Malaya Journal of Matematik, Vol. S, No. 1, 250-256, 2019

https://doi.org/10.26637/MJM0S01/0047

$(\in, \in \vee q_k)$-fuzzy soft near-rings and $(\in, \in \vee q_k)$-fuzzy soft ideals over near-rings

T. Manikantan[1]* and P. Ramasamy[1,2]

Abstract

The concept of quasi-coincidence of a fuzzy point with a fuzzy subset has played a vital role in generating various fuzzy algebraic substructures. In this article, we introduce the notions of $(\in, \in \vee q_k)$-fuzzy soft near-ring and $(\in, \in \vee q_k)$-fuzzy soft ideal over a near-ring which are generalization of $(\in, \in \vee q)$-fuzzy soft near-ring and $(\in, \in \vee q)$-fuzzy soft ideal respectively and study some of their properties with examples. We also introduce the notion of $(\in, \in \vee q_k)$-fuzzy soft subnear-ring (resp. ideal) of an $(\in, \in \vee q_k)$-fuzzy soft near-ring and obtain some related results.

Keywords

$(\in, \in \vee q_k)$-fuzzy soft near-ring, $(\in, \in \vee q_k)$-fuzzy soft ideal, $(\in, \in \vee q_k)$-fuzzy soft subnear-ring.

AMS Subject Classification

16Y30, 08A72, 06D72.

[1] *Post Graduate and Research Department of Mathematics, Thiruvalluvar Government Arts College (Affl. to Periyar University), Rasipuram - 637 401, Namakkal District, Tamilnadu, India.*

[2] *Post Graduate and Research Department of Mathematics, Government Arts College (Autonomous) (Affl. to Periyar University), Salem - 636 007, Tamilnadu, India.*

*__Corresponding author__: [1] manikantan_slm@rediffmail.com, manikantan@tgac.ac.in; [2] 73ramasamy@gmail.com

Article History: Received **22** November **2018**; Accepted **09** May **2019**

Contents

1 **Introduction** 250

2 **Preliminaries** 251

3 **$(\in, \in \vee q_k)$-fuzzy soft near-rings and $(\in, \in \vee q_k)$-fuzzy soft ideals** 252

4 **Conclusion** 255

References 255

1. Introduction

Zadeh [24] initiated the fundamental concept of fuzzy set which provides a framework for generalization of many basic concepts of algebraic structures. The quasi-coincident concept on a fuzzy point with a fuzzy subset was introduced by Ming and Ming in [17]. This concept has played an important role to generate various fuzzy algebraic substructures. In particular, Sandeep Kumar Bhakat and Pratulanada Das [7] initiated the notion of an $(\in, \in \vee q)$-fuzzy subgroup which is an important generalization of a fuzzy subgroup defined in [6]. In [8], they have extended this notion to rings. Narayanan and Manikantan [19] have introduced the notions of an $(\in, \in \vee q)$-fuzzy subnear-ring and $(\in, \in \vee q)$-fuzzy ideals of near-rings. In [9], Dheena and Coumaressane defined $(\in, \in \vee q_k)$-fuzzy subnear-ring and $(\in, \in \vee q_k)$-fuzzy ideals of near-rings. For more details on $(\in, \in \vee q_k)$-fuzzy algebraic structures, we refer the readers to [5, 10, 11, 13–15, 23]. In 1999, Russian researcher Molodtsov initiated the concept of a soft set [18] which can be known as an effective mathematical tool to deal with uncertainity. Maji et al. [16] presented the definitions of fuzzy soft set and fuzzy soft set operations. Aygünoğlu and Aygün [4] gave the concept of fuzzy soft group and discussed some of its properties. İnan and Öztürk [12] defined the concepts of fuzzy soft ring and $(\in, \in \vee q)$-fuzzy soft ring over a ring. In [20], Öztürk and İnan have extended these concepts to near-rings. In this article, we introduce the notions of $(\in, \in \vee q_k)$-fuzzy soft near-ring and $(\in, \in \vee q_k)$-fuzzy soft ideal over a near-ring which are generalization of $(\in, \in \vee q)$-fuzzy soft near-ring and $(\in, \in \vee q)$-fuzzy soft ideal respectively and study some of their properties with examples. We also introduce the definitions of $(\in, \in \vee q_k)$-fuzzy soft subnear-ring (resp. ideal) of an $(\in, \in \vee q_k)$-fuzzy soft near-ring and obtain some related results.

2. Preliminaries

In this section, the basic concepts which will be used in the subsequent sections are given.

A *near-ring* [22] is a non-empty set N with two binary operations "$+$" and "$\cdot$" satisfying the axioms: (i) $(N,+)$ is a group, (ii) $(N,\cdot)$ is a semigroup and (iii) $(u+v)\cdot w = u\cdot w + v\cdot w$ for all $u,v,w \in N$. Since it satisfies the right distributive law, we call it a right near-ring. We will use the word "near-ring" to mean "right near-ring". In what follows, N denotes the near-ring unless otherwise specified.

Let X be a non-empty set. A *fuzzy subset* γ of X is a mapping from X to $[0,1]$ (see [24]).

A fuzzy subset γ of X is of the form

$$\gamma(v) = \begin{cases} r \in (0,1] & \text{if } v = u, \\ 0 & \text{if } v \neq u, \end{cases}$$

is called a *fuzzy point* with support u and value r and is denoted by u_r (see [17]). For a fuzzy point u_r and a fuzzy subset γ in X, we say that

(i) $u_r \in \gamma$ if $\gamma(u) \geq r$. In this case, u_r is said to *belong to* γ (see [17]).

(ii) $u_r q \gamma$ if $\gamma(u) + r > 1$. In this case, u_r is said to be *quasi-coincident* with γ (see [17]).

(iii) $u_r q_k \gamma$ if $\gamma(u) + r > 1-k$, where $k \in [0,1)$. In this case, u_r is said to be *k-quasi-coincident* with γ (see [9]).

We say that, $u_r \in \vee q_k \gamma$ (resp. $x_t \in \wedge q_k \gamma$) means $x_t \in \gamma$ or $x_t q_k \gamma$ (resp. $x_t \in \gamma$ and $x_t q_k \gamma$) and $x_t \overline{\in} \lambda$, $x_t \overline{\in \vee q_k} \gamma$ will respectively mean $x_t \in \gamma$ and $x_t \in \vee q_k \gamma$ does not hold.

