5): It is evident from Theorem 2.15 in [10]. $SSI_{\tau_1}(f^{-1}(\mu),\ r)\geq f^{-1}(SSC_{\tau_2}(\mu,\ r))$ We know that by Theorem 2.15 in [10],

$$SSI_{\tau}(\lambda,\ r)=\overline{1}-SSC_{\tau}(\overline{1}-\lambda,\ r).$$

Hence, we have $\overline{1}-SSC_{\tau_1}(\overline{1}-f^{-1}(\mu),\ r)\geq f^{-1}(\overline{1}-SSI_{\tau_2}(\overline{1}-\mu,\ r))$ $\overline{1}-SSC_{\tau_1}(\overline{1}-f^{-1}(\mu),\ r)\geq\overline{1}-f^{-1}(SSI_{\tau_2}(\overline{1}-\mu,\ r))$ $-SSC_{\tau_1}(f^{-1}(\overline{1}-\mu),\ r)\geq-f^{-1}(SSI_{\tau_2}(\overline{1}-\mu,\ r))$ $SSC_{\tau_1}(f^{-1}(\overline{1}-\mu),\ r)\leq f^{-1}(SSI_{\tau_2}(\overline{1}-\mu,\ r))$ $f^{-1}(SSI_{\tau_2}(\overline{1}-\mu,\ r))\geq SSC_{\tau_1}(f^{-1}(\overline{1}-\mu),\ r)$. Thus (5) is proved.

(5)⇒(1): Let μ be r-fsso set of Y. From Theorem **??**(3), $\mu=SSI_{\tau_2}(\mu,r)$. By (5), $f^{-1}(SSI_{\tau_2}(\mu,r))\geq SSC_{\tau_1}(f^{-1}(\mu),r)$ $\Rightarrow$ $f^{-1}(\mu)\geq SSC_{\tau_1}(f^{-1}(\mu),r)$ Also, by Theorem 2.15 in [10], $f^{-1}(\mu)\leq SSC_{\tau_1}(f^{-1}(\mu),r)$ Thus $f^{-1}(\mu)=SSC_{\tau_1}(f^{-1}(\mu),r)$. (i.e)$f^{-1}(\mu)$ is r-fssc set in X. $\Rightarrow$ f is FcS-ICts.
Thus proved (1). □

Theorem 2.3. *For a $f:(X,\tau_1)\to(Y,\tau_2)$, the below representation are equivalent.*

(1) A map f is FcsSCts.

(2) For each r-fc set $\mu\in I^Y$, $f^{-1}(\mu)$ is r-fsso in X.

(3) $f(SSI_{\tau_1}(\lambda,\ r))\geq C_{\tau_2}(f(\lambda),\ r)$, $\forall\ \lambda\in I^X$ and $r\in I_0$.

(4) $SSI_{\tau_1}(f^{-1}(\mu),\ r) \geq f^{-1}(C_{\tau_2}(\mu,\ r))$, $\forall\ \mu \in I^Y$ *and* $r \in I_0$.

Proof. (1)⇔(2): It is evident from Definition of r- fssc and $f^{-1}(\overline{1}-\mu) = \overline{1} - f^{-1}(\mu)$. Let μ be r- F closed in Y. Then $\overline{1}-\mu$ is r- F open in Y. By (1), $f^{-1}(\overline{1}-\mu)$ is r-fssc set in X. $\Rightarrow$ $f^{-1}(\overline{1}-\mu) = \overline{1} - f^{-1}(\mu)$ is r-fssc in X. $\Rightarrow$ $f^{-1}(\mu)$ is r-fsso in X. Thus proved (2).

