Malaya Journal of Matematik, Vol. S, No. 1, 234-242, 2019

https://doi.org/10.26637/MJM0S01/0045

Fuzzy M-open sets in $\hat{S}$ostak's fuzzy topological spaces

B. Vijayalakshmi[1]*, S. Bamini[2], M. Saraswathi[3], and A. Vadivel[4]

Abstract

We introduce the concept of ι-fuzzy θ-open (resp. ι-fuzzy θ-closed), ι-fuzzy M-open (resp. ι-fuzzy M-closed) sets in fuzzy topological spaces in the sense of $\hat{S}$ostak's. Also, we introduce ι-fuzzy θ-interior (resp. ι-fuzzy θ-closure), ι-fuzzy M-interior (resp. ι-fuzzy M-closure) and investigate some of their properties. Moreover, we investigate the relationships between ι-fuzzy open, ι-fuzzy θ-semiopen, ι-fuzzy θ-open, ι-fuzzy δ-semiopen, ι-fuzzy δ-preopen, ι-fuzzy a-open, ι-fuzzy e-open and ι-fuzzy e^*-open in $\hat{S}$ostak's fuzzy topological spaces.

Keywords

ι-fuzzy M-open, ι-fuzzy M-closed, ι-fuzzy M-interior, ι-fuzzy M-closure, ι-fuzzy θ-interior, ι-fuzzy θ-closure, ι-fuzzy θ-semiopen, ι-fuzzy θ-open, ι-fuzzy δ-semiopen, ι-fuzzy δ-preopen, ι-fuzzy a-open, ι-fuzzy e-open and ι-fuzzy e^*-open.

AMS Subject Classification

54A40, 54C05, 03E72.

[1] *Department of Mathematics, Government Arts College, Chidambaram -608102; Mathematics Section (FEAT), Annamalai University, Annamalai Nagar - 608002, India.*

[2, 3] *Department of Mathematics, Kandaswamy Kandara's College, P-Velur, India-638182.*

[4] *Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002; Department of Mathematics, Government Arts College(Autonomous), Karur-639005, India.*

***Corresponding author**: [1]mathvijaya2006au@gmail.com; [4]avmaths@gmail.com

Article History: Received **22** November **2018**; Accepted **09** May **2019**

Contents

1. Introduction

$\hat{S}$ostak [27] introduced the fuzzy topology as an extension of Chang's fuzzy topology [5]. It has been developed in many directions [12, 13, 25]. Weaker forms of fuzzy continuity between fuzzy topological spaces have been considered by many authors [2, 3, 6, 9, 11, 19, 22] using the concepts of fuzzy semi-open sets [2], fuzzy regular open sets [2], fuzzy preopen sets, fuzzy strongly semiopen sets [3], fuzzy γ-open sets [11], fuzzy δ-semiopen sets [1], fuzzy δ-preopen sets [1], fuzzy semi δ-preopen sets [29] and fuzzy e-open sets [26]. Ganguly and Saha [10] introduced the notions of fuzzy δ-cluster points in Chang's [5]fuzzy topological spaces. Kim and Park [14] introduced r-δ-cluster points and δ-closure operators in $\hat{S}$ostak's fuzzy topological spaces. It is a good extension of the notions of Ganguly and Saha [10]. Park et al. [20] introduced the fuzzy semi-preopen. In 2008, the initiations of e-open sets, e^*-open sets and a-open sets in topological spaces are due to Erdal Ekici[[7],[8]]. Sobana et al. [28] defined r-fuzzy e-open sets in $\hat{S}$ostak's fuzzy topological space. Vadivel et al. [30] introduced r-fuzzy e^*-open sets in $\hat{S}$ostak's fuzzy topological space. In 1968, Velicko studied θ-open sets [31] and δ-open sets for the purpuse of investigating the characherizations of H-closed topological spaces. semi-open set [17] were initiated by Levine in 1963. In 1993, Raychaudhuri and Mukherjee defined δ-preopen sets [24]. In 1997, δ-semiopen sets was obtained by Park [21] and θ-semi-open sets were obtained by Caldas in 2008 [4]. Shafei introduced fuzzy θ-closed [32] and fuzzy θ-open sets in 2006. Maghrabi et al.[18] introduced the notion of M-open sets in topological spaces in 2011.

In this paper, we introduce the concept of ι-fuzzy θ-

open (resp. ι-fuzzy θ-closed),ι-fuzzy M-open (resp. ι-fuzzy M-closed) sets in fuzzy topological spaces in the sense of $\hat{S}$ostak's. Also, we introduce ι-fuzzy θ-interior (resp. ι-fuzzy θ-closure), ι-fuzzy M-interior (resp. ι-fuzzy M-closure) and investigate some of their properties. Moreover, we investigate the relationships between ι-fuzzy open, ι-fuzzy θ-semiopen, ι-fuzzy θ-open, ι-fuzzy δ-semiopen, ι-fuzzy δ-preopen, ι-fuzzy a-open, ι-fuzzy e-open and ι-fuzzy e^*-open in $\hat{S}$ostak's fuzzy topological spaces.

2. Preliminaries

Throughout this article, we denote nonempty sets by X, Y etc., $I = [0,\ 1]$ and $I_0 = (0,\ 1]$. For $\alpha \in I$, $\overline{\alpha}(x) = \alpha$, $\forall x \in X$. A fuzzy point x_t for $t \in I_0$ is an element of I^X such that $x_t(y) = \begin{cases} t & \text{if y is equal to x} \\ 0 & \text{if y is not equal to x.} \end{cases}$
Let $Pt(X)$ denote the set of all fuzzy points in X. A fuzzy point $x_t \in \mu$ iff $t < \mu(x)$. $\mu \in I^X$ is quasi-coincident with ν, denoted by $\mu q \nu$, if $\exists\, x \in X$ such that $\mu(x) + \nu(x) > 1$.
If μ is not quasi-coincident with ν, we denoted $\mu \bar{q} \nu$. If A is a subset of X, we define the characteristic function χ_A on X by $\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$ All notations and definitions will be standard in the fuzzy set theory.

Lemma 2.1. *[27] Consider X be a nonempty set and $\mu,\ \nu \in I^X$. Then*

(i) $\mu q \nu$ *iff there exists* $x_t \in \mu$ *such that* $x_t q \nu$.

(ii) $\mu q \nu$, *then* $\mu \wedge \nu \neq \overline{0}$.

(iii) $\mu \bar{q} \nu$ *iff* $\mu \leq \overline{1} - \nu$.

(iv) $\mu \leq \nu$ *iff* $x_t \in \mu$ *implies* $x_t \in \nu$ *iff* $x_t q \mu$ *implies* $x_t q \nu$ *implies* $x_t \bar{q} \mu$.

(v) $x_t \bar{q} \bigvee_{i \in \mu} \nu_i$ *iff there exists* $i_0 \in \mu$ *such that* $x_t \bar{q} \nu_{i_0}$.