Definition 2.1. *[1] A fuzzy subset γ of N is called a fuzzy subnear-ring of N if for all $u,v \in N$,*

(i) $\gamma(u-v) \geq \min\{\gamma(u), \gamma(v)\}$,

(ii) $\gamma(uv) \geq \min\{\gamma(u), \gamma(v)\}$.

Definition 2.2. *[1] A fuzzy subset γ of N is called a fuzzy ideal of N if for all $u,v,w \in N$,*

(i) $\gamma(u-v) \geq \min\{\gamma(u), \gamma(v)\}$,

(ii) $\gamma(v+u-v) \geq \gamma(u)$,

(iii) $\gamma(uv) \geq \gamma(u)$,

(iv) $\gamma(u(v+w)-uv) \geq \gamma(w)$.

Definition 2.3. *[9] A fuzzy subset γ of N is called an $(\in, \in \vee q_k)$-fuzzy subnear-ring (briefly, $(\in, \in \vee q_k)$-FN) of N if*

(i) $u_r \in \gamma$, $v_s \in \gamma \Rightarrow (u+v)_{\min\{r,s\}} \in \vee q_k \gamma$,

(ii) $u_r \in \gamma \Rightarrow (-u)_r \in \vee q_k \gamma$,

(iii) $u_r \in \gamma$, $v_s \in \gamma \Rightarrow (uv)_{\min\{r,s\}} \in \vee q_k \gamma$

for all $u,v \in N$ and $r,s \in (0,1]$.

An $(\in, \in \vee q_k)$-fuzzy subnear-ring of N with $k=0$ is an $(\in, \in \vee q)$-fuzzy subnear-ring of N (see [19]).

Definition 2.4. *[9] A fuzzy subset γ of N is called an $(\in, \in \vee q_k)$-fuzzy ideal (briefly, $(\in, \in \vee q_k)$-FI) of N if*

(i) $u_r \in \gamma$, $v_s \in \gamma \Rightarrow (u-v)_{\min\{r,s\}} \in \vee q_k \gamma$,

(ii) $u_r \in \gamma$, $v \in N \Rightarrow (v+u-v)_r \in \vee q_k \gamma$,

(iii) $u_r \in \gamma$, $v \in N \Rightarrow (uv)_r \in \vee q_k \gamma$,

(iv) $w_r \in \gamma$, $u,v \in N \Rightarrow (u(v+w)-uv)_r \in \vee q_k \gamma$

for all $u,v,w \in N$ and $r,s \in (0,1]$.

An $(\in, \in \vee q_k)$-fuzzy ideal of N with $k=0$ is an $(\in, \in \vee q)$-fuzzy ideal of N (see [19]).

In what follows, U is a basic universe set, E is a set of parameters, $P \subseteq E$ and $\mathscr{P}(U)$ is the power set of U unless otherwise specified.

Definition 2.5. *[18] A pair (H,P) is called a soft set over U, where $H : P \to \mathscr{P}(U)$ is a mapping and $P \subseteq E$.*

Definition 2.6. *[16] Let $\mathscr{F}(U)$ be a set of all fuzzy subsets of U and $P \subseteq E$. A pair (Δ, P) is called a fuzzy soft set (FSS, in short) over U, where $\Delta : P \to \mathscr{F}(U)$ is a mapping. That is, for each $\rho \in P$, $\Delta(\rho) = \Delta_\rho : U \to [0,1]$ is a fuzzy subset of U.*

Definition 2.7. *[21] Let (Δ,P) be a FSS over U. Then the set $Supp(\Delta,P) = \{\rho \in P | \Delta(\rho) \neq \emptyset\}$ is called the support of (Δ,P). A FSS (Δ,P) is called non-null if $Supp(\Delta,P) \neq \emptyset$. It is denoted by $(\Delta,P) \neq \Phi$.*

Definition 2.8. *[16] Let (Δ,P_1) and (Θ,P_2) be FSSs over U. Then,*

(i) "(Δ,P_1) AND (Θ,P_2)", shown by $(\Delta,P_1) \wedge (\Theta,P_2)$, is defined as $(\Delta,P_1) \wedge (\Theta,P_2) = (\Psi,P)$, where $P = P_1 \times P_2$ and $\Psi(\rho,\sigma) = \Delta(\rho) \cap \Theta(\sigma)$ for all $(\rho,\sigma) \in P_1 \times P_2$.

(ii) "(Δ,P_1) OR (Θ,P_2)", denoted by $(\Delta,P_1) \vee (\Theta,P_2)$, is defined as $(\Delta,P_1) \vee (\Theta,P_2) = (\Psi,P)$, where $P = P_1 \times P_2$ and $\Delta(\rho) \cup \Theta(\sigma)$ for all $(\rho,\sigma) \in P_1 \times P_2$.

Definition 2.9. *[16] The union of FSSs (Δ,P_1) and (Θ,P_2) over U is expressed as a FSS (Ψ,P), where $P = P_1 \cup P_2$ and for all $\rho \in P$,*

$$\Psi(\rho) = \begin{cases} \Delta(\rho) & \text{if } \rho \in P_1 \backslash P_2, \\ \Theta(\rho) & \text{if } \rho \in P_2 \backslash P_1, \\ \Delta(\rho) \cup \Theta(\rho) & \text{if } \rho \in P_1 \cap P_2. \end{cases}$$

This is denoted by $(\Delta,P_1) \sqcup (\Theta,P_2) = (\Psi,P)$.

Definition 2.10. *[3] The restricted intersection of FSSs (Δ,P_1) and (Θ,P_2) over U, shown by $(\Delta,P_1) \sqcap_r (\Theta,P_2)$, is defined as $(\Delta,P_1) \sqcap_r (\Theta,P_2) = (\Psi,P)$, where $P = P_1 \cap P_2 \neq \emptyset$ and $\Psi(\rho) = \Delta(\rho) \cap \Theta(\rho)$ for all $\rho \in P$.*

Definition 2.11. *[3] The extended intersection of FSSs (Δ,P_1) and (Θ,P_2) over U is expressed as a FSS (Ψ,P), where $P=P_1\cup P_2$ and for all $\rho\in P$,*

$$\Psi(\rho)=\begin{cases}\Delta(\rho) & \text{if } \rho\in P_1\backslash P_2,\\ \Theta(\rho) & \text{if } \rho\in P_2\backslash P_1,\\ \Delta(\rho)\cup\Theta(\rho) & \text{if } \rho\in P_1\cap P_2.\end{cases}$$

This is denoted by $(\Delta,P_1)\sqcap_e(\Theta,P_2)=(\Psi,P)$.