(2)⇒(3): Suppose $\exists\ \lambda \in I^X$ and $r \in I_0 \ni$

$$f(SSI_{\tau_1}(\lambda,\ r)) \ngeq C_{\tau_2}(f(\lambda),\ r).$$

There exists $y \in Y$ and $t \in I_0 \ni f(SSI_{\tau_1}(\lambda,\ r))(y) < t < C_{\tau_2}(f(\lambda),\ r)(y)$. If $f^{-1}(\{y\}) = \phi$, it is a contradiction, since

$$f(SSI_{\tau_1}(\lambda,\ r))(y) = 0.$$

If $f^{-1}(\{y\}) \neq \phi$, $\exists\, x \in f^{-1}(\{y\}) \ni$

$$\begin{aligned} f(SSI_{\tau_1}(\lambda,\ r))(y) &\leq SSI_{\tau_1}(\lambda,\ r)(x) \\ &< t \\ &< C_{\tau_2}(f(\lambda),\ r)(f(x)). \end{aligned} \tag{2.3}$$

Since $C_{\tau_2}(f(\lambda),\ r)(f(x)) > t$, $\exists\ r$- F closed set $\mu \in I^Y$ with $f(\lambda) \geq \mu \ni$

$$C_{\tau_2}(f(\lambda),\ r)(f(x)) \geq \mu(f(x)) > t.$$

However,

$$f(\lambda) \geq \mu \ \ \Rightarrow \ \ \lambda \geq f^{-1}(\mu). \tag{2.4}$$

From (2), $f^{-1}(\mu)$ is r-fsso. Thus

$$SSI_{\tau_1}(\lambda, r)(x) \geq f^{-1}(\mu)(x) = \mu(f(x)) > t.$$

It is a contradiction for (2.2). Hence, $f(SSI_{\tau_1}(\lambda,\ r)) > C_{\tau_2}(f(\lambda),\ r)$. Hence (3) is proved.

(3)⇒(4): $\forall\ \mu \in I^Y$, $r \in I_0$, put $\lambda = f^{-1}(\mu)$. From (3), we have

$$f(SSI_{\tau_1}(\lambda,\ r)) \geq C_{\tau_2}(f(\lambda),\ r)$$

(i.e)

$$\begin{aligned} f(SSI_{\tau_1}(f^{-1}(\mu),\ r)) &\geq C_{\tau_2}(f(f^{-1}(\mu)),\ r) \\ &\geq C_{\tau_2}(\mu,\ r) \end{aligned}$$

It implies

$$\begin{aligned} SSI_{\tau_1}(f^{-1}(\mu),\ r) &\geq f^{-1}(f(SSI_{\tau_1}(f^{-1}(\mu),\ r))) \\ &\geq f^{-1}(C_{\tau_2}(\mu,\ r)) \end{aligned}$$

Thus proved (4).

(4)⇒(2): Let μ be r- fc in Y. Then by (4), $SSI_{\tau_1}(f^{-1}(\mu),\ r) \geq f^{-1}(\mu)$. Also, by Theorem 2.15 in [10](5), $f^{-1}(\mu) \geq SSI_{\tau_1}(f^{-1}\ (\mu),\ r)$.
Thus $f^{-1}(\mu) = SSI_{\tau_1}(f^{-1}(\mu),\ r)$. (i.e) $f^{-1}(\mu)$ is r-fsso. This proves (2). □

Theorem 2.4. *For a bijective* $f : (X,\ \tau) \to (Y,\ \eta)$ *the below representation are equivalent.*

(1) f *is FcS-ICts.*

(2) $SSI_\eta(f(\lambda),\ r) \geq f(SSC_\tau(\lambda,\ r))$, $\forall\ \lambda \in I^X$ & $r \in I_0$.

Proof. (1)⇔(2): Let f be a FcS-ICts mapping and $\lambda \in I^X$ and $r \in I_0$. Then $f^{-1}(SSI_\eta(f(\lambda), r))$ is r-fssc set in X. Since f is 1-1, using Theorem 2.2 we have,

$$f^{-1}(SSI_\eta(\mu,\ r)) \geq SSC_\tau(f^{-1}(\mu),\ r).$$

Let $\mu = f(\lambda)$. Then, $f^{-1}(SSI_\eta(f(\lambda),\ r)) \geq SSC_\tau(f^{-1}(f(\lambda)),\ r) = SSC_\tau(\lambda,\ r)$. Again since f is onto, we have $SSI_\eta(f(\lambda),\ r) = ff^{-1}(SSI_\eta(f(\lambda),\ r) \geq f(SSC_\tau(\lambda,\ r)$. Thus (2) is proved.