Definition 2.2. *[27] A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:*

(1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$,

(2) $\tau(\bigvee_{i \in \Gamma} \nu_i) \geq \bigwedge_{i \in \Gamma} \tau(\nu_i)$, *for any* $\{\nu_i\}_{i \in \Gamma} \subset I^X$,

(3) $\tau(\nu_1 \wedge \nu_2) \geq \tau(\nu_1) \wedge \tau(\nu_2)$, *for any* $\nu_1,\ \nu_2 \in I^X$.

The pair $(X,\ \tau)$ is called a fuzzy topological space or $\hat{S}$ostak's fuzzy topological space or smooth topological space (for short, fts, sfts, sts).

Remark 2.3. *[23] Let $(X,\ \tau)$ be a sfts. Then, for every $\iota \in I_0$, $\tau_r = \{\nu \in I^X : \tau(\nu) \geq \iota\}$ is a Change's fuzzy topology on X.*

Theorem 2.4. *[25] Let $(X,\ \tau)$ be a sfts. Then for each $\mu \in I^X$, $\iota \in I_0$, we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows:*
$C_\tau(\mu,\ \iota) = \bigwedge\{\nu \in I^X : \mu \leq \nu,\ \tau(\overline{1} - \nu) \geq \iota\}$.
For $\mu,\ \nu \in I^X$ and $\iota, s \in I_0$, the operator C_τ satisfies the following conditions:

(1) $C_\tau(\overline{0}, \iota) = \overline{0}$,

(2) $\mu \leq C_\tau(\mu,\ \iota)$,

(3) $C_\tau(\mu,\ \iota) \vee C_\tau(\nu,\ \iota) = C_\tau(\mu \vee \nu,\ \iota)$,

(4) $C_\tau(\mu,\ \iota) \leq C_\tau(\mu,\ s)$ *if* $\iota \leq s$,

(5) $C_\tau(C_\tau(\mu,\ \iota),\ \iota) = C_\tau(\mu,\ \iota)$.

Theorem 2.5. *[25] Let $(X,\ \tau)$ be a sfts. Then for each $\iota \in I_0$, $\mu \in I^X$ we define an operator $I_\tau : I^X \times I_0 \to I^X$ as follows:*
$I_\tau(\mu,\ \iota) = \bigvee\{\nu \in I^X : \mu \geq \nu,\ \tau(\nu) \geq \iota\}$.
For $\mu,\ \nu \in I^X$ and $\iota, s \in I_0$, the operator I_τ satisfies the following conditions:

(1) $I_\tau(\overline{1}, \iota) = \overline{1}$,

(2) $\mu \geq I_\tau(\mu,\ \iota)$,

(3) $I_\tau(\mu,\ \iota) \wedge I_\tau(\nu,\ \iota) = I_\tau(\mu \wedge \nu,\ \iota)$,

(4) $I_\tau(\mu,\ \iota) \leq I_\tau(\mu,\ s)$ *if* $s \leq \iota$,

(5) $I_\tau(I_\tau(\mu,\ \iota),\ \iota) = I_\tau(\mu,\ \iota)$,

(6) $I_\tau(\overline{1} - \mu, \iota) = \overline{1} - C_\tau(\mu,\ \iota)$ *and* $C_\tau(\overline{1} - \mu, \iota) = \overline{1} - I_\tau(\mu,\ \iota)$

Definition 2.6. *[15] Let $(X,\ \tau)$ be a sfts. Then for each $\nu \in I^X$, $x_t \in P_t(X)$ and $\iota \in I_0$, ν is called*

(i) ι-open Q_τ-neighbourhood of x_t if $x_t q \nu$ with $\tau(\nu) \geq \iota$.

(ii) ι-open R_τ-neighbourhood of x_t if $x_t q \nu$ with $\nu = I_\tau(C_\tau(\mu,\ \iota),\ \iota)$.

We denote $Q_\tau(x_t,\ \iota) = \{\nu \in I^X : x_t q \nu, \tau(\nu) \geq \iota\}$, $R_\tau(x_t,\ \iota) = \{\nu \in I^X : x_t q \nu = I_\tau(C_\tau(\mu,\ \iota),\ \iota)\}$.

Definition 2.7. *[15] Let $(X,\ \tau)$ be a sfts. Then for each $\mu \in I^X$, $x_t \in P_t(X)$ and $\iota \in I_0$, x_t is called*

(i) ι-τ cluster point of μ if for every $\nu \in Q_\tau(x_t,\ \iota)$, we have $\nu q \mu$.

(ii) ι-δ cluster point of μ if for every $\nu \in R_\tau(x_t,\ \iota)$, we have $\nu q \mu$.

(iii) An δ-closure operator is a mapping $D_\tau : I^X \times I \to I^X$ defined as follows: $\delta C_\tau(\mu,\ \iota)$ or $D_\tau(\mu,\ \iota) = \bigvee\{x_t \in P_t(X) : x_t$ is r-δ-cluster point of $\mu\}$

Definition 2.8. *Let $(X,\ \tau)$ be a sfts. For $\mu,\ \nu \in I^X$ and $\iota \in I_0$, μ is called an*

(i) ι-fuzzy δ-semiopen (resp. ι-fuzzy δ-semiclosed)[28] set if $\mu \leq C_\tau(\delta I_\tau(\mu,\ \iota),\ \iota)$
(resp. $I_\tau(\delta C_\tau(\mu,\ \iota),\ \iota) \leq \mu$).

(ii) *ι-fuzzy δ-preopen (resp. ι-fuzzy δ-preclosed)[28] set if $\mu \leq I_\tau(\delta C_\tau(\mu,\ \iota),\ \iota)$ (resp. $C_\tau(\delta I_\tau(\mu,\ \iota),\ \iota) \leq \mu$).*

(iii) *ι-fuzzy a-open (resp. ι-fuzzy a-closed)[28] set if $\mu \leq I_\tau(C_\tau(\delta I_\tau(\mu,\ \iota),\ \iota),\ \iota)$ (resp. $C_\tau(I_\tau(\delta C_\tau(\mu,\ \iota),\ \iota),\ \iota) \leq \mu$).*

(iv) *ι-fuzzy e-open (resp. ι-fuzzy e-closed)[28] set if $\mu \leq C_\tau(\delta I_\tau(\mu,\ \iota),\ \iota) \vee I_\tau(\delta C_\tau(\mu,\ \iota),\ \iota)$ (resp. $C_\tau(\delta I_\tau(\mu,\ \iota),\ \iota) \wedge I_\tau(\delta C_\tau(\mu,\ \iota),\ \iota) \leq \mu$).*

(v) *ι-fuzzy e^*-open (resp. ι-fuzzy e^*-closed)[30] set if $\mu \leq C_\tau(I_\tau(\delta C_\tau(\mu,\ \iota),\ \iota),\ \iota)$ (resp. $I_\tau(C_\tau(\delta I_\tau(\mu,\ \iota),\ \iota),\ \iota) \leq \mu$).*

(vi) *ι-fuzzy semiopen (resp. ι-fuzzy semi-closed) [16] set if $\mu \leq C_\tau(I_\tau(\mu,\ \iota),\ \iota)$ (resp. $I_\tau(C_\tau(\mu,\ \iota),\ \iota) \leq \mu$).*

3. ι-fuzzy θ-open and ι-fuzzy θ-closed sets

In the following section, we introduce the concepts of ι-fuzzy θ-interior (resp. ι-fuzzy θ-closure), ι-fuzzy θ-open (resp. ι-fuzzy θ-closed), ι-fuzzy θ-semiopen (resp. ι-fuzzy θ-semiclosed), ι-fuzzy θ-preopen (resp. ι-fuzzy θ-preclosed) sets to the sfts.