Definition 2.12. *[2] For the family of FSSs $\{(\Delta_i,P_i)|i\in\Omega\}$ over U, where Ω is an index set,*

(i) the restricted intersection of these FSSs is a FSS $\{\sqcap_r(\Delta_i,P_i)|i\in\Omega\}=(\Delta,P)$, where $P=\bigcap_{i\in\Omega}P_i\neq\emptyset$ and $\Delta(\rho)=\bigcap_{i\in\Omega}\Delta_i(\rho)$ for all $\rho\in P$.

(ii) the union of these FSSs is a FSS $\{\sqcup(\Delta_i,P_i)|i\in\Omega\}=(\Delta,P)$, where $P=\bigcup_{i\in\Omega}P_i$ and $\Delta(\rho)=\bigcup_{i\in\Omega(\rho)}\Delta_i(\rho)$ for all $\rho\in P$ and $\Omega(\rho)=\{i\in\Omega:\rho\in P_i\}$.

(iii) the AND of these FSSs is a FSS $\{\bigwedge(\Delta_i,P_i)|i\in\Omega\}=(\Delta,P)$, where $P=\Pi_{i\in\Omega}P_i$ and $\Delta(\rho)=\bigcap_{i\in\Omega}\Delta_i(\rho_i)$ for all $\rho=(\rho_i)_{i\in\Omega}\in P$.

Definition 2.13. *[20] Let (Δ,P) be a FSS over N. Then (Δ,P) is called a fuzzy soft near-ring (FSN, for short) over N if $\Delta(\rho)$ is a fuzzy subnear-ring of N for all $\rho\in P$.*

Definition 2.14. *[20] Let η and τ be fuzzy subnear-rings of N. Then η is called a fuzzy subnear-ring of τ if $\eta(u)\leq\tau(u)$ for all $u\in N$.*

Definition 2.15. *[20] Let η be a fuzzy subset of N and τ be a fuzzy subnear-ring of N. Then η is called a fuzzy ideal of τ if $\eta(u)\leq\tau(u)$ for all $u\in N$ and η is a fuzzy ideal of N.*

Definition 2.16. *[20] Let (Δ,P_1) and (Θ,P_2) be FSNs over N. Then (Δ,P_1) is called a fuzzy soft subnear-ring of (Θ,P_2) if $P_1\subseteq P_2$ and $\Delta(\rho)$ is a fuzzy subnear-ring of $\Theta(\rho)$ for all $\rho\in P_1$.*

Definition 2.17. *[20] Let (η,Q) be a FSS and (Δ,P) be a FSN over N. Then (η,Q) is called a fuzzy soft ideal (FSI, for short) of (Δ,P) if $Q\subseteq P$ and $\eta(\rho)$ is a fuzzy ideal of $\Delta(\rho)$ for all $\rho\in Q$.*

3. $(\in,\in\vee q_k)$-fuzzy soft near-rings and $(\in,\in\vee q_k)$-fuzzy soft ideals

In this section, we introduce the notions of $(\in,\in\vee q_k)$-fuzzy soft near-ring and $(\in,\in\vee q_k)$-fuzzy soft ideal over a near-ring N. Further we define the notion of $(\in,\in\vee q_k)$-fuzzy soft subnear-ring (resp. ideal) of an $(\in,\in\vee q_k)$-fuzzy soft near-ring and investigate some of their properties.

Definition 3.1. *Let (Δ,P) be a FSS over N. Then (Δ,P) is called an $(\in,\in\vee q_k)$-fuzzy soft near-ring (briefly, $(\in,\in\vee q_k)$-FSN) over N if for each $\rho\in P$, $\Delta(\rho)=\Delta_\rho$ is an $(\in,\in\vee q_k)$-FN of N, that is,*

(i) $u_r\in\Delta_\rho$, $v_s\in\Delta_\rho\Rightarrow(u+v)_{\min\{r,s\}}\in\vee q_k\Delta_\rho$,

(ii) $u_r\in\Delta_\rho\Rightarrow(-u)_r\in\vee q_k\Delta_\rho$,

(iii) $u_r\in\Delta_\rho$, $v_s\in\Delta_\rho\Rightarrow(uv)_{\min\{r,s\}}\in\vee q_k\Delta_\rho$,

for all $u,v\in N$ and $r,s\in(0,1]$.

We know that a fuzzy subset Δ_ρ of N satisfies two conditions (i) and (ii) if and only if it satisfies: (iv) $u_r,v_s\in\Delta_\rho\Rightarrow(u-v)_{\min\{r,s\}}\in\vee q_k\Delta_\rho$ for all $u,v\in N$ and $r,s\in(0,1]$.

An $(\in,\in\vee q_k)$-fuzzy soft near-ring over N with $k=0$ is an $(\in,\in\vee q)$-*fuzzy soft near-ring* over N (see [20]).

Lemma 3.2. *The conditions $(i)-(iv)$ in Definition 3.1 are equivalent to*

(i) $\Delta_\rho(u+v)\geq\min\{\Delta_\rho(u),\Delta_\rho(v),\frac{1-k}{2}\}$,

(ii) $\Delta_\rho(-u)\geq\min\{\Delta_\rho(u),\frac{1-k}{2}\}$,

(iii) $\Delta_\rho(uv)\geq\min\{\Delta_\rho(u),\Delta_\rho(v),\frac{1-k}{2}\}$,

(iv) $\Delta_\rho(u-v)\geq\min\{\Delta_\rho(u),\Delta_\rho(v),\frac{1-k}{2}\}$.

for all $u,v\in N$, respectively.

Proof. From Lemma 3.2 and Lemma 3.3 of [9], the proof can be achieved. □

Remark 3.3. *The condition (i) with the condition (ii) in Lemma 3.2 is equivalent to the condition (iv).*

Theorem 3.4. *Let (Δ,P) be a FSS over N. Then (Δ,P) is called an $(\in,\in\vee q_k)$-FSN over N if and only if for all $\rho\in P$ and $u,v\in N$,*

(i) $\Delta_\rho(u-v)\geq\min\{\Delta_\rho(u),\Delta_\rho(v),\frac{1-k}{2}\}$,

(ii) $\Delta_\rho(uv)\geq\min\{\Delta_\rho(u),\Delta_\rho(v),\frac{1-k}{2}\}$.