(2) ⇒(1): Let μ be r-fsso set of Y. Then by Theorem 2.15 in [10] (3), $\mu = SSI_\eta(\mu, r)$. By (2),

$$f(SSC_\tau(f^{-1}(\mu),\ r) \leq SSI_\eta(f(f^{-1}(\mu),\ r)) = SSI_\eta(\mu,\ r) = \mu$$

and $SSC_\tau(f^{-1}(\mu),\ r) = f^{-1}f(SSC_\tau(f^{-1}(\mu),\ r)) \leq f^{-1}(\mu)$. Also, by Theorem 2.15 in [10](5), $SSC_\tau(f^{-1}(\mu),\ r) \geq f^{-1}(\mu)$. Thus, $f^{-1}(\mu) = SSC_\tau(f^{-1}(\mu),\ r)$. (i.e) $f^{-1}(\mu)$ is r-fssc. Thus (1). □

Theorem 2.5. *Let* $(X,\ \tau)$ *and* $(Y,\ \eta)$ *be fts's and let* $f : X \to Y$ *be a mapping. Then the below representation are equivalent.*

(1) f *is called FcS-IO.*

(2) $f(SSI_\tau(\lambda,\ r)) \leq SSC_\eta(f(\lambda),\ r)$, $\forall\ \lambda \in I^X$ *and* $r \in I_0$,

(3) $SSI_\tau(f^{-1}(\mu),\ r) \leq f^{-1}(SSC_\eta(\mu,\ r))$, $\forall\ \mu \in I^Y$ *and* $r \in I_0$,

(4) For any $\mu \in I^Y$ *and any r-fssc with* $f^{-1}(\mu) \leq \lambda$, $\exists$ *a r-fsso* $\rho \in I^Y$ *with* $\mu \leq \rho \ni f^{-1}(\rho) \leq \lambda$.

Proof. (1)⇒(2): For each $\lambda \in I^X$, since, $SSI_\tau(\lambda,\ r) \leq \lambda$ from Theorem **??**(5), we have

$$f(SSI_\tau(\lambda,\ r)) \leq f(\lambda).$$

From (1), $f(SSI_\tau(\lambda,\ r))$ is r-fssc. Hence

$$f(SSI_\tau(\lambda,\ r)) \leq SSC_\eta(f(\lambda),\ r).$$

Thus proved (2).

(2)⇒(3): For all $\mu \in I^Y$, $r \in I_0$ put $\lambda = f^{-1}(\mu)$ from (2). Then, $f(SSI_\tau(f^{-1}(\mu),\ r)) \leq SSC_\eta(f(f^{-1}(\mu)),\ r) \leq SSC_\eta(\mu,\ r)$. It implies

$$SSI_\tau(f^{-1}(\mu),\ r) \leq f^{-1}(SSC_\eta(\mu,\ r)).$$

(3)⇒(4): Let λ be r-fssc set of $X \ni f^{-1}(\mu) \leq \lambda$. Since $\overline{1}-\lambda \leq f^{-1}(\overline{1}-\mu)$ and $SSI_\tau(\overline{1}-\lambda,\ r) = \overline{1}-\lambda$. $SSI_\tau(\overline{1}-\lambda,\ r) = \overline{1}-\lambda \leq SSI_\tau(f^{-1}(\overline{1}-\mu,\ r)$. From (3), $\overline{1}-\lambda \leq SSI_\tau(f^{-1}(\overline{1}-\mu,\ r) \leq f^{-1}(SSC_\eta(\overline{1}-\mu,\ r))$. It implies $\lambda \geq \overline{1}-f^{-1}(SSC_\eta(\overline{1}-\mu,\ r)) = f^{-1}(\overline{1}-SSC_\eta(\overline{1}-\mu,\ r))$ (i.e) $\lambda \geq$

$f^{-1}(SSI_\eta(\mu,\ r))$ Hence $\exists$ a r-fsso $SSI_\eta(\mu,\ r) \in I^Y$ with $\mu \leq SSI_\eta(\mu,\ r) \ni$

$$f^{-1}(SSI_\eta(\mu,\ r)) \leq \lambda.$$

Thus (4) is proved.