Definition 3.1. (i) *ι-fuzzy θ-interior (resp. ι-fuzzy θ-semi-interior and ι-fuzzy θ-pre-interior) of a subset μ in a sfts $(X,\ \tau)$, $\forall\ \iota \in I_0$, denoted by $\theta I_\tau(\mu,\iota)$ (resp. $\theta sI_\tau(\mu,\iota)$ and $\theta pI_\tau(\mu,\iota)$) defined as $\theta I_\tau(\mu,\iota) = \bigvee\{I_\tau(\nu) : \mu \geq \nu,\ \tau(\overline{1}-\nu) \geq \iota\}$; $\theta sI_\tau(\mu,\iota) = \bigvee\{sI_\tau(\nu) : \mu \geq \nu,\ \nu$ is r-fsc$\}$; $\theta pI_\tau(\mu,\iota) = \bigvee\{pI_\tau(\nu) : \mu \geq \nu,\ \nu$ is r-fpc $\}$.*

(ii) *ι-fuzzy θ-closure (resp. ι-fuzzy θ-semi-closure and ι-fuzzy θ-pre-closure) of a subset μ in a sfts $(X,\ \tau)$, $\forall\ \iota \in I_0$, denoted by $\theta C_\tau(\mu,\iota)$ (resp. $\theta sC_\tau(\mu,\iota)$ and $\theta pC_\tau(\mu,\iota)$) defined as $\theta C_\tau(\mu,\iota) = \bigwedge\{C_\tau(\nu) : \mu \leq \nu,\ \tau(\nu) \geq \iota\}$; $\theta sC_\tau(\mu,\iota) = \bigwedge\{sC_\tau(\nu) : \mu \leq \nu,\ \nu$ is r-fso $\}$; $\theta pC_\tau(\mu,\iota) = \bigwedge\{pC_\tau(\nu) : \mu \leq \nu,\ \nu$ is r-fpo $\}$.*

Definition 3.2. *Let $(X,\ \tau)$ be a sfts. For $\mu,\ \nu \in I^X$ and $\iota \in I_0$, μ is called an*

(i) *ι-fuzzy θ-open (resp. ι-fuzzy θ-closed) set if $\mu = \theta I_\tau(\mu,\iota)$ (resp. $\mu = \theta C_\tau(\mu,\iota)$).*

(ii) *ι-fuzzy θ-semiopen (resp. ι-fuzzy θ-semiclosed) set if $\mu \leq C_\tau(\theta I_\tau(\mu,\iota),\iota)$ (resp. $I_\tau(\theta C_\tau(\mu,\iota),\iota) \leq \mu$).*

(iii) *ι-fuzzy θ-preopen (resp. ι-fuzzy θ-preclosed) set if $\mu \leq I_\tau(\theta C_\tau(\mu,\iota),\iota)$ (resp. $C_\tau(\theta I_\tau(\mu,\iota),\iota) \leq \mu$).*

The family of all ι-fuzzy θ-open (resp. ι-fuzzy θ-closed), ι-fuzzy θ-semiopen, (resp. ι-fuzzy θ-semiclosed), ι-fuzzy θ-preopen (resp. ι-fuzzy θ-preclosed) sets will be denoted by ι-fθo (resp. ι-fθc), ι-fθso(resp. ι-fθsc), ι-fθpo(resp. ι-fθpc) sets.

Lemma 3.3. *Let $\mu,\ \nu \in I^X$ and $\iota \in I_0$ in a sfts $(X,\ \tau)$, then*

(i) *μ is ι-fuzzy θ-open iff $\mu = \theta I_\tau(\mu,\iota)$.*

(ii) *If $\mu < \nu$, then $\theta I_\tau(\mu,\iota) < \theta I_\tau(\nu,\iota)$.*

(iii) *$\theta I_\tau(\theta I_\tau(\mu,\iota)) < \theta I_\tau(\mu,\iota)$.*

(iv) *For any subset μ of X, $\mu \leq C_\tau(\mu,\iota) \leq \delta C_\tau(\mu,\iota) \leq \theta C_\tau(\mu,\iota)$ (resp. $\theta I_\tau(\mu,\iota) \leq \delta I_\tau(\mu,\iota) \leq I_\tau(\mu,\iota) \leq \mu$.*

(v) *$\overline{1} - (\theta I_\tau(\mu,\iota)) = \theta C_\tau(\overline{1}-\mu,\iota)$.*

(vi) *$\overline{1} - (\theta C_\tau(\mu,\iota)) = \theta I_\tau(\overline{1}-\mu,\iota)$.*

(vii) *$\theta I_\tau(\mu \wedge \nu,\iota) = \theta I_\tau(\mu,\iota) \wedge \theta I_\tau(\nu,\iota)$ and $\theta I_\tau(\mu,\iota) \vee \theta I_\tau(\nu,\iota) < \theta I_\tau(\mu \vee \nu,\iota)$.*

(viii) *$\theta C_\tau(\mu \wedge \nu,\iota) = \theta C_\tau(\mu,\iota) \wedge \theta C_\tau(\nu,\iota)$ and $\theta C_\tau(\mu \vee \nu,\iota) = \theta C_\tau(\mu,\iota) \vee \theta C_\tau(\nu,\iota)$.*

Proposition 3.4. *Let $\mu \in I^X$ and $\iota \in I_0$ in a sfts $(X,\ \tau)$, then*

(i) *$\theta sC_\tau(\mu,\iota) = \mu \vee I_\tau(\theta C_\tau(\mu,\iota),\iota)$ and $\theta sI_\tau(\mu,\iota) = \mu \wedge C_\tau(\theta I_\tau(\mu,\iota),\iota)$.*

(ii) *$\delta pC_\tau(\mu,\iota) = \mu \vee C_\tau(\delta I_\tau(\mu,\iota),\iota)$ and $\delta pI_\tau(\mu,\iota) = \mu \wedge I_\tau(\delta C_\tau(\mu,\iota),\iota)$.*

(iii) *$\overline{1} - (\delta I_\tau(\mu,\iota)) = \delta C_\tau(\overline{1}-\mu,\iota)$ and $\delta I_\tau(\overline{1}-\mu,\iota) = \overline{1} - \delta C_\tau(\mu,\iota)$*