Proof. The proof follows from Lemma 3.2. □

Example 3.5. *Let $N=\{0,u,v,w\}$ be a near-ring with $(N,+)$ and $(N,\cdot)$ as defined below (Scheme 20 : (7,8,1,2) see [22], p. 408).*

$+$	0	u	v	w
0	0	u	v	w
u	u	0	w	v
v	v	w	0	u
w	w	v	u	0

$\cdot$	0	u	v	w
0	0	0	0	0
u	u	u	u	u
v	0	u	v	w
w	u	0	w	v

Define a FSS (Δ,P) over N, where $P=\{a,b,c\}$, as follows:
$\Delta_a=\{(0,0.7),(u,0.5),(v,0.6),(w,0.5)\}$,
$\Delta_b=\{(0,0.6),(u,0.5),(v,0.2),(w,0.2)\}$and
$\Delta_c=\{(0,0.4),(u,0.1),(v,0.3),(w,0.1)\}$.
We can easily check that (Δ,P) is an $(\in,\in\vee q_k)$-FSN over N with $k=0.4$.

Definition 3.6. *Let (η,Q) be a FSS over N. Then (η,Q) is called an $(\in,\in\vee q_k)$-fuzzy soft ideal (briefly, $(\in,\in\vee q_k)$-FSI) over N if for each $\rho\in Q$, $\eta(\rho)=\eta_\rho$ is an $(\in,\in\vee q_k)$-FI of N, that is,*

(i) $u_r\in\eta_\rho,\ v_s\in\eta_\rho\Rightarrow(u-v)_{\min\{r,s\}}\in\vee q_k\eta_\rho$,

(ii) $u_r\in\eta_\rho,\ v\in N\Rightarrow(v+u-v)_r\in\vee q_k\eta_\rho$,

(iii) $u_r\in\eta_\rho,\ v\in N\Rightarrow(uv)_r\in\vee q_k\eta_\rho$,

(iv) $u_r\in\eta_\rho,\ v,w\in N\Rightarrow(v(w+u)-vw)_r\in\vee q_k\eta_\rho$,

for all $u,v,w\in N$ and $r,s\in(0,1]$.

An $(\in,\in\vee q_k)$-fuzzy soft ideal over N with $k=0$ is called an $(\in,\in\vee q)$-*fuzzy soft ideal* over N.

Lemma 3.7. *The conditions $(i)-(iv)$ in Definition 3.6 are equivalent to*

(i) $\eta_\rho(u-v)\geq\min\{\eta_\rho(u),\eta_\rho(v),\frac{1-k}{2}\}$,

(ii) $\eta_\rho(v+u-v)\geq\min\{\eta_\rho(u),\frac{1-k}{2}\}$,

(iii) $\eta_\rho(uv)\geq\min\{\eta_\rho(u),\frac{1-k}{2}\}$,

(iv) $\eta_\rho(u(v+w)-uv)\geq\min\{\eta_\rho(w),\frac{1-k}{2}\}$.

for all $u,v,w\in N$, respectively.

Proof. From Lemma 3.11 of [9], the proof can be achieved. □

Theorem 3.8. *Let (η,Q) be a FSS over N. Then (η,Q) is an $(\in,\in\vee q_k)$-FSI over N if and only if for all $\rho\in P$ and $u,v,w\in N$,*

(i) $\eta_\rho(u-v)\geq\min\{\eta_\rho(u),\eta_\rho(v),\frac{1-k}{2}\}$,

(ii) $\eta_\rho(v+u-v)\geq\min\{\eta_\rho(u),\frac{1-k}{2}\}$,

(iii) $\eta_\rho(uv)\geq\min\{\eta_\rho(u),\frac{1-k}{2}\}$,

(iv) $\eta_\rho(u(v+w)-uv)\geq\min\{\eta_\rho(w),\frac{1-k}{2}\}$.

Proof. The proof follows from Lemma 3.7. □

Example 3.9. *Let $N=\{0,u,v,w\}$ be a near-ring as in Example 3.5. Define a FSS (η,Q) over N, where $Q=\{a,b\}$, as shown below*
$\eta_a=\{(0,0.7),(u,0.5),(v,0.2),(w,0.2)\}$ *and*
$\eta_b=\{(0,0.6),(u,0.3),(v,0.3),(w,0.5)\}$.
We can easily verify that (η,Q) is an $(\in,\in\vee q_k)$-FSN and an $(\in,\in\vee q_k)$-FSI over N with $k=0.4$.

Remark 3.10. *It is obvious that every $(\in,\in\vee q_k)$-FSI over N is an $(\in,\in\vee q_k)$-FSN over N, but the converse is not true in general as explained below.*

Example 3.11. *Consider the $(\in,\in\vee q_k)$-FSN as defined in Example 3.5. Since $\Delta_c(vu)=\Delta_c(u)=0.1\ngeq 0.3=min\{0.3,0.3\}=min\{\Delta_c(v),\frac{1-k}{2}\}$, (Δ,P) is not an $(\in,\in\vee q_k)$-FSI over N.*

Remark 3.12. *For any $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) (Δ,P) over N, we can conclude that*

(i) if $\Delta_\rho(u)<\frac{1-k}{2}$ for all $\rho\in P$ and $u\in N$, then (Δ,P) is a FSN over N,

(ii) if $k=0$, then (Δ,P) is an $(\in,\in\vee q)$-fuzzy soft near-ring (resp. ideal) over N.

Remark 3.13. *Every FSN (resp. FSI) and $(\in,\in\vee q)$-FSN (resp. $(\in,\in\vee q)$-FSI) is an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N, but in genreal the converses are not true as demonstrated below.*

Example 3.14. *Consider the near-ring N as in Example 3.5. Define a FSS (Ψ,R) over N, where $R=\{a,b\}$, is given by*
$\Psi_a=\{(0,0.4),(u,0.7),(v,0.4),(w,0.6)\}$ *and*
$\Psi_b=\{(0,0.6),(u,0.6),(v,0.3),(w,0.3)\}$.
Then (Ψ,R) is an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N with $k=0.2$. But

(i) (Ψ,R) is not a FSN over N, since $0.4=\Psi_a(0)=\Psi_a(u-u)\ngeq\min\{\Psi_a(u),\Psi_a(u)\}=0.7$, Ψ_a is not a fuzzy subnear-ring of N.

(ii) (Ψ,R) is not an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N, since $u_{0.7}\in\Psi_a$ and $(u-u)_{0.4}\overline{\in\vee q}\Psi_a$, Ψ_a is not an $(\in,\in\vee q)$-fuzzy subnear-ring (resp. ideal) of N.