(4)$\Rightarrow$(1): Let ω be r-fsso set of X. Now, we have to prove that $f(\omega)$ is r-fssc set of Y. Put $\mu = \overline{1} - f(\omega)$ and $\lambda = \overline{1} - \omega \ni \lambda$ is r-fssc. We obtain $f^{-1}(\mu) = f^{-1}(\overline{1} - f(\omega)) = \overline{1} - f^{-1}(f(\omega)) \leq \overline{1} - \omega = \lambda \Rightarrow f^{-1}(\mu) \leq \lambda$. From (4), $\exists$ a r-fsso set ρ with $\mu \leq \rho \ni$

$$f^{-1}(\rho) \leq \lambda = \overline{1} - \omega.$$

It implies $\omega \leq \overline{1} - f^{-1}(\rho) = f^{-1}(\overline{1} - \rho)$. Thus,

$$f(\omega) \leq f(f^{-1}(\overline{1} - \rho)) \leq \overline{1} - \rho \tag{2.5}$$

Also, since $\mu \leq \rho$,

$$f(\omega) = \overline{1} - \mu \geq \overline{1} - \rho \tag{2.6}$$

Hence from equations (2.5) and (2.6), we get, $f(\omega) = \overline{1} - \rho$. (i.e)$f(\omega)$ is r-fssc. Thus (1) is proved. □

Theorem 2.6. *Let $(X,\ \tau)$ and $(Y,\ \eta)$ be fts's and $f : X \to Y$ be a mapping. The below representation are equivalent.*

(1) f is called F c S-I closed.

(2) $f(SSC_\tau(\lambda,\ r)) \geq SSI_\eta(f(\lambda),\ r)$, $\forall\ \lambda \in I^X$ and $r \in I_0$.

(3) $SSC_\tau(f^{-1}(\mu),\ r) \geq f^{-1}(SSI_\eta(\mu,\ r))$, $\forall\ \mu \in I^Y$ and $r \in I_0$.

(4) For any $\mu \in I^Y$ and r-fsso $\lambda \in I^X$ with $f^{-1}(\mu) \leq \lambda$, $\exists$ a r-fssc, $\rho \in I^Y$ with $\mu \geq \rho \ni f^{-1}(\rho) \geq \lambda$.

Proof. (1)$\Rightarrow$(2): Let f be F c S-I closed. For each $\lambda \in I^X$ and $r \in I_0$, since $SSC_\tau(\lambda, r) \geq \lambda$ from Theorem 2.15 in [10](5), we have

$$f(SSC_\tau(\lambda, r)) \geq f(\lambda).$$

From (1), $f(SSC_\tau(\lambda, r))$ is r-fsso. Hence

$$f(SSC_\tau(\lambda, r)) \geq SSI_\eta(f(\lambda), r).$$

Thus (2) is proved.

(2)$\Rightarrow$(3): For all $\mu \in I^Y$, $r \in I_0$ put $\lambda = f^{-1}(\mu)$ from (2). Then,

$$f(SSC_\tau(f^{-1}(\mu), r)) \geq SSI_\eta(f(f^{-1}(\mu)), r) \geq SSI_\eta(\mu, r).$$

It implies $SSC_\tau(f^{-1}(\mu), r) \geq f^{-1}(SSI_\eta(\mu, r))$. Thus proved (3) .