Lemma 3.5. *Let $\nu \in I^X$ and $\iota \in I_0$ in a sfts $(X,\ \tau)$, then*

(i) *$\delta pC_\tau(\nu,\iota) = \nu \vee C_\tau(\delta I_\tau(\nu,\iota),\iota)$ and $\delta pI_\tau(\nu,\iota) = \nu \wedge I_\tau(\delta C_\tau(\nu,\iota),\iota)$.*

(ii) *$\delta pC_\tau(\delta pI_\tau(\nu,\iota),\iota) = \delta pI_\tau(\nu,\iota) \vee C_\tau(\delta I_\tau(\nu,\iota),\iota)$*

(iii) *$\delta pI_\tau(\delta pC_\tau(\nu,\iota),\iota) = \delta pC_\tau(\nu,\iota) \wedge I_\tau(\delta C_\tau(\nu,\iota),\iota)$.*

(iv) *$\delta sI_\tau(\nu,\iota) = \nu \wedge C_\tau(\delta I_\tau(\nu,\iota),\iota)$ and $\delta sC_\tau(\nu,\iota) = \nu \vee I_\tau(\delta C_\tau(\nu,\iota),\iota)$.*

Lemma 3.6. *Let $\nu \in I^X$ and $\iota \in I_0$ in a sfts $(X,\ \tau)$, then*

(i) *$C_\tau(\delta I_\tau(\nu,\iota),\iota) = \delta C_\tau(\delta I_\tau(\nu,\iota),\iota)$.*

(ii) *$I_\tau(\delta C_\tau(\nu,\iota),\iota) = \delta I_\tau(\delta C_\tau(\nu,\iota),\iota)$.*

Definition 3.7. *A subset μ in a sfts $(X,\ \tau)$ is called a ι-fuzzy locally closed set $\forall\ \iota \in I_0$, if $\mu = \nu \bigwedge \alpha$, where $\tau(\nu) \geq \iota$, α is ι-fuzzy closed in X.*

Definition 3.8. *Let $(X,\ \tau)$ be a sfts. For $\mu \in I^X$ and $\iota \in I_0$, then A sfts $(X,\ \tau)$ is ι-fuzzy extremely disconnected (briefly, ι-FED) if the ι-fuzzy closure of every ι-fuzzy open set of X is ι-fuzzy open.*

4. ι-fuzzy M-open and ι-fuzzy M-closed sets

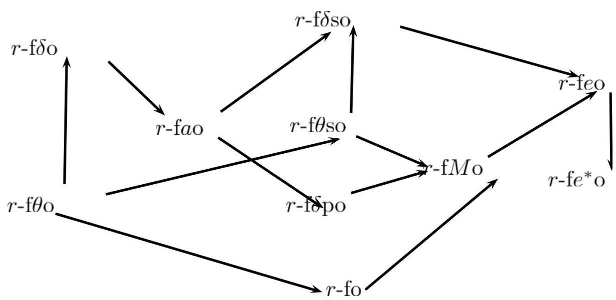
Definition 4.1. *A subset $\mu \in I^X$, & $\forall\ \iota \in I_0$ in a sfts $(X,\ \tau)$ is called an ι-fuzzy*

(i) *M-open set if $\mu \leq C_\tau(\theta I_\tau(\mu,\iota),\iota) \vee I_\tau(\delta C_\tau(\mu,\iota),\iota)$.*

(ii) *M-closed set if $\mu \geq I_\tau(\theta C_\tau(\mu,\iota),\iota) \wedge C_\tau(\delta I_\tau(\mu,\iota),\iota)$.*

Definition 4.2. *ι-fuzzy M-interior (resp. ι-fuzzy M-closure) of μ in a sfts $(X,\ \tau)$. $\forall\ \iota \in I_0$, denoted by $MI_\tau(\mu,\iota)$ (resp. $MC_\tau(\mu,\iota)$) defined as $MI_\tau(\mu,\iota) = \bigvee\{\nu \in I^X : \mu \geq \nu,\ \nu$ is a ι-fMo set$\}$; $MC_\tau(\mu,\iota) = \bigwedge\{\nu \in I^X : \mu \leq \nu,\ \nu$ is a ι-fMc set$\}$.*

Remark 4.3. *The implications are true $\forall\ \mu$ in a sfts X and $\iota \in I_0$.*



where ι-fo,ι-fθso, ι-fθsc, ι-fθo, ι-fθc, ι-fδso, ι-fδsc, ι-fδpo, ι-fδpc, ι-fao, ι-fac,ι-fMo, ι-fMc, ι-feo, ι-fec, ι-fe^*o and ι-fe^*c, are abbreviated by ι-fuzzy open, ι-fuzzy θ-semiopen, ι-fuzzy θ-semiclosed, ι-fuzzy θ-open, ι-fuzzy θ-closed, ι-fuzzy δ-semiopen, ι-fuzzy δ-semiclosed, ι-fuzzy δ-preopen, ι-fuzzy δ-preclosed, ι-fuzzy a-open, ι-fuzzy a-closed, ι-fuzzy M-open, ι-fuzzy M-closed, ι-fuzzy e-open, ι-fuzzy e-closed, ι-fuzzy e^*-open and ι-fuzzy e^*-closed respectively.

From the above definitions, it is clear that every ι-fδpo is ι-fMo and every ι-fθso is ι-fMo. Also, it is clear that every ι-fMo set is ι-feo set and ι-fe*o set. Also, every ι-fθo, ι-fδo, ι-fao set is ι-fMo set. The converses need not be true in general.

Example 4.4. *Let $X = \{x,\ y,\ z\}$ and $\mu,\nu \in I^X$ given by $\mu(x) = 0.4$, $\mu(y) = 0.5$, $\mu(z) = 0.2$; $\nu(x) = 0.5$, $\nu(y) = 0.4$, $\nu(z) = 0.7$. and $\tau : I^X \to I$ is*

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\,\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & \mu = \mu,\\ 0, & otherwise,\end{cases}$$

*is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ν is $\frac{1}{2}$-fe*o but ν is not $\frac{1}{2}$-fMo set.*

Example 4.5. *Let $X = \{x,\ y,\ z\}$ and $\mu,\nu \in I^X$ given by $\mu(x) = 0.5$, $\mu(y) = 0.3$, $\mu(z) = 0.2$; $\nu(x) = 0.5$, $\nu(y) = 0.4$, $\nu(z) = 0.4$. and $\tau : I^X \to I$ is*

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\,\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & \mu = \mu,\\ 0, & otherwise,\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ν is $\frac{1}{2}$-feo set but ν is not $\frac{1}{2}$-fMo set.

Example 4.6. *Let $X = \{x,\ y,\ z\}$ and $\mu,\nu \in I^X$ given by $\mu(x) = 0.1$, $\mu(y) = 0.1$, $\mu(z) = 0.1$; $\nu(x) = 0.9$, $\nu(y) = 0.9$, $\nu(z) = 0.9$. and $\tau : I^X \to I$ is*

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & if\ \mu = \mu,\\ 0, & otherwise,\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ν is $\frac{1}{2}$-fMo but ν is neither $\frac{1}{2}$-fδpo nor $\frac{1}{2}$-fδo nor$\frac{1}{2}$-fao set.