Theorem 3.15. *The restricted intersection of two $(\in,\in\vee q_k)$-FSNs (resp. $(\in,\in\vee q_k)$-FSIs) over N is an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N when it is non-null.*

Proof. Let (Δ,P_1) and (Θ,P_2) be $(\in,\in\vee q_k)$-FSNs of N and let $(\Delta,P_1)\sqcap_r(\Theta,P_2)=(\Psi,P)$, where $P=P_1\cap P_2$ and $\Psi(\rho)=\Delta(\rho)\cap\Theta(\rho)$ for all $\rho\in Supp(\Psi,P)$. Since $\Delta(\rho)$ and $\Theta(\rho)$ are $(\in,\in\vee q_k)$-FNs of N, by Theorem 3.16 of [9], $\Delta(\rho)\cap\Theta(\rho)=\Psi(\rho)$ is $(\in,\in\vee q_k)$-FN of N for all $\rho\in Supp(\Psi,P)$. Hence (Ψ,P) is an $(\in,\in\vee q_k)$-FSN over N. Similarly, we can prove for $(\in,\in\vee q_k)$-FSIs. □

Theorem 3.16. *The extended intersection of two $(\in,\in\vee q_k)$-FSNs (resp. $(\in,\in\vee q_k)$-FSIs) over N is an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N when it is non-null.*

Proof. Let (Δ,P_1) and (Θ,P_2) be two $(\in,\in\vee q_k)$-FSNs over N and let $(\Delta,P_1)\sqcap_e(\Theta,P_2)=(\Psi,P)$ be the extended intersection of (Δ,P_1) and (Θ,P_2). Then $P=P_1\cup P_2$ and for any $\rho\in Supp(\Psi,P)$, if $\rho\in P_1\backslash P_2$, then $\Psi(\rho)=\Delta(\rho)$ is an $(\in,\in\vee q_k)$-FN of N, if $\rho\in P_2\backslash P_1$, then $\Psi(\rho)=\Theta(\rho)$ is an $(\in,\in\vee q_k)$-FN of N, and if $\rho\in P_1\cap P_2$, then $\Psi(\rho)=\Delta(\rho)\cap\Theta(\rho)$ is an $(\in,\in\vee q_k)$-FN of N, since the intersection of two $(\in,\in\vee q_k)$-FNs of N is an $(\in,\in\vee q_k)$-FN of N. Therefore $\Psi(\rho)$ is an $(\in,\in\vee q_k)$-FN of N for all $\rho\in Supp(\Psi,P)$. Hence (Ψ,P) is an $(\in,\in\vee q_k)$-FSN over N. Similarly, we can show for $(\in,\in\vee q_k)$-FSIs. □

Theorem 3.17. *If (Δ,P_1) and (Θ,P_2) are $(\in,\in\vee q_k)$-FSNs (resp. $(\in,\in\vee q_k)$-FSIs) over N such that $P_1\cap P_2=\emptyset$, then their union is an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N.*

Proof. Using Definition 2.7, we can write $(\Delta,P_1)\sqcup(\Theta,P_2)=(\Psi,P)$, where $P=P_1\cup P_2$. Since $P_1\cap P_2=\emptyset$, we have either $\rho\in P_1\backslash P_2$ or $\rho\in P_2\backslash P_1$ for all $\rho\in Supp(\Psi,P)$. For any $\rho\in Supp(\Psi,P)$, if $\rho\in P_1\backslash P_2$, then $\Psi(\rho)=\Delta(\rho)$ is an $(\in,\in\vee q_k)$-FN of N, and if $\rho\in P_2\backslash P_1$, then $\Psi(\rho)=\Theta(\rho)$ is an $(\in,\in\vee q_k)$-FN of N. Thus $\Psi(\rho)$ is an $(\in,\in\vee q_k)$-FN of N for all $\rho\in Supp(\Psi,P)$. Hence (Ψ,P) is an $(\in,\in\vee q_k)$-FSN over N. Similarly, we can prove for $(\in,\in\vee q_k)$-FSIs. □

Theorem 3.18. *Let (Δ,P_1) and (Θ,P_2) be $(\in,\in\vee q_k)$-FSNs (resp. $(\in,\in\vee q_k)$-FSIs) over N. Then $(\Delta,P_1)\wedge(\Theta,P_2)$ is an $(\in,\in\vee q_k)$-FSN (resp. $(\in,\in\vee q_k)$-FSI) over N when it is non-null.*

Proof. By Definition 2.8, we can write $(\Delta,P_1)\wedge(\Theta,P_2)=(\Psi,P)$, where $P=P_1\times P_2$ and $\Psi(\rho,\sigma)=\Delta(\rho)\cap\Theta(\sigma)$ for all $(\rho,\sigma)\in P_1\times P_2$. Since $\Delta(\rho)$ and $\Theta(\sigma)$ are $(\in,\in\vee q_k)$-FNs of N for all $\rho\in P_1$ and $\sigma\in P_2$, $\Delta(\rho)\cap\Theta(\sigma)=\Psi(\rho,\sigma)$ is also an $(\in,\in\vee q_k)$-FN of N for all $(\rho,\sigma)\in P_1\times P_2$. Therefore (Ψ,P) is an $(\in,\in\vee q_k)$-FSN over N. Similarly, we can show for $(\in,\in\vee q_k)$-FSIs.

□

In the subsequent example, we illustrate that $(\Delta,P_1)\vee(\Theta,P_2)$ is not an $(\in,\in\vee q_k)$-FSN over N, in general.

Example 3.19. *Consider the near-ring N as given in Example 3.5. Define FSSs (Δ,P_1) and (Θ,P_2) over N, where $P_1=\{a,b,c\}$ and $P_2=\{b,c\}$, as follows:*
$\Delta_a=\{(0,0.7),(u,0.5),(v,0.6),(w,0.8)\}$,
$\Delta_b=\{(0,0.4),(u,0.5),(v,0.2),(w,0.2)\}$,
$\Delta_c=\{(0,0.4),(u,0.1),(v,0.3),(w,0.1)\}$,
$\Theta_b=\{(0,0.5),(u,0.3),(v,0.7),(w,0.3)\}$ *and*
$\Theta_c=\{(0,0.8),(u,0.4),(v,0.1),(w,0.1)\}$.