(3)$\Rightarrow$(4): Let λ be r-fsso set of $X \ni f^{-1}(\mu) \geq \lambda$. Since $\overline{1} - \lambda \geq f^{-1}(\overline{1} - \mu)$ and $SSC_\tau(\overline{1} - \lambda,\ r) = \overline{1} - \lambda$ $SSC_\tau(\overline{1} - \lambda,\ r) = \overline{1} - \lambda \geq SSC_\tau(f^{-1}(\overline{1} - \mu,\ r)$. From (3), $\overline{1} - \lambda \geq SSC_\tau(f^{-1}(\overline{1} - \mu),\ r) \geq f^{-1}(SSI_\eta(\overline{1} - \mu,\ r))$ It implies $\lambda \leq \overline{1} - f^{-1}(SSI_\eta(\overline{1} - \mu,\ r)) = f^{-1}(\overline{1} - SSI_\eta(\overline{1} - \mu,\ r))$ $\lambda \leq f^{-1}(SSC_\eta(\mu,\ r))$ Hence $\exists$ a r-fssc set $SSC_\eta(\mu,\ r) \in I^Y$ with $\mu \geq SSC_\eta(\mu,\ r) \ni f^{-1}(SSC_\eta(\mu,\ r)) \geq \lambda$.

(4)$\Rightarrow$(1): Let ω be r-fssc set of X. Put $\mu = \overline{1} - f(\omega)$ and $\lambda = \overline{1} - \omega \ni \lambda$ is r-fsso. We obtain $f^{-1}(\mu) = f^{-1}(\overline{1} - f(\omega)) = \overline{1} - f^{-1}(f(\omega)) \leq \overline{1} - \omega = \lambda$. From (4), $\exists$ a r-fssc set $\rho \in I^Y$ with $\mu \geq \rho \ni$

$$f^{-1}(\rho) \geq \lambda = \overline{1} - \omega.$$

It implies $\omega \geq \overline{1} - f^{-1}(\rho) = f^{-1}(\overline{1} - \rho)$. Thus,

$$f(\omega) \geq f(f^{-1}(\overline{1} - \rho)) = \overline{1} - \rho. \tag{2.7}$$

Also, since $\mu \leq \rho$,

$$f(\omega) = \overline{1} - \mu \leq \overline{1} - \rho \tag{2.8}$$

Hence from (2.7) and (2.8), we have $f(\omega) = \overline{1} - \rho$. (i.e)$f(\omega)$ is r-fsso. Thus (1). □

Theorem 2.7. *Let $(X,\ \tau)$ and $(Y,\ \eta)$ be fts's and let $f : X \to Y$ be a bijective mapping. Then the below representation hold:*

(1) f is a F c S-I closed iff $f^{-1}(SSI_\eta(\mu,\ r) \leq SSC_\tau(f^{-1}(\mu),\ r)$, $\forall\ \mu \in I^Y$ and $r \in I_0$,

(2) f is F c S-I closed iff f if F c S-I open.

Proof. (1)$\Rightarrow$: Let f be F c S-I closed. From Theorem 2.6(2), $\forall\ \lambda \in I^X$ and $r \in I_0$,

$$f(SSC_\tau(\lambda,\ r)) \geq SSI_\eta(f(\lambda),\ r).$$

For all $\mu \in I^Y$, $r \in I_0$. Put $\lambda = f^{-1}(\mu)$, since f is onto, $f(f^{-1}(\mu)) = \mu$. Thus,

$$f(SSC_\tau(f^{-1}(\mu),\ r)) \geq SSI_\tau(f(f^{-1}(\mu)),\ r) = SSI_\eta(\mu,\ r).$$

It implies

$$\begin{aligned} SSC_\tau(f^{-1}(\mu),\ r) &= f^{-1}(f(SSC_\tau(f^{-1}(\mu),\ r))) \\ &\geq f^{-1}(SSI_\eta(\mu,\ r)). \end{aligned}$$