Example 4.7. *Let $X = \{x,\ y,\ z\}$ and $\mu,\nu \in I^X$ given by $\mu(x) = 0.1$, $\mu(y) = 0.1$, $\mu(z) = 0.1$; $\nu(x) = 0.9$, $\nu(y) = 0.9$, $\nu(z) = 0.9$. and $\tau : I^X \to I$ is*

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\,\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & if\ \mu = \mu,\\ 0, & otherwise,\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ν is $\frac{1}{2}$-fδsoset but ν is not $\frac{1}{2}$-fδo set.

Example 4.8. *Let $X = \{x,\ y,\ z\}$ and $\mu,\nu,\omega \in I^X$ given by $\mu(x) = 0.3$, $\mu(y) = 0.4$, $\mu(z) = 0.5$; $\nu(x) = 0.6$, $\nu(y) = 0.9$, $\nu(z) = 0.5$; $\omega(x) = 0.7$, $\omega(y) = 1$, $\omega(z) = 0.5$. and $\tau : I^X \to I$ is*

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\,\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & if\ \mu = \mu,\ \nu,\\ 0, & otherwise,\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ω is $\frac{1}{2}$-fMo but ω is neither $\frac{1}{2}$-fθso set nor $\frac{1}{2}$-fδso set.

Example 4.9. *Let $X = \{x,\ y,\ z\}$ and $\mu,\nu,\omega \in I^X$ given by $\mu(x) = 0.3$, $\mu(y) = 0.4$, $\mu(z) = 0.5$; $\nu(x) = 0.6$, $\nu(y) = 0.5$, $\nu(z) = 0.5$; $\omega(x) = 0.7$, $\omega(y) = 0.6$, $\omega(z) = 0.5$. and $\tau : I^X \to I$ is*

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\,\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & if\ \mu = \mu,\ \nu,\\ 0, & otherwise,\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ω is $\frac{1}{2}$-fMo and $\frac{1}{2}$-fθso set but ω is not $\frac{1}{2}$-fθo set.

Example 4.10. *Let $X = \{x,\ y,\ z\}$ and $\mu, \nu, \omega \in I^X$ given by*
$\mu(x) = 0.3$, $\mu(y) = 0.5$, $\mu(z) = 0.5$;
$\nu(x) = 0.5$, $\nu(y) = 0.5$, $\nu(z) = 0.5$;
$\omega(x) = 0.7$, $\omega(y) = 0.6$, $\omega(z) = 0.5$.
and $\tau : I^X \to I$ is

$$\tau(\mu) = \begin{cases} 1, & \mu \in \{\,\overline{0}\,,\,\overline{1}\},\\ \frac{1}{2}, & \text{if } \mu = \mu,\ \nu,\\ 0, & \text{otherwise},\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then μ is $\frac{1}{2}$-fuzzy open set but μ is neither $\frac{1}{2}$-fuzzy θ-open set nor $\frac{1}{2}$-fuzzy δ-open set.

Proposition 4.11. *If μ is an ι-fuzzy M-open subset of a sfts $(X,\ \tau)$ and $\theta I_\tau(\mu, \iota) = \overline{0}$, then μ is ι-fuzzy δ-preopen.*

Proof. Obvious. □

Theorem 4.12. *Arbitrary union (resp. intersection) of ι-fMo (resp. ι-fMc) sets is an ι-fMo(resp. ι-fMc) set.*

Proof. We prove only for ι-fMo sets, let $\{\mu_\alpha : \alpha \in \Gamma\}$ be a family of ι-fMo sets. For each $\alpha \in \Gamma$,
$\mu_\alpha \leq C_\tau(\theta I_\tau(\mu_\alpha, \iota), \iota) \vee I_\tau(\delta C_\tau(\mu_\alpha, \iota), \iota)$.

$$\begin{aligned}\bigvee_{\alpha\in\Gamma} \mu_\alpha &\leq \bigvee_{\alpha\in\Gamma} (C_\tau(\theta I_\tau(\mu_\alpha, \iota), \iota) \vee I_\tau(\delta C_\tau(\mu_\alpha, \iota), \iota))\\ &= \bigvee_{\alpha\in\Gamma} (C_\tau(\theta I_\tau(\mu_\alpha, \iota), \iota)) \vee \bigvee_{\alpha\in\Gamma} (I_\tau(\delta C_\tau(\mu_\alpha, \iota), \iota))\\ &\leq C_\tau(\theta I_\tau(\bigvee_{\alpha\in\Gamma} (\mu_\alpha, \iota), \iota)) \vee I_\tau(\delta C_\tau(\bigvee_{\alpha\in\Gamma} (\mu_\alpha, \iota), \iota)).\end{aligned}$$

□

Remark 4.13. *The intersection (resp. union) of two ι-fMo (resp. ι-fMc) sets need not be ι-fMo (resp. ι-fMc) as illustrated in the forthcoming example.*

Example 4.14. *Let $X = \{x,\ y,\ z\}$ and $\mu, \nu, \omega, \delta \in I^X$ given by Let μ, ν, ω and δ be fuzzy subsets of $X = \{x,\ y,\ z\}$ defined as follows*
$\mu(x) = 0.4$, $\mu(y) = 0.5$, $\mu(z) = 0.2$;
$\nu(x) = 0.6$, $\nu(y) = 0.6$, $\nu(z) = 0.8$;
$\omega(x) = 0.7$, $\omega(y) = 0.4$, $\omega(z) = 0.8$;
$\delta(x) = 0.6$, $\delta(y) = 0.4$, $\delta(z) = 0.8$;
and $\tau : I^X \to I$ is

$$\tau(\mu) = \begin{cases} 1, & \text{if } \mu = \overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu = \mu,\\ 0, & \text{otherwise},\end{cases}$$

is a fuzzy topology on X. For ι=$\frac{1}{2}$, then ν and ω is $\frac{1}{2}$-fMo but $\nu \wedge \omega = \delta$ is not a $\frac{1}{2}$-fMo set.