Then (Δ,P_1) and (Θ,P_2) are $(\in,\in\vee q_k)$-FSNs over N with $k=0.2$.
Let $P=P_1\times P_2=\{(a,b),(a,c),(b,b),(b,c),(c,b),(c,c)\}$. Define a FSS $(\Psi,P)=(\Delta,P_1)\wedge(\Theta,P_2)$ over N as follows:
$\Psi_{(a,b)}=\{(0,0.5),(u,0.3),(v,0.6),(w,0.3)\}$,
$\Psi_{(a,c)}=\{(0,0.7),(u,0.4),(v,0.1),(w,0.1)\}$,
$\Psi_{(b,b)}=\{(0,0.4),(u,0.3),(v,0.2),(w,0.2)\}$,
$\Psi_{(b,c)}=\{(0,0.4),(u,0.4),(v,0.1),(w,0.1)\}$,
$\Psi_{(c,b)}=\{(0,0.4),(u,0.1),(v,0.3),(w,0.1)\}$ *and*
$\Psi_{(c,c)}=\{(0,0.4),(u,0.1),(v,0.1),(w,0.1)\}$.

Then (Ψ,P) is an $(\in,\in\vee q_k)$-FSN and also an $(\in,\in\vee q_k)$-FSI over N with $k=0.2$.

Define a FSS $(\Upsilon,P)=(\Delta,P_1)\vee(\Theta,P_2)$ as follows:
$\Upsilon_{(a,b)}=\{(0,0.7),(u,0.5),(v,0.7),(w,0.8)\}$,
$\Upsilon_{(a,c)}=\{(0,0.8),(u,0.5),(v,0.6),(w,0.8)\}$,
$\Upsilon_{(b,b)}=\{(0,0.5),(u,0.5),(v,0.7),(w,0.3)\}$
$\Upsilon_{(b,c)}=\{(0,0.8),(u,0.5),(v,0.2),(w,0.2)\}$,
$\Upsilon_{(c,b)}=\{(0,0.5),(u,0.3),(v,0.7),(w,0.3)\}$ *and*
$\Upsilon_{(c,c)}=\{(0,0.8),(u,0.4),(v,0.3),(w,0.1)\}$.

Then (Υ,P) is not an $(\in,\in\vee q_k)$-FSN over N with $k=0.2$, because $\Upsilon_{(c,c)}(u-v)=\Upsilon_{(c,c)}(w)=0.1\not\geq 0.4=\max\{0.4,0.3,0.4\}=\max\{\Upsilon_{(c,c)}(u),\Upsilon_{(c,c)}(v),\frac{1-k}{2}\}$, $\Upsilon_{(c,c)}$ is not an $(\in,\in\vee q_k)$-FN of N.

In the following theorem, we give an additional condition for $(\Delta,P_1)\vee(\Theta,P_2)$ to be an $(\in,\in\vee q_k)$-FSN.

Theorem 3.20. *Let (Δ,P_1) and (Θ,P_2) be $(\in,\in\vee q_k)$-FSNs over N and let $\Delta(\rho)\subseteq\Theta(\sigma)$ or $\Theta(\sigma)\subseteq\Delta(\rho)$ for all $(\rho,\sigma)\in P_1\times P_2$. Then $(\Delta,P_1)\vee(\Theta,P_2)$ is an $(\in,\in\vee q_k)$-FSN over N.*

Proof. Using Definition 2.8, we can write $(\Delta,P_1)\vee(\Theta,P_2)=(\Psi,P)$, where $P=P_1\times P_2$ and $\Psi(\rho,\sigma)=\Delta(\rho)\cup\Theta(\sigma)$ for all $(\rho,\sigma)\in P_1\times P_2$. For any $(\rho,\sigma)\in P_1\times P_2$, if $\Delta(\rho)\subseteq\Theta(\sigma)$, then $\Psi(\rho,\sigma)=\Delta(\rho)\cup\Theta(\sigma)=\Theta(\sigma)$ is an $(\in,\in\vee q_k)$-FN of N and if $\Theta(\sigma)\subseteq\Delta(\rho)$, then $\Psi(\rho,\sigma)=\Delta(\rho)\cup\Theta(\sigma)=\Delta(\rho)$ is an $(\in,\in\vee q_k)$-FN of N. Hence (Ψ,P) is an $(\in,\in\vee q_k)$-FSN over N. □

Theorem 3.21. *Let $\{(\Delta_i,P_i)|i\in\Omega\}$ be a family of $(\in,\in\vee q_k)$-FSNs over N. Then*

(i) $\{\sqcap_r(\Delta_i,P_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSN over N if $\{\sqcap_r(\Delta_i,P_i)|i\in\Omega\}\neq\Phi$,

(ii) $\{\bigwedge(\Delta_i,P_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSN over N if $\{\bigwedge(\Delta_i,P_i)|i\in\Omega\}\neq\Phi$,

(iii) if $P_i\cap P_j=\emptyset$ for all $i,j\in\Omega$, $i\neq j$, then $\{\sqcup(\Delta_i,P_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSN over N.

Proof. (i) Let $\{\sqcap_r(\Delta_i,P_i)|i\in\Omega\}=(\Delta,P)$, where $P=\bigcap_{i\in\Omega}P_i$ and for each $\rho\in P$, $\Delta(\rho)=\bigcap_{i\in\Omega}\Delta_i(\rho)$. Suppose $(\Delta,P)\neq\Phi$. If $\rho\in P$, then $\Delta(\rho)=\bigcap_{i\in\Omega}\Delta_i(\rho)\neq\emptyset$. Since (Δ_i,P_i) is an $(\in,\in\vee q_k)$-FSN for all $i\in\Omega$, $\Delta_i(\rho)$ is an $(\in,\in\vee q_k)$-FN of N for all $\rho_i\in Supp(\Delta_i,P_i)$ and $i\in\Omega$ and so by Theorem 3.16 in [9], $\bigcap_{i\in\Omega}\Delta_i(\rho)$ is an $(\in,\in\vee q_k)$-FN of N for all $\rho\in Supp(\Delta,P)$. Hence $(\Delta,P)=\{\sqcap_r(\Delta_i,P_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSN over N.

(ii) Let $\{\bigwedge(\Delta_i,P_i)|i\in\Omega\}=(\Delta,P)$, where $P=\prod_{i\in\Omega}P_i$ and for each $\rho=(\rho_i)_{i\in\Omega}\in P$, $\Delta(\rho)=\bigcap_{i\in\Omega}\Delta_i(\rho_i)$. Suppose $(\Delta,P)\neq\Phi$. If $\rho\in P$, then $\Delta(\rho)=\bigcap_{i\in\Omega}\Delta_i(\rho_i)\neq\emptyset$. Since (Δ_i,P_i) is an $(\in,\in\vee q_k)$-FSN for all $i\in\Omega$, $\Delta_i(\rho_i)$ is an $(\in,\in\vee q_k)$-FN of N for all $\rho_i\in Supp(\Delta_i,P_i)$ and $i\in\Omega$ and so by Theorem 3.16 in [9], $\bigcap_{i\in\Omega}\Delta_i(\rho_i)$ is an $(\in,\in\vee q_k)$-FN of N for all $\rho_i\in Supp(\Delta_i,P_i)$. Hence $(\Delta,P)=\{\bigwedge(\Delta_i,P_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSN over N.