($\Leftarrow$) Put $\mu = f(\lambda)$. Since f is injective, $f^{-1}(SSI_\eta(f(\lambda),\ r)) \leq SSC_\tau(f^{-1}(f(\lambda)),\ r) = SSC_\tau(\lambda,\ r)$. Since f is onto,

$$SSI_\eta(f(\lambda),\ r) \leq f(SSC_\tau(\lambda,\ r)).$$

$\Rightarrow$ f is F c S-I closed. (2) It is evident from : $f^{-1}(SSI_\eta(\mu,\ r)) \leq SSC_\tau(f^{-1}(\mu),\ r)$ $\Leftrightarrow$ $f^{-1}(\overline{1} - SSC_\eta(\overline{1} - \mu,\ r)) \leq \overline{1} - SSI_\tau(\overline{1} - f^{-1}(\mu),\ r)$ $\Leftrightarrow$ $\overline{1} - f^{-1}(SSC_\eta(\overline{1} - \mu,\ r)) \leq \overline{1} - SSI_\tau(f^{-1}(\overline{1} - \mu),\ r)$ $\Leftrightarrow$ $f^{-1}(SSC_\eta(\overline{1} - \mu,\ r)) \geq SSI_\tau(f^{-1}(\overline{1} - \mu),\ r)$ $\Leftrightarrow$ Hence f is F c S-I open. □

Theorem 2.8. *Let $f : (X,\ \tau) \to (Y,\ \eta)$ be a bijective mapping from an fts $(X,\ \tau)$ into an fts $(Y,\ \eta)$. Then the below representation are equivalent.*

(1) f is F c S-I homeomorphism.

(2) f is F c S-I Cts and F c S-I closed.

(3) f is F c S-I Cts and F c S-I open.

(4) $f(SSI_\tau(\lambda,\ r)) = SSC_\eta(f(\lambda),\ r)$, $\forall\ \lambda \in I^X$ and $r \in I_0$.

(5) $f(SSC_\tau(\lambda,\ r)) = SSI_\eta(f(\lambda),\ r)$, $\forall\ \lambda \in I^X$ and $r \in I_0$.

(6) $SSI_\tau(f^{-1}(\mu),\ r) = f^{-1}(SSC_\eta(\mu,\ r))$, $\forall\ \mu \in I^Y$ and $r \in I_0$.

(7) $SSC_\tau(f^{-1}(\mu),\ r) = f^{-1}(SSI_\eta(\mu,\ r))$, $\forall\ \mu \in I^Y$ and $r \in I_0$.

Proof. (1)⇒(2): Since f is c S-I homeomorphism, f and f^{-1} are c S-I Cts. Let μ be any r-fsso set in Y. As f is c S-I Cts, $f^{-1}(\mu)$ is r-fssc set of X. Also f^{-1} is c S-I Cts implies that $f(f^{-1}(\mu)) = \mu$ is r-fsso set in Y. This implies that f is c S-I closed map.

(2)⇒(1): Let μ be any r-fsso set of Y. Since f is F c S-I Cts, $f^{-1}(\mu)$ is r-fssc set of Y. Since f is F c S-I closed map, $f(f^{-1}(\mu)) = \mu$ is r-fsso in X. Thus f^{-1} is F c S-I Cts.

(2)⇔(3): It is clear from Theorem 2.7(2).

(2)⇔(5): For each $\lambda \in I^X$ and $r \in I_0$, since f is c S-I closed, using Theorem2.6(2), we have,

$$f(SSC_\tau(\lambda,\ r)) \geq SSI_\eta(f(\lambda),\ r).$$

Also f is F c S-I Cts, so by Theorem2.4,

$$SSI_\eta(f(\lambda),\ r) \geq f(SSC_\tau(\lambda,\ r)),$$

$\forall\ \lambda \in I^X$ and $r \in I_0$. Hence, $f(SSC_\tau(\lambda,\ r)) = SSI_\eta(f(\lambda),\ r)$.