Theorem 4.15. *In a sfts $(X,\ \tau)$ μ is an ι-fMo set iff $\mu = \theta s I_\tau(\mu, \iota) \vee \delta p I_\tau(\mu, \iota)$.*

Proof. Let μ be an ι-fMo set. Then

$$\mu \leq C_\tau(\theta I_\tau(\mu, \iota), \iota) \vee I_\tau(\delta C_\tau(\mu, \iota), \iota). \tag{4.1}$$

Hence by Proposition 3.4, Lemma 3.5 and by using (4.1), we have,
$\theta s I_\tau(\mu, \iota) \vee \delta p I_\tau(\mu, \iota)$

$$\begin{aligned}&= (\mu \wedge C_\tau(\theta I_\tau(\mu, \iota), \iota)) \vee (\mu \wedge I_\tau(\delta C_\tau(\mu, \iota), \iota))\\ &= \mu \wedge (C_\tau(\theta I_\tau(\mu, \iota), \iota) \vee I_\tau(\delta C_\tau(\mu, \iota), \iota))\\ &= \mu.\end{aligned}$$

Conversely, suppose that $\mu = \theta s I_\tau(\mu, \iota) \vee \delta p I_\tau(\mu, \iota)$. Then by Proposition 3.4 and Lemma 3.5,

$$\begin{aligned}\mu &= (\mu \wedge C_\tau(\theta I_\tau(\mu, \iota), \iota)) \vee (\mu \wedge I_\tau(\delta C_\tau(\mu, \iota), \iota))\\ &\leq C_\tau(\theta I_\tau(\mu, \iota), \iota) \vee I_\tau(\delta C_\tau(\mu, \iota), \iota).\end{aligned}$$

Therefore, μ is an ι-fMo set. □

Proposition 4.16. *In a sfts $(X,\ \tau)$ μ is an ι-fMc set iff $\mu = \theta s C_\tau(\mu, \iota) \vee \delta p C_\tau(\mu, \iota)$.*

Theorem 4.17. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)&(ii) are hold.*

(i) $MC_\tau(\mu, \iota) = \theta s C_\tau(\mu, \iota) \wedge \delta p C_\tau(\mu, \iota)$.

(ii) $MI_\tau(\mu, \iota) = \theta s I_\tau(\mu, \iota) \vee \delta p I_\tau(\mu, \iota)$.

Proof. (i) Since $MC_\tau(\mu, \iota) \leq \theta s C_\tau(\mu, \iota) \wedge \delta p C_\tau(\mu, \iota)$.
Also, $\theta s C_\tau(\mu, \iota) \wedge \delta p C_\tau(\mu, \iota)$

$$\begin{aligned}&= (\mu \vee I_\tau(\theta C_\tau(\mu, \iota), \iota)) \wedge (\mu \vee C_\tau(\delta I_\tau(\mu, \iota), \iota))\\ &= \mu \vee (I_\tau(\theta C_\tau(\mu, \iota), \iota) \wedge C_\tau(\delta I_\tau(\mu, \iota), \iota)).\end{aligned}$$

But, $MC_\tau(\mu, \iota)$ is ι-fMc, hence,
$MC_\tau(\mu, \iota)$

$$\begin{aligned}&> I_\tau(\theta C_\tau(MC_\tau(\mu, \iota), \iota), \iota) \wedge C_\tau(\delta I_\tau(MC_\tau(\mu, \iota), \iota), \iota)\\ &> I_\tau(\theta C_\tau(\mu, \iota), \iota) \wedge C_\tau(\delta I_\tau(\mu, \iota), \iota).\end{aligned}$$

Thus $\mu \bigvee I_\tau(\theta C_\tau(\mu, \iota), \iota) \wedge C_\tau(\delta I_\tau(\mu, \iota), \iota)) <$
$\mu \vee MC_\tau(\mu, \iota) = MC_\tau(\mu, \iota)$
Therefore, $\theta s C_\tau(\mu, \iota) \wedge \delta p C_\tau(\mu, \iota) < MC_\tau(\mu, \iota)$.
So,

$$MC_\tau(\mu, \iota) = \theta s C_\tau(\mu, \iota) \wedge \delta p C_\tau(\mu, \iota).$$

(ii) Obvious. □

Theorem 4.18. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)&(ii) are hold.*

(i) *μ is ι-fMo iff $\mu = MI_\tau(\mu, \iota)$.*

(ii) *μ is ι-fMc iff $\mu = MC_\tau(\mu, \iota)$.*

Proof. (i) Let μ be an ι-fMo set. Then by Theorem 4.15,

$$\mu = \theta sI_\tau(\mu,\iota) \vee \delta pI_\tau(\mu,\iota)$$

and by Theorem 4.17, we have $\mu = MI_\tau(\mu,\iota)$.

Conversely, let $\mu = MI_\tau(\mu,\iota)$. Then by Theorem 4.17, $\mu = \theta sI_\tau(\mu,\iota) \vee \delta pI_\tau(\mu,\iota)$ and by Theorem 4.15, μ is an ι-fMo set.

(ii) Obvious. □

Theorem 4.19. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)&(ii) are hold.*

(i) $MI_\tau(1-\mu,\iota) = 1-(MC_\tau(\mu,\iota))$.

(ii) $MC_\tau(1-\mu,\iota) = 1-(MI_\tau(\mu,\iota))$.

Proof. (i) $\forall\ \mu \in I^X$, $\iota \in I_0$

$$\begin{aligned}1-(MC_\tau(\mu,\iota)) &= 1-\bigwedge\{\nu : \nu \geq \mu,\ \nu \text{ is } r\text{-fMc}\}\\ &= \bigvee\{1-\nu : 1-\nu \leq 1-\mu,\ 1-\nu \text{ is } r\text{-fMo}\}\\ &= MI_\tau(1-\mu,\iota).\end{aligned}$$

(ii) same as (i). □

Theorem 4.20. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)–(vi) are hold.*

(i) $MC_\tau(0,\iota) = 0$ *and* $MI_\tau(1,\iota) = 1$.

(ii) $I_\tau(\mu,\iota) \leq MI_\tau(\mu,\iota) \leq \mu \leq MC_\tau(\mu,\iota) \leq C_\tau(\mu,\iota)$.

(iii) $\mu \leq \nu \Rightarrow MI_\tau(\mu,\iota) \leq MI_\tau(\nu,\iota)$ *and* $MC_\tau(\mu,\iota) \leq MC_\tau(\nu,\iota)$.

(iv) $MC_\tau(MC_\tau(\mu,\iota),\iota) = MC_\tau(\mu,\iota)$ *and* $MI_\tau(MI_\tau(\mu,\iota),\iota) = MI_\tau(\mu,\iota)$.

(v) $MC_\tau(\mu,\iota) \vee MC_\tau(\nu,\iota) < MC_\tau(\mu \vee \nu,\iota)$ *and* $MI_\tau(\mu,\iota) \vee MI_\tau(\nu,\iota) < MI_\tau(\mu \vee \nu,\iota)$.

(vi) $MC_\tau(\mu \wedge \nu,\iota) < MC_\tau(\mu,\iota) \wedge MC_\tau(\nu,\iota)$ *and* $MI_\tau(\mu \wedge \nu,\iota) < MI_\tau(\mu,\iota) \wedge MI_\tau(\nu,\iota)$.

Proof. (i) It is trivial from the definitions of MC_τ and MI_τ.