Similarly, we can prove (iii). □

Definition 3.22. *Let η and τ be two $(\in,\in\vee q_k)$-FNs of N. Then η is called an $(\in,\in\vee q_k)$-FN of τ if $\eta(u)\leq\tau(u)$ for all $u\in N$.*

Definition 3.23. *Let (Δ,P_1) and (Θ,P_2) be $(\in,\in\vee q_k)$-FSNs over N. Then (Δ,P_1) is called an $(\in,\in\vee q_k)$-fuzzy soft subnear-ring of (Θ,P_2) if $P_1\subseteq P_2$ and $\Delta(\rho)$ is an $(\in,\in\vee q_k)$-FN of $\Theta(\rho)$ for all $\rho\in Supp(\Delta,P_1)$.*

Theorem 3.24. *If (Δ,P_1) and (Θ,P_2) are $(\in,\in\vee q_k)$-FSNs over N and if $\Delta(\rho)\subseteq\Theta(\rho)$ for all $\rho\in Supp(\Delta,P_1)$, then (Δ,P_1) is an $(\in,\in\vee q_k)$-fuzzy soft subnear-ring of (Θ,P_2).*

Proof. Straightforward. □

Definition 3.25. *Let η be a fuzzy subset of N and τ be an $(\in,\in\vee q_k)$-FNs of N. Then η is called an $(\in,\in\vee q_k)$-FI of τ if $\eta(u)\leq\tau(u)$ for all $u\in N$ and η is an $(\in,\in\vee q_k)$-FI of N.*

Definition 3.26. *Let (Δ,P) be an $(\in,\in\vee q_k)$-FSN over N. Then a FSS (η,Q) over N is called an $(\in,\in\vee q_k)$-fuzzy soft ideal of (Δ,P) expressed by $(\eta,Q)\trianglelefteq(\Delta,P)$ if $Q\subseteq P$ and $\eta(\rho)$ is an $(\in,\in\vee q_k)$-FI of $\Delta(\rho)$, denoted by $\eta(\rho)\lhd\Delta(\rho)$, for all $\rho\in Supp(\eta,Q)$.*

Example 3.27. *Let (Δ,P) be an $(\in,\in\vee q_k)$-FSN as in Example 3.5. Define a FSS (η,Q) over N, where $Q=\{a,b\}$, as follows:*
$\eta_a=\{(0,0.4),(u,0.5),(v,0.6),(w,0.4)\}$ *and*
$\eta_b=\{(0,0.3),(u,0.2),(v,0.1),(w,0.1)\}$.
We can easily verify that (η,Q) is an $(\in,\in\vee q_k)$-FSI of (Δ,P) over N with $k=0.3$.

Theorem 3.28. *The restricted intersection of two $(\in,\in\vee q_k)$-FSIs of an $(\in,\in\vee q_k)$-FSN (Δ,P) over N is an $(\in,\in\vee q_k)$-FSI of (Δ,P) when it is non-null.*

Proof. Let $(\xi,Q_1)\trianglelefteq(\Delta,P)$ and $(\tau,Q_2)\trianglelefteq(\Delta,P)$. By Definition 2.10, we can write $(\xi,Q_1)\sqcap_r(\tau,Q_2)=(\eta,Q)$, where $Q=Q_1\cap Q_2\neq\emptyset$ and $\eta(\rho)=\xi(\rho)\cap\tau(\rho)$ for all $\rho\in Supp$ (η,Q). Since $Q_1\subseteq P$ and $Q_2\subseteq P$, we have $Q_1\cap Q_2=Q\subseteq P$. Suppose that (η,Q) is non-null. Since $(\xi,Q_1)\trianglelefteq(\Delta,P)$ and $(\tau,Q_2)\trianglelefteq(\Delta,P)$, we have $\xi(\rho)\lhd\Delta(\rho)$ and $\tau(\rho)\lhd\Delta(\rho)$ for all $\rho\in Supp(\eta,Q)$. So $\xi(\rho)\cap\tau(\rho)=\eta(\rho)\lhd\Delta(\rho)$ for all $\rho\in Supp(\eta,Q)$. Hence $(\xi,Q_1)\sqcap_r(\tau,Q_2)=(\eta,Q)$ is an $(\in,\in\vee q_k)$-FSI of (Δ,P) over N. □

Theorem 3.29. *The union of two $(\in,\in\vee q_k)$-FSIs of an $(\in,\in\vee q_k)$-FSN (Δ,P) over N is an $(\in,\in\vee q_k)$-FSI of (Δ,P).*

Proof. Let $(\xi,Q_1)\trianglelefteq(\Delta,P)$ and $(\tau,Q_2)\trianglelefteq(\Delta,P)$. By Definition 2.9, we can write $(\xi,Q_1)\sqcup(\tau,Q_2)=(\eta,Q)$, where $Q=Q_1\cup Q_2$ and for all $\rho\in Supp(\eta,Q)$,

$$\eta(\rho)=\begin{cases}\xi(\rho) & \text{if } \rho\in Q_1\backslash Q_2,\\ \tau(\rho) & \text{if } \rho\in Q_2\backslash Q_1,\\ \xi(\rho)\cup\tau(\rho) & \text{if } \rho\in Q_1\cap Q_2.\end{cases}$$

Obviously $Q_1\cup Q_2=Q\subseteq P$. Since $Q_1\cap Q_2=\emptyset$, either $\rho\in Q_1\backslash Q_2$ or $\rho\in Q_2\backslash Q_1$ for all $\rho\in Supp(\eta,Q)$. If $\rho\in Q_1\backslash Q_2$, then $\eta(\rho)=\xi(\rho)\lhd\Delta(\rho)$ and if $\rho\in Q_2\backslash Q_1$, then $\eta(\rho)=\tau(\rho)\lhd\Delta(\rho)$. Thus $\eta(\rho)\lhd\Delta(\rho)$ for all $\rho\in Supp(\eta,Q)$. Therefore (η,Q) is an $(\in,\in\vee q_k)$-FSI of (Δ,P) over N. □

Theorem 3.30. *Let $\{(\eta_i,Q_i)|i\in\Omega\}$ be a family of $(\in,\in\vee q_k)$-FSIs of an $(\in,\in\vee q_k)$-FSN (Δ,P) over N. Then*

(i) $\{\sqcap_r(\eta_i,Q_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSI of (Δ,P) if $\{\sqcap_r(\eta_i,Q_i)|i\in\Omega\}\neq\Phi$.