(3)⇔(4): For each $\lambda \in I^X$ and $r \in I_0$, since f is c S-I open, using Theorem 2.5(2), we have,

$$f(SSI_\tau(\lambda,\ r)) \leq SSC_\eta(f(\lambda),\ r).$$

Also f is F c S-I Cts, so by Theorem 2.2, $f(SSI_\tau(\lambda,\ r)) \geq SSC_\eta(f(\lambda),\ r)$, $\forall\ \lambda \in I^X$ and $r \in I_0$. Hence, $f(SSI_\tau(\lambda,\ r)) = SSC_\eta(f(\lambda),\ r)$.

(2)⇔(7): Let f be F c S-I Cts and $\lambda \in I^X$, $r \in I_0$. Then from Theorem 2.4,

$$SSI_\eta(f(\lambda),\ r) \geq f(SSC_\tau(\lambda,\ r)).$$

Since f is c S-I closed, by Theorem 2.6(3),

$$SSC_\tau(f^{-1}(\mu),\ r) \geq f^{-1}(SSI_\eta(\mu,\ r)),$$

$\forall\ \mu \in I^Y$ and $r \in I_0$. By putting $\lambda = f^{-1}(\mu)$, $\forall\ \mu \in I^Y$ and $r \in I_0$. Thus proves (7).

(3)⇔(6): Let f be F c S-I Cts and $\lambda \in I^X$, $r \in I_0$. Then from Theorem 2.2,

$$f(SSI_\tau(\lambda,\ r)) \geq SSC_\eta(f(\lambda),\ r).$$

Since f is c S-I open, by Theorem 2.5,

$$SSI_\tau(f^{-1}(\mu),\ r) \leq f^{-1}(SSC_\eta(\mu,\ r)),$$

$\forall\ \mu \in I^Y$ and $r \in I_0$. By putting $\lambda = f^{-1}(\mu)$, $\forall\ \mu \in I^Y$ and $r \in I_0$. Thus proves (6). □

Remark 2.9. *For a mapping $f : X \to Y$, the below representation are valid:*

(1) f is FcS-ICts ⇒ f is FcsSCts.

(2) f is FcS-ICts ⇒ f is FcCts.

(3) f is FcS-IO (resp. FcS-IC) ⇒ f is FcsSO (resp. FcsSC).

(4) f is FcS-IO (resp. FcS-IC) ⇒ f is FcO (resp. FcC).

But the converses need not be true as shown by the following examples.

3. Example

Example 3.1. *Consider the fts's (X,τ) &(Y,η) with $X = Y = \{x,y,z\}$ and*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \lambda = \nu_1 \\ 0, & \text{otherwise} \end{cases} \qquad \eta(\nu) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \nu = \nu_2 \\ 0, & \text{otherwise} \end{cases}$$

where $\nu_1(x) = 0.3,\ \nu_1(y) = 0.3,\ \nu_1(z) = 0.5$; $\nu_2(x) = 0.7,\ \nu_2(y) = 0.7,\ \nu_2(z) = 0.5$; $\nu_3(x) = 0.8,\ \nu_3(y) = 0.8,\ \nu_3(z) = 0.5$. *Then the identity function $f : X \to Y$ is*

(i) FcsSCts but not FcS-ICts because ν_3 is $\frac{1}{2}$-fsso in $(Y,\ \eta)$ but $f^{-1}(\nu_3) = \nu_3$ is not $\frac{1}{2}$-fssc in $(X,\ \tau)$.

(ii) FcCts but not FcS-ICts because ν_3 is $\frac{1}{2}$-fsso in $(Y,\ \eta)$ but $f^{-1}(\nu_3) = \nu_4$ is not $\frac{1}{2}$-fssc in $(X,\ \tau)$.