(iii) Since, by Theorem 4.15,
$MC_\tau(\mu,\iota) = \theta sC_\tau(\mu,\iota) \wedge \delta pC_\tau(\mu,\iota)$ and $\mu \leq \nu$,

$$\begin{aligned}MC_\tau(\mu,\iota) &= \theta sC_\tau(\mu,\iota) \wedge \delta pC_\tau(\mu,\iota)\\ &\leq \theta sC_\tau(\nu,\iota) \wedge \delta pC_\tau(\nu,\iota)\\ &= MC_\tau(\nu,\iota).\end{aligned}$$

Similarly, by Theorem 4.15,
$MI_\tau(\mu,\iota) = \theta sI_\tau(\mu,\iota) \vee \delta pI_\tau(\mu,\iota)$ and $\mu \leq \nu$,

$$\begin{aligned}MI_\tau(\mu,\iota) &= \theta sI_\tau(\mu,\iota) \wedge \delta pI_\tau(\mu,\iota)\\ &\leq \theta sI_\tau(\nu,\iota) \wedge \delta pI_\tau(\nu,\iota)\\ &= MC_\tau(\nu,\iota).\end{aligned}$$

(iv) Since, by Theorem 4.18,
$MC_\tau(MC_\tau(\mu,\iota),\iota)$

$$\begin{aligned}&= \theta sC_\tau(MC_\tau(\mu,\iota),\iota) \wedge \delta pC_\tau(MC_\tau(\mu,\iota),\iota)\\ &= [\theta sC_\tau(\theta sC_\tau(\mu,\iota) \wedge \delta pC_\tau(\mu,\iota),\iota) \wedge\\ &\quad \delta pC_\tau(\theta sC_\tau(\mu,\iota) \wedge \delta pC_\tau(\mu,\iota),\iota)]\\ &\leq [(\theta sC_\tau(\mu,\iota) \wedge \theta sC_\tau(\delta pC_\tau(\mu,\iota),\iota)) \wedge\\ &\quad (\delta pC_\tau(\theta sC_\tau(\mu,\iota),\iota) \wedge \delta pC_\tau(\mu,\iota))]\\ &= \theta sC_\tau(\mu,\iota) \wedge \delta pC_\tau(\mu,\iota)\\ &= MC_\tau(\mu,\iota).\end{aligned}$$

Hence, $MC_\tau(MC_\tau(\mu,\iota),\iota) \leq MC_\tau(\mu,\iota)$.

But $MC_\tau(\mu,\iota) \leq MC_\tau(MC_\tau(\mu,\iota),\iota)$. Therefore,

$$MC_\tau(MC_\tau(\mu,\iota),\iota) = MC_\tau(\mu,\iota).$$

(v) and (vi): Obvious. □

Lemma 4.21. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)–(iv) are hold.*

(i) $pI_\tau(\delta pC_\tau(\mu,\iota),\iota) = \delta pC_\tau(\mu,\iota) \wedge I_\tau(C_\tau(\mu,\iota),\iota)$ *and* $pC_\tau(\delta pI_\tau(\mu,\iota),\iota) = \delta pI_\tau(\mu,\iota) \vee C_\tau(I_\tau(\mu,\iota),\iota)$.

(ii) $\theta pI_\tau(\delta pC_\tau(\mu,\iota),\iota) = \delta pC_\tau(\mu,\iota) \wedge I_\tau(\theta C_\tau(\mu,\iota),\iota)$ *and* $\theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota) = \delta pI_\tau(\mu,\iota) \vee C_\tau(\theta I_\tau(\mu,\iota),\iota)$.

(iii) $\theta sC_\tau(\theta I_\tau(\mu,\iota),\iota) = sC_\tau(\theta I_\tau(\mu,\iota),\iota)$ $=I_\tau(C_\tau(\theta I_\tau(\mu,\iota),\iota),\iota)$.

(iv) $\theta sI_\tau(\theta C_\tau(\mu,\iota),\iota) = sI_\tau(\theta C_\tau(\mu,\iota),\iota)$ $=C_\tau(I_\tau(\theta C_\tau(\mu,\iota),\iota),\iota)$.

Proof. Obvious. □

Proposition 4.22. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)&(ii) are hold.*

(i) $MC_\tau(\mu,\iota) = \mu \vee \theta pI_\tau(\delta pC_\tau(\mu,\iota),\iota)$.

(ii) $MI_\tau(\mu,\iota) = \mu \wedge \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$.

Proof. (i) By Lemma 4.21,
$\mu \vee \theta pI_\tau(\delta pC_\tau(\mu,\iota),\iota)$

$$\begin{aligned}&= \mu \vee (\delta pC_\tau(\mu,\iota) \wedge I_\tau(\theta C_\tau(\mu,\iota),\iota))\\ &= (\mu \vee \delta pC_\tau(\mu,\iota)) \wedge (\mu \vee I_\tau(\theta C_\tau(\mu,\iota),\iota))\\ &= \delta pC_\tau(\mu,\iota) \wedge \theta sC_\tau(\mu,\iota)\\ &= MC_\tau(\mu,\iota).\end{aligned}$$

(ii) Obvious. □

Theorem 4.23. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)–(iii) are equivalent.*

(i) μ *is an ι-fMo set.*

(ii) $\mu \leq \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$.

(iii) $\theta pC_\tau(\mu,\iota) = \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$.

Proof. (i) $\Rightarrow$ (ii): Let μ be an fMo set. Then by Theorem 4.18, $M = MI_\tau(\mu,\iota)$ and by Proposition 4.22,

$$\mu = \mu \wedge \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$$

and hence $\mu \leq \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$.

(ii) $\Rightarrow$ (i): Let $\mu \leq \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$. Then by Proposition 4.22,

$$\begin{aligned}\mu &\leq \mu \wedge \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)\\ &= MI_\tau(\mu,\iota).\end{aligned}$$

So $M \leq MI_\tau(\mu,\iota)$. Then $M = MI_\tau(\mu,\iota)$ and hence, μ is an ι-fMo set.

(ii) $\Rightarrow$ (iii): Let $\mu \leq \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$. Then

$$\theta pC_\tau(\mu,\iota) \leq \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$$

and $\theta pC_\tau(\mu,\iota) = \theta pC_\tau(\delta pI_\tau(\mu,\iota),\iota)$.

(iii) $\Rightarrow$ (ii): Obvious. $\square$

Theorem 4.24. *In a sfts $(X,\ \tau)$, $\forall\ \mu \in I^X$ and $\iota \in I_o$, (i)–(iii) are equivalent.*

(i) μ is an ι-fMc set.

(ii) $\theta pI_\tau(\delta pC_\tau(\mu,\iota),\iota) \leq \mu$.

(iii) $\theta pI_\tau(\mu,\iota) = \theta pI_\tau(\delta pC_\tau(\mu,\iota),\iota)$.

Theorem 4.25. *If μ is a fuzzy subset of an extremely disconnected sfts $(X,\ \tau)$ and $\iota \in I_o$. Then (i) & (ii) are equivalent :*

(i) μ is an ι-fuzzy open set.

(ii) μ is an ι-fMo set and ι-fuzzy locally closed.

Proof. (i) $\Rightarrow$ (ii): Obvious from Definitions 3.8 and 4.1.

