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(T,S)-Intuitionistic fuzzy ideals in near-rings

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Abstract

In this paper, the concept of intuitionistic fuzzy ideals in near-rings with respect to a t-norm and t-co-norm has been studied and some of their properties are investigated. Using (T,S)-intuitionistic fuzzy ideals, characterizations of quotient near-rings are established.

Keywords

Near-ring, subnear-ring (ideal), fuzzy ideals, intuitionistic fuzzy ideals, (T,S)-intuitionistic fuzzy ideals.

AMS Subject Classification

03E72, 16Y30.

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Contents

1. Introduction

The concept of a fuzzy set was formulated by Zadeh in [15], since then, the theory of fuzzy sets developed by Zadeh and others has evoked tremendous interest among researchers working in different branches of mathematics. In 1971, Rosenfeld [12] extended the concept of fuzzy set theory to group theory and defined fuzzy group and derived some properties. Fuzzy semigroups were introduced by Kuroki [9] as a generalization of classical semigroups. Many classes of semigroups were studied by Kuroki using fuzzy ideals in [10]. Mordeson et. al. [11] gave a systematic exposition of fuzzy semigroups, where one can find theoretical results on fuzzy semigroups and their use in fuzzy coding, fuzzy finite state machines and fuzzy languages. In 1991, Abou-Zaid [1] introduced the notion of fuzzy subnear-rings and ideals in near-rings. Atanassov introduced intuitionistic fuzzy sets which constitute a generalization of the notion of fuzzy sets [2, 3]. The degree of membership of an element is given in a fuzzy set, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. Biswas [4] introduced the notion of intuitionistic fuzzy subgroup of a group by using the notion of intuitionistic fuzzy sets. Kim and Jun [7] introduced the concept of intuitionistic fuzzy ideals of semigroups and in [8], Kim and Lee studied intuitionistic fuzzy bi-ideals of semigroups. Kim and Lee [6] gave the concept of intuitionistic (T,S) normed fuzzy ideals of -rings. Zhan Jianming and Ma Xueling [16], also discussed the various properties on intuitionistic fuzzy ideals of near-rings. Also Cho et al in [5] the notion of a normal intuitionistic fuzzy N-subgroup in a near-ring is introduced and related properties are investigated. In this paper we introduce the notion of intuitionistic fuzzy subnear-ring (ideal) of a near-ring with respect to $t$norm T and $t$conorm S. Then we characterize all of them based on special kind of level sets $U(A;[t,s])$ and $L(A;[t,s])$, which is a generalization of classic level subsets. At the following the behaviour of these structures under homomorphisms is investigated. In particular, by the help of the congruence relations on near-rings, we construct (T,S)-intuitionistic fuzzy subnear-ring (ideal) on near-ring of quotient.

2. Preliminaries

In this section, we review some elementary aspects that are necessary for this paper.

Definition 2.1. An algebra $(N,+,\cdot)$ is said to be a near-ring if it satisfies the following conditions:
(i) $(N,+)$ is a (not necessarily abelian) group.

(ii) $(N,\cdot)$ is a semigroup.
(iii) for all $x,y,z \in N, x(y+z) = xy+xz$.

Definition 2.2. A subset I of a near-ring N is said to be a subnear-ring if $(I,\cdot)$ is also a near-ring.

Proposition 2.3. A subset I of a near-ring N is a subnear-ring of N if and only if $x-y, xy \in I$ for all $x,y \in I$.

Definition 2.4. A mapping $f : N_1 \to N_2$ is called a near-ring homomorphism if $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$ for all $c,y \in N_1$.

Definition 2.5. An ideal I of a near-ring $(R,+,\cdot)$ is a subset of N such that
$(a).(I,+)$ is a normal subgroup of $(N,+)$,
$(b).NI \subset I$,
$(c).(r+i)s-rs \in I$ for all $i \in I$ and $r,s \in N$.
Note that I is a left ideal of N if I satisfies (a) and (b), and I is a right ideal of N if I satisfies (a) and (c). If I is both left and right ideal, I is called an ideal of N.

Definition 2.6. A quotient near-ring (also called a residue-class near-ring) is a near-ring that is the quotient of a near-ring and one of its ideals I, denoted N/I. If I is an ideal of a near-ring N and $a \in N$, then a coset of I is a set of the form $a+I = \{a+s/s \in I\}$. The set of all cosets is denoted by N/I.

Theorem 2.7. If I is an ideal of a near-ring N, the set N/I is a near-ring under the operations $(a+I)+(b+I) = (a+b)+I$ and $(a+I).(b+I) = (a.b)+I$.

Definition 2.8. A mapping $\mu : X \to [0,1]$, where X is an arbitrary nonempty set and is called a fuzzy set in X.

