k -Prime labeling of certain cycle connected graphs

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Abstract

A k - prime labeling of a graph G is an injective function $f : V \rightarrow \{k, k+1, k+2, \dots, k+|V|-1\}$ for some positive integer k that induces a function $f^+ : E(G) \rightarrow N$ of the edges of G defined by $f^+(uv) = \gcd(f(u), f(v)), \forall e = uv \in E(G)$ such that $\gcd(f(u), f(v)) = 1$. A graph G that admits k - prime labeling is called a k - prime graph. In this paper, we investigate some cycle related graphs which admit k - prime labeling.

Keywords

k - prime labeling, Cycle graph, Tadpole graph, Barycentric subdivision, Friendship graph.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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Article History: Received 22 November 2018; Accepted 09 May 2019

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1. Introduction

In our study we consider simple, finite, undirected and non - trivial graph $G = (V, E)$ with the vertex set V and the edge set E . The idea of prime labeling was introduced by Entringer and the problem was discussed by A. Tout, A.N. Dabboucy and K. Howalla. [1]. Later Vaidya and Prajapati [2] studied the concept of k - prime labeling. They proved that P_m is k - prime for each positive integer k . In our work we have discussed k - prime labeling for all positive integer $k \geq 2$.

2. Preliminaries

We begin with the definition of k - prime labeling.

Definition 2.1. A k - prime labeling of a graph G is an injective function $f : V \rightarrow \{k, k+1, k+2, \dots, k+|V|-1\}$ for some positive integer k that induces a function $f^+ : E(G) \rightarrow N$ of the edges of G defined by $f^+(uv) = \gcd(f(u), f(v)), \forall e = uv \in E(G)$ such that $\gcd(f(u), f(v)) = 1$. A graph G that admits k - prime labeling is called a k - prime graph.

The definition of Tadpole Graph $T_{m,n}$, Barycentric Subdivision of cycles C_n and Friendship Graph F_n are available in literature.

3. Main Results

Theorem 3.1. Even Cycle C_{2n} is k - prime for $n \geq 2$.

Proof. Let us denote the vertices and edges of C_{2n} as $V = \{v_1, v_2, \dots, v_{2n}\}$

$E = \{v_i v_{i+1} : 1 \leq i \leq 2n-1\} \cup \{v_{2n} v_1\}$

Refer Figure 1(a) and 2(a),

Now let us label the vertices of C_{2n} as follows:

Case 1: $(2n-1)$ is prime.

$f(v_i) = k+i-1$, for $1 \leq i \leq 2n$ and $k \not\equiv 0 \pmod{(2n-1)}$.

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and $f^+(v_{2n} v_1) = \gcd(f(v_{2n}), f(v_1)) = 1 \quad \forall e \in E(C_{2n})$.

This is clear from Figure 1(b).

Case 2: $(2n-1)$ is not prime.

$f(v_i) = k+i-1$, for $1 \leq i \leq 2n$ and k not a multiple factor of $(2n-1)$.

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and $f^+(v_{2n} v_1) = \gcd(f(v_{2n}), f(v_1)) = 1 \quad \forall e \in E(C_{2n})$.

This is clear from Figure 2(b). Hence C_{2n} admits k - prime labeling. □

Figure 1. (a) C_6 (b) k - prime labeling of C_6 for $k = 4$

Figure 2. (a) C_{16} (b) k - prime labeling of C_{16} for $k = 7$

Theorem 3.2. Odd Cycle C_{2n+1} is k - prime for k odd and $n = 1, 2, 4$.

Proof. Let us denote the vertices and edges of C_{2n+1} as

$$V = \{v_1, v_2, \dots, v_{2n+1}\}$$

$$E = \{v_i v_{i+1} : 1 \leq i \leq 2n\} \cup \{v_{2n+1} v_1\}$$

Refer Figure 3(a),

Now let us label the vertices of C_{2n+1} as follows:

$$f(v_i) = k + i - 1, \text{ for } 1 \leq i \leq 2n + 1.$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and $f^+(v_{2n+1} v_1) = \gcd(f(v_{2n+1}), f(v_1)) = 1 \quad \forall e \in E(C_{2n+1})$.

This is clear from Figure 3(b).

Hence C_{2n+1} admits k - prime labeling.

□

Figure 3. (a) C_5 (b) k - prime labeling of C_5 for $k = 11$

Theorem 3.3. Tadpole graph $T_{n,m}$ is k - prime for all even positive integer $n \geq 4$ and $m \geq 1$.

Proof. Let us denote the vertices and edges of $T_{n,m}$ as

$$V = \{v_1, v_2, \dots, v_{n+m}\}$$

$$E = \{v_i v_{i+1} : 1 \leq i \leq n + m - 1\} \cup \{v_n v_1\}$$

Refer Figure 4(a),

Now let us label the vertices of $T_{n,m}$ as follows:

Case 1: $(n - 1)$ is prime.

$$f(v_i) = k + i - 1, \text{ for } 1 \leq i \leq n + m \text{ and } k \not\equiv 0 \pmod{(n - 1)}.$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and $f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(T_{n,m})$.