(ii)$\Rightarrow$ (i): Let μ be an ι-fuzzy open and a ι-fuzzy locally closed subset of X. Then $\mu = \nu \wedge C_\tau(\mu,\iota)$ and

$$\mu \leq C_\tau(\theta I_\tau(\mu,\iota),\iota) \vee I_\tau(\delta C_\tau(\mu,\iota),\iota).$$

Hence,

$$\begin{aligned}\mu &\leq \nu \wedge [C_\tau(\theta I_\tau(\mu,\iota),\iota) \vee I_\tau(\delta C_\tau(\mu,\iota),\iota)]\\ &\leq (\nu \wedge C_\tau(\theta I_\tau(\mu,\iota),\iota)) \vee (I_\tau(\nu \wedge \delta C_\tau(\mu,\iota),\iota))\\ &\leq (\nu \wedge C_\tau(I_\tau(\mu,\iota),\iota)) \vee (I_\tau(\nu \wedge C_\tau(\mu,\iota),\iota))\\ &\leq (\nu \wedge I_\tau(C_\tau(\mu,\iota),\iota)) \vee (I_\tau(\nu \wedge C_\tau(\mu,\iota),\iota))\\ &\leq I_\tau(\nu \wedge C_\tau((\mu,\iota),\iota) \vee I_\tau(\nu \wedge C_\tau(\mu,\iota),\iota)\\ &= I_\tau(\mu,\iota) \vee I_\tau(\mu,\iota)\\ &= I_\tau(\mu,\iota).\end{aligned}$$

Hence μ is ι-fuzzy open. $\square$

Theorem 4.26. *In a sfts $(X,\ \tau)$, $\forall\ \mu,\nu \in I^X$ and $\iota \in I_o$, (i) & (ii) are hold.*

(i) If $\tau(\nu) \geq \iota$ where ν is a crisp subset and μ is an ι-fMo set, then $\mu \wedge \nu$ is an ι-fMo set.

(ii) If $\tau(\overline{1}-\nu) \geq \iota$ where ν is a crisp subset and μ is an ι-fMc set, then $\mu \vee \nu$ is an ι-fMc set.

Proof. (i) Let μ be ι-fMo and $\nu \in I^X$ with $\tau(\nu) \geq \iota$ which is a crisp subset. Then

$$\begin{aligned}\mu \wedge \nu &\leq (C_\tau(\theta I_\tau(\mu,\iota),\iota) \vee I_\tau(\delta C_\tau(\mu,\iota),\iota)) \wedge \nu\\ &= (C_\tau(\theta I_\tau(\mu,\iota),\iota) \wedge \nu) \vee (I_\tau(\delta C_\tau(\mu,\iota),\iota) \wedge \nu)\\ &\leq (C_\tau(\theta I_\tau(\mu,\iota) \wedge \nu,\iota)) \vee (I_\tau(\delta C_\tau(\mu,\iota) \wedge \nu,\iota))\\ &\leq (C_\tau(\theta I_\tau(\mu \wedge \nu,\iota),\iota)) \vee (I_\tau(\delta C_\tau(\mu \wedge \nu,\iota),\iota)).\end{aligned}$$

Hence $\mu \wedge \nu$ is ι-fMo.

(ii) Similar to the proof of (i). $\square$

Theorem 4.27. *Let $(X,\ \tau)$ be a sfts, $\mu,\ \nu \in I^X$ and $\iota \in I_0$.*

(i) If μ is ι-fMo with $\tau(\overline{1}-\mu) \geq \iota$, then μ is r-fθso.

(ii) If μ is ι-fMc with $\tau(\mu) \geq \iota$, then μ is ι-fθsc.

(iii) If μ is ι-fao with $\tau(\mu) \geq \iota$, then μ is ι-fMo.

(iv) If μ is ι-fac with $\tau(\overline{1}-\mu) \geq \iota$, then μ is ι-fMc.

Proof. We prove (i) and (iii).

Other results have similar proofs.

(i) Let μ be an ι-fMo set and $\tau(\overline{1}-\mu) \geq \iota$. Then

$$\begin{aligned}\mu &\leq C_\tau(\theta I_\tau(\mu,\iota),\iota) \vee I_\tau(\delta C_\tau(\mu,\iota),\iota)\\ &= C_\tau(\theta I_\tau(\mu,\iota),\iota) \vee I_\tau(\mu,\iota)\\ &\leq C_\tau(\theta I_\tau(\mu,\iota),\iota).\end{aligned}$$

Hence μ is an ι-fuzzy θ-semiopen set.

(iii) Let μ be an ι-fao set and $\tau(\mu) \geq \iota$. Then $\delta I_\tau(\mu,\iota) = \mu$.

$$\begin{aligned}\mu &\leq I_\tau(C_\tau(\delta I_\tau(\mu,\iota),\iota),\iota)\\ &\leq I_\tau(C_\tau(\mu,\iota),\iota)\\ &\leq I_\tau(C_\tau(\mu,\iota),\iota) \vee C_\tau(\theta I_\tau(\mu,\iota),\iota)\\ &\leq I_\tau(\delta C_\tau(\mu,\iota),\iota) \vee C_\tau(\theta I_\tau(\mu,\iota),\iota)\end{aligned}$$

Thus μ is an ι-fuzzy M-open set. $\square$

Theorem 4.28. *Let $(X,\ \tau)$ be a sfts. For $\mu,\ \nu \in I^X$ and $\iota \in I_0$.*

(i) μ is ι-fMo iff $1-\mu$ is ι-fMc.

(ii) If $\tau(\mu) \geq \iota$, then μ is ι-fMo set.

(iii) then $I_\tau(\mu,\ \iota)$ is an ι-fMo set.

(iv) then $C_\tau(\mu,\ \iota)$ is an ι-fMc set.

Proof. (i) & (ii) are trivial.

(iii) From the Definition of I_τ of Theorem 2.5 and Definition 2.2, since $\tau(I_\tau(\mu,\ \iota)) \geq \iota$, by (ii) $I_\tau(\mu,\ \iota)$ is an ι-fMo set.

(iv) Since $\overline{1}-C_\tau(\mu,\ \iota) = I_\tau(\overline{1}-\mu,\ \iota)$ from Theorem 2.5, hence by (iii), we have $\tau(\overline{1}-C_\tau(\mu,\ \iota)) \geq \iota$.

Hence $\overline{1}-C_\tau(\mu,\ \iota)$ is ι-fMo. By (i) $C_\tau(\mu,\ \iota)$ is an ι-fMc set. $\square$

5. Conclusion

We have introduced the concept of ι-fuzzy θ-open (resp. ι-fuzzy θ-closed), ι-fuzzy M-open (resp. ι-fuzzy M-closed) sets in fuzzy topological spaces in the sense of $\hat{S}$ostak's. Also, we have introduced ι-fuzzy θ-interior (resp. ι-fuzzy θ-closure), ι-fuzzy M-interior (resp. ι-fuzzy M-closure) and investigated some of their properties. Moreover, we investigated the relationships between ι-fuzzy open, ι-fuzzy θ-semiopen, ι-fuzzy θ-open, ι-fuzzy δ-semiopen, ι-fuzzy δ-preopen, ι-fuzzy a-open, ι-fuzzy e-open and ι-fuzzy e^*-open in $\hat{S}$ostak's fuzzy topological spaces.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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