Example 3.2. *Consider the fts's (X,τ) &(Y,η) with $X = Y = \{x,y,z\}$ and*

$$\tau(\nu) = \begin{cases} 1, & \text{if } \nu = \overline{0} \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \nu = \nu_1 \\ 0, & \text{otherwise} \end{cases}$$

$$\eta(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1} \\ \frac{1}{2}, & \text{if } \mu = \nu_2 \\ 0, & \text{otherwise} \end{cases}$$

$\nu_1(x) = 0.7,\ \nu_1(y) = 0.7,\ \nu_1(z) = 0.5$;
$\nu_2(x) = 0.3,\ \nu_2(y) = 0.3,\ \nu_2(z) = 0.5$;
$\nu_3(x) = 0.8,\ \nu_3(y) = 0.8,\ \nu_3(z) = 0.5$.
Then the identity function $f : X \to Y$ is FcsSO but not FcS-IO because ν_3 is $\frac{1}{2}$-fsso in $(X,\ \tau)$ but $f(\nu_3) = \nu_3$ is not $\frac{1}{2}$-fssc in $(Y,\ \eta)$.

Theorem 3.3. *If $f : X \to Y$ is FcsSO and $g : Y \to Z$ is FcS-IC, then $g \circ f : X \to Z$ is FsSO map.*

Proof. Let λ be r- fo set in X. As f is FcsSO, $f(\lambda)$ is r-fssc set of Y. Also since $g : Y \to Z$ is FcS-IC, $g(f(\lambda))$ is r-fsso in Z. (i.e) $(g \circ f)(\lambda) = g(f(\lambda))$ is r-fsso in Z. Thus $g \circ f$ is FsSO map. □

Theorem 3.4. *If $f: X \to Y$ is FcS-IO and $g: Y \to Z$ is FcS-IC, then $g \circ f: X \to Z$ is FS-IO map.*

Proof. Let λ be r-fsso set in X. As f is FcS-IO, $f(\lambda)$ is r-fssc set of Y. Also since $g: Y \to Z$ is F c S-I closed, $g(f(\lambda))$ is r-fsso in Z. (i.e)

$$(g \circ f)(\lambda) = g(f(\lambda))$$

is r-fsso in Z. Thus $g \circ f$ is FS-IO map. □

Theorem 3.5. *If $f: X \to Y$ is FcO and $g: Y \to Z$ is FcsSC, then $g \circ f: X \to Z$ is FsSO map.*

Proof. Let μ be r- fo set in X. As f is FcO, $f(\mu)$ is r-fc set of Y. Also since $g: Y \to Z$ is FcsSC, $g(f(\mu))$ is r-fsso in Z. (i.e) $(g \circ f)(\mu) = g(f(\mu))$ is r-fsso in Z. Thus $g \circ f$ is FsSO map. □

Theorem 3.6. *If $f: X \to Y$ is FcC and $g: Y \to Z$ is FcsSO, then $g \circ f: X \to Z$ is FsSC map.*

Proof. Let μ be r- fc set in X. As f is FcC, $f(\mu)$ is r- fo set of Y. Also since $g: Y \to Z$ is FcsSO, $g(f(\mu))$ is r-fssc in Z. (i.e) $(g \circ f)(\mu) = g(f(\mu))$ is r-fssc in Z. Thus $g \circ f$ is FsSC map. □

Theorem 3.7. *If $f: X \to Y$ is FcS-ICts and $: Y \to Zg$ is FS-ICts, then $g \circ f: X \to Z$ is FcS-ICts.*

Proof. Let μ be r-fsso set in Z. As g is FS-ICts, $g^{-1}(\mu)$ is r-fsso set of Y. Also since $f: Y \to Z$ is FcS-ICts, $f^{-1}(g^{-1}(\mu))$ is r-fssc in X. (i.e) $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ is r-fssc set in X. Thus $g \circ f$ is FcS-I. □

4. Conclusion

Šostak's fuzzy topology has been recently of major interest among fuzzy topologies. The concepts of FcS-ICts, FcsS-Cts,FcS-IO, FcsSO, FcS-IC, FcsSC and FcS-H in fuzzy topological space in Šostak's sense are introduced and studied. Some of their characteristic properties are considered. Also a comparison between these new types of functions are established and counter examples are also given. These results will help to extend the some generalized continuous mappings, compactness and hence it will help to improve smooth topological and bi-topological spaces.

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