Case 2: $(n - 1)$ is not prime.

$$f(v_i) = k + i - 1, \text{ for } 1 \leq i \leq n + m \text{ and } k \text{ not a multiple factor of } (n - 1).$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and $f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(T_{n,m})$. This is clear from Figure 4(b).

Hence $T_{n,m}$ admits k - prime labeling.

□

Figure 4. (a) $T_{6,3}$ (b) k - prime labeling of $T_{6,3}$ for $k = 6$

Theorem 3.4. Tadpole graph $T_{n,m}$ is k - prime for k odd, $n = 3, 5, 9$ and $m \geq 1$.

Proof. Let us denote the vertices and edges of $T_{n,m}$ as

$$V = \{v_1, v_2, \dots, v_{n+m}\}$$

$$E = \{v_i v_{i+1} : 1 \leq i \leq n + m - 1\} \cup \{v_n v_1\}$$

Refer Figure 5(a),

Now let us label the vertices of $T_{n,m}$ as follows:

$$f(v_i) = k + i - 1, \text{ for } 1 \leq i \leq n + m.$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and $f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(T_{n,m})$.

This is clear from Figure 5(b).

Hence $T_{n,m}$ admits k - prime labeling.

□

Figure 5. (a) $T_{5,3}$ (b) k - prime labeling of $T_{5,3}$ for $k = 11$

Theorem 3.5. Friendship graph F_n ($n \geq 3$) is k - prime for k odd.

Proof. Let us denote the vertices and edges of F_n as

$$V = \{v, v_1, v_2, \dots, v_{2n}\}$$

$$E = \{v v_i : 1 \leq i \leq 2n\} \cup \{v_i v_{i+1} : i = 1, 3, 5, \dots, 2n - 1\}$$

Refer Figure 6(a),

Now let us label the vertices of F_n as follows:

$f(v) = p$, where p is largest prime less than or equal to $k + 2n$ and remaining $2n$ vertices are distinctly labeled from $\{k, k + 1, \dots, k + 2n\} - p$.

$f(v_i) = k + i - 1$, for $1 \leq i \leq 2n$.

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(vv_i) = \gcd(f(v), f(v_i)) = 1$ and $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1 \quad \forall e \in E(F_n)$.

This is clear from Figure 6(b).

Hence F_n admits k - prime labeling.

Figure 6. (a) F_5 (b) k - prime labeling of F_5 for $k = 5$

Theorem 3.6. The barycentric subdivision $C_n(C_n)$ of cycle C_n is k - prime for $n \geq 3$.

Proof. **Case 1:** $(2n - 1)$ is prime.

Sub Case (a): When k is odd

Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n . Let $v_1, v_2, \dots, v_n$ be the newly inserted vertices of $C_n(C_n)$ where v_{i+1} subdivides the edge $u_i u_{i+1}$ for $1 \leq i \leq n - 1$ and v_1 subdivides the edge $u_n u_1$.

Now let us label the vertices of $C_n(C_n)$ as follows:

Let us denote the vertices and edges of $C_n(C_n)$ as

$$V = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$$

$$E = \{v_i u_i : 1 \leq i \leq n\} \cup \{u_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{u_n v_1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_n v_1\}$$

Refer Figure 7(a),

Now let us label the vertices of $C_n(C_n)$ as follows:

$$f(v_1) = k$$

$$f(u_1) = k + 1$$

$$f(v_i) = k + 2(i - 1), 2 \leq i \leq n,$$

$$f(u_i) = k + 2i - 1, 2 \leq i \leq n,$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i u_i) = \gcd(f(v_i), f(u_i)) = 1$

$$f^+(u_i v_{i+1}) = \gcd(f(u_i), f(v_{i+1})) = 1$$

$$f^+(u_n v_1) = \gcd(f(u_n), f(v_1)) = 1$$

$$f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1 \text{ and}$$

$$f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(C_n(C_n)).$$

This is clear from Figure 7(b).

Sub Case (b): When k is even

Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n . Let $v_1, v_2, \dots, v_n$ be the newly inserted vertices of $C_n(C_n)$ where v_i subdivides the edge $u_i u_{i+1}$ for $1 \leq i \leq n - 1$ and v_n subdivides the edge $u_n v_1$.