(ii) $\{\bigwedge(\eta_i,Q_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSI of (Δ,P) if $\{\bigwedge(\eta_i,Q_i)|i\in\Omega\}\neq\Phi$.

(iii) if $Q_i\cap Q_j=\emptyset$ for all $i,j\in\Omega$, $i\neq j$, then $\{\sqcup(\eta_i,Q_i)|i\in\Omega\}$ is an $(\in,\in\vee q_k)$-FSI of (Δ,P).

Proof. The proofs of $(i)-(iii)$ are similar to the proofs of Theorem 3.21 and Theorem 3.29 and so we omit the proofs. □

4. Conclusion

In this article, as a generalization of $(\in,\in\vee q)$-fuzzy soft near-ring and $(\in,\in\vee q)$-fuzzy soft ideal, we have introduced the notions of $(\in,\in\vee q_k)$-fuzzy soft near-ring and $(\in,\in\vee q_k)$-fuzzy soft ideal over a near-ring respectively. We have also introduced the concept of $(\in,\in\vee q_k)$-fuzzy soft subnear-ring (resp. ideal) of an $(\in,\in\vee q_k)$-fuzzy soft near-ring and studied some of their properties with illustrated examples. We have applied the operations of fuzzy soft sets to $(\in,\in\vee q_k)$-fuzzy soft near-rings and $(\in,\in\vee q_k)$-fuzzy soft ideals.

Acknowledgment

The authors are highly grateful to anonymous reviewers for their valuable comments and suggestions.

References

[1] S. Abou-Zaid, On fuzzy subnear-rings and ideals, Fuzzy Sets and Systems **44** (1991) 139-146.
[2] B. Ahmat and A. Kharal, On fuzzy soft sets, *Adv. Fuzzy Syst.*, Article ID 586507(2009), 3458-3463.
[3] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.*, 57(2009), 1547-1553.
[4] A. Aygünoğlu and H. Aygün, Introduction to fuzzy soft groups, *Comput. Math. Appl.*, 58(2009), 1279-1286.
[5] M. Azhar, N. Yaqoob, M. Gulistan and M.M. Khalaf, On $(\in,\in\vee q_k)$-fuzzy hyperideals in ordered LA-semihypergroups, *Discrete Dyn. Nat. Soc.*, Vol. 2018, Article ID 9494072(2018), 13 pages.
[6] S.K. Bhakat and P. Das, On the definition of a fuzzy subgroup, *Fuzzy Sets and Systems*, 51(1992), 235-241.
[7] S.K. Bhakat and P. Das, $(\in,\in\vee q)$-fuzzy subgroup, *Fuzzy Sets and Systems*, 80(1996), 359-368.
[8] S.K. Bhakat and P. Das, Fuzzy subrings and ideals redefined, *Fuzzy Sets and Systems*, 81(1996), 383-393.
[9] P. Dheena and S. Coumaressane, Generalization of $(\in,\in\vee q)$-fuzzy subnear-ring and ideals, *Iran. J. Fuzzy Syst.*, 5(1)(2008), 79-97.
[10] Faisal, N. Yaqoob and R. Chinram, A Note on $(\in,\in\vee q_k)$–Fuzzy Γ-Ideals of Γ-Semigroups, *Appl. Math. Sci.*, 6(101)(2012), 5013-5028.
[11] M. Gulistan, M. Shahzad, N. Yaqoob, On $(\in,\in\vee q_k)$-fuzzy KU-ideals of KU-algebras, *Acta Univ. Apulensis Math. Inform.*, 39(2014), 75-83.
[12] E. İnan and M.A. Öztürk, Fuzzy soft rings and ideals, *Neural Comput. Appl.*, 21(Suppl. 1)(2012), s1-s8.

[13] Y.B. Jun, Generalizations of $(\in, \in \vee q)$-fuzzy subalgebrs in BCK/BCI-algebras, *Comput. Math. Appl.*, 58(2009), 1383-1390.

[14] Y.B. Jun, M.S. Kang and C.H. Park, Fuzzy subgroups based on fuzzy points, *Commun. Korean Math. Soc.*, 26(3)(2011), 349-371.

[15] M. Khan, B. Davvaz, N. Yaqoob, M. Gulistan and M.M. Khalaf, On $(\in, \in \vee q_k)$-intuitionistic (fuzzy ideals, fuzzy soft ideals) of subtraction algebras, *Songklanakarin J. Sci. Technol.*, 37(4)(2015), 465-475.

[16] P.K. Maji, R. Biswas and P.R. Roy, Fuzzy soft sets, *J. Fuzzy Math.*, 9(3)(2001), 589-602.

[17] P.P. Ming and L.Y. Ming, Fuzzy topology I. Neighbourhood structures of a fuzzy point and Moore-Smith convergence, *J. Math. Anal. Appl.*, 76(1980), 571-579.

[18] S. Molodtsov, Soft set theory-first results, *Comput. Math. Appl.*, 37(1999), 19-31.

[19] AL. Narayanan and T. Manikantan, $(\in, \in \vee q)$-fuzzy subnear-rings and $(\in, \in \vee q)$-fuzzy ideals of near-rings, *J. Appl. Math. Comput.*, 18(1-2) (2005), 419-430.

[20] M.A. Öztürk and E. İnan, Fuzzy soft subnear-rings and $(\in, \in \vee q)$-fuzzy soft subnear-rings, *Comput. Math. Appl.*, 63(2012), 617-628.

[21] B. Pazar Varol, A. Aygünoğlu and H. Aygün, On fuzzy soft rings, *J. Hyperstruct.*, 1(2)(2002), 1-15.

[22] G. Pilz, Near-Rings, North-holland, Amsterdam, 1983.

[23] M. Shabir, Y.B. Jun and Y. Nawaz, Semigroups characterized by $(\in, \in \vee q_k)$-fuzzy ideals, *Comput. Math. Appl.*, 60(2010), 1473-1493.

[24] L.A. Zadeh, Fuzzy sets, *Inf. Control*, 8(1965), 338-353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆

ISSN(P):2319 − 3786

Malaya Journal of Matematik

ISSN(O):2321 − 5666

⋆⋆⋆⋆⋆⋆⋆⋆⋆