Now let us label the vertices of $C_n(C_n)$ as follows:

Let us denote the vertices and edges of $C_n(C_n)$ as

$$V = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$$

$$E = \{u_i v_i : 1 \leq i \leq n\} \cup \{v_i u_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_n u_1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_n v_1\}$$

Refer Figure 8(a),

Let us label the vertices of $C_n(C_n)$ as follows:

$$f(u_1) = k$$

$$f(v_1) = k + 1$$

$$f(u_i) = k + 2(i - 1), 2 \leq i \leq n,$$

$$f(v_i) = k + 2i - 1, 2 \leq i \leq n,$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(u_i v_i) = \gcd(f(u_i), f(v_i)) = 1$

$$f^+(v_i u_{i+1}) = \gcd(f(v_i), f(u_{i+1})) = 1$$

$$f^+(v_n u_1) = \gcd(f(v_n), f(u_1)) = 1$$

$$f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1 \text{ and}$$

$$f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(C_n(C_n)).$$

This is clear from Figure 8(b).

Case 2: $(2n - 1)$ is not prime.

Sub Case (a): When k is odd

Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n . Let $v_1, v_2, \dots, v_n$ be the newly inserted vertices of $C_n(C_n)$ where v_{i+1} subdivides the edge $u_i u_{i+1}$ for $1 \leq i \leq n - 1$ and v_1 subdivides the edge $u_n u_1$.

Now let us label the vertices of $C_n(C_n)$ as follows:

Let us denote the vertices and edges of $C_n(C_n)$ as

$$V = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$$

$$E = \{v_i u_i : 1 \leq i \leq n\} \cup \{u_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{u_n v_1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_n v_1\}$$

Refer Figure 9(a),

Now let us label the vertices of $C_n(C_n)$ as follows:

$$f(v_1) = k$$

$$f(u_1) = k + 1$$

$$f(v_i) = k + 2(i - 1), 2 \leq i \leq n,$$

$$f(u_i) = k + 2i - 1, 2 \leq i \leq n,$$

It is easy to check that the induced function $f^+ : E(G) \rightarrow N$ of edges defined by $f^+(v_i u_i) = \gcd(f(v_i), f(u_i)) = 1$

$$f^+(u_i v_{i+1}) = \gcd(f(u_i), f(v_{i+1})) = 1$$

$$f^+(u_n v_1) = \gcd(f(u_n), f(v_1)) = 1$$

$$f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1 \text{ and}$$

$$f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(C_n(C_n)).$$

This is clear from Figure 9(b).

Sub Case (b): When k is even

Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n . Let $v_1, v_2, \dots, v_n$ be the newly inserted vertices of $C_n(C_n)$ where v_i subdivides the edge $u_i u_{i+1}$ for $1 \leq i \leq n - 1$ and v_n subdivides the edge $u_n v_1$.

Now let us label the vertices of $C_n(C_n)$ as follows:

Let us denote the vertices and edges of $C_n(C_n)$ as

$$V = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$$

$$E = \{u_i v_i : 1 \leq i \leq n\} \cup \{v_i u_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_n u_1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_n v_1\}$$

Refer Figure 10(a),

Let us label the vertices of $C_n(C_n)$ as follows:

$$f(u_1) = k$$

$$f(v_1) = k + 1$$

$f(u_i) = k + 2(i - 1), 2 \leq i \leq n$,
 $f(v_i) = k + 2i - 1, 2 \leq i \leq n$,
 It is easy to check that the induced function $f^+ : E(G) \rightarrow N$
 of edges defined by $f^+(u_i v_i) = \gcd(f(u_i), f(v_i)) = 1$
 $f^+(v_i u_{i+1}) = \gcd(f(v_i), f(u_{i+1})) = 1$
 $f^+(v_n u_1) = \gcd(f(v_n), f(u_1)) = 1$
 $f^+(v_i v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1$ and
 $f^+(v_n v_1) = \gcd(f(v_n), f(v_1)) = 1 \quad \forall e \in E(C_n(C_n))$.
 This is clear from Figure 10(b).
 Hence $C_n(C_n)$ admits k - prime labeling.

Figure 10. (a) $C_8(C_8)$ (b) k - prime labeling of $C_8(C_8)$ for $k = 14$

□

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Figure 7. (a) $C_3(C_3)$ (b) k - prime labeling of $C_3(C_3)$ for $k = 7$

Figure 8. (a) $C_4(C_4)$ (b) k - prime labeling of $C_4(C_4)$ for $k = 6$

Figure 9. (a) $C_{13}(C_{13})$ (b) k - prime labeling of $C_{13}(C_{13})$ for $k = 13$

4. Conclusion

In this paper we have proved that Cycle graph C_n , Tadpole graph $T_{n,m}$, Friendship graph F_n and Barycentric subdivision $C_n(C_n)$ is k - prime. We have discussed k - prime labeling of these graphs for any positive integer k . Further our study is extended to snake related graphs.

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 ISSN(P):2319 – 3786
 Malaya Journal of Matematik
 ISSN(O):2321 – 5666
 ★★★★★★