Definition 2.9. Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is an object having the form $A = \{< x,\mu_A,\ \nu_A(x) > | x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x)+\nu_A(x) \le 1$.

Notation: *For the sake of simplicity, we shall use the symbol* $A =< \mu_A, \nu_A >$ *for the IFS,*
$A = \{< x,\mu_A(x),\ \nu_A(x) > | x \in X\}$.

Definition 2.10. A fuzzy subset μ in a near-ring N is said to be fuzzy subnear-ring of N if it satisfies the following conditions:
$(F1)$ for all $x,y \in N$, $\mu(x-y) \ge min\{\mu(x),\mu(y)\}$,
$(F2)$ for all $x,y \in N$, $\mu(xy) \ge min\{\mu(x),\mu(y)\}$.

Definition 2.11. A fuzzy subnearring μ of N is said to be fuzzy ideal if it satisfies the following conditions:
$(F3)$ for all $x,y \in N, \mu(y+x-y) \ge \mu(x)$,
$(F4)$ for all $x,y \in N, \mu(xy) \ge \mu(y)$,
$(F5)$ for all $x,y \in N, \mu((x+z)y-xy) \ge \mu(z)$.
A fuzzy subnear-ring satisfying (F_3) and (F_4) is called fuzzy left ideal and it is a right ideal if it satisfy (F_3) and (F_5)

Definition 2.12. An intuitionistic fuzzy set $A = (\mu_A,\ \lambda_A)$ of a near-ring N is called an intuitionistic fuzzy subnear-ring of N if
(i) $\mu_A(x-y) \ge min\{\mu_A(x),\mu_A(y)\}$
and $\lambda_A(x-y) \le max\{\lambda_A(x),\lambda_A(y)\}$,
(ii) $\mu_A(xy) \ge min\{\mu_A(x),\mu_A(y)\}$
and $\lambda_A(xy) \le max\{\lambda_A(x),\lambda_A(y)\}$,
for all $x,y \in N$.

Definition 2.13. An intuitionistic fuzzy set $A = (\mu_A,\lambda_A)$ in a near-ring N is called an intuitionistic fuzzy ideal of N if
(i) $\mu_A(x-y) \ge \mu_A(x) \wedge \mu_A(y)$ and $\lambda_A(x-y) \le \lambda_A(x) \vee \lambda_A(y)$,
(ii) $\mu_A(y+x-y) = \mu_A(x)$ and $\lambda_A(y+x-y) = \lambda_A(x)$.
(iii) $\mu_A(rx) \ge \mu_A(x)$ and $\lambda(rx) \le \lambda(x)$,
(iv) $\mu((x+i)y-xy) \ge \mu_A(i)$ and $\lambda((x+i)y-xy) \le \lambda_A(i)$ for all $x,y,i \in N$.
If $A = (\mu_A,\lambda_A)$ satisfies (i), (ii) and (iii) then A is called an intuitionistic fuzzy left ideal of N and if $A = (\mu_A,\ \lambda_A)$ satisfies (i), (ii) and (iv) then A is called an intuitionistic fuzzy right ideal of N.

Lemma 2.14. If μ is an intuitionistic fuzzy ideal of N, then $\mu(0) \ge \mu(x)$ and $\nu(0) \le \nu(x)$ for all $x \in N$.

Definition 2.15. A t-norm is a function $T : [0,1]\times[0,1] \to [0,1]$ that satisfies the following conditions for all $x,y,z \in [0,1]$:
$(T1) T(x,1) = x$,
$(T2) T(x,y) = T(y,x)$
$(T3) T(x,T(y,z)) = T(T(x,y),z)$.
$(T4) T(x,y) \le T(x,z)$, whenever $y \le z$.
A simple example of such defined t-norm is a function $T(x,y) = min(x,y)$. In general case, $T(x,y) \le min(x,y)$ and $T(x,0) = 0$ for all $x,y \in [0,1]$.

Definition 2.16. A mapping $S : [0,1]\times[0,1] \to [0,1]$ is called a t-co-norm if the following conditions are satisfied:
$(S1)$ $S(x,0) = x$, for all $x \in [0,1]$
$(S2)$ $S(x,y) = S(y,x)$ for all $x,y \in [0,1]$
$(S3)$ $S(x,S(y,z)) = S(S(x,y),z)$, for all $x,y,z \in [0,1]$
$(S4)$ If $x \le p$ and $y \le q$, then $S(x,y) \le S(p,q)$,
for all $x,y,p,q \in [0,1]$

Proposition 2.17. Let S be a t-co-norm. Then the following conditions are satisfied:
$(1) S(x,1) = 1$, for all $x \in [0,1]$
$(2) S(x,y) \ge max(x,y)$ for all $x,y \in [0,1]$.

Definition 2.18. Let $A =< \mu_A, \nu_A >$ be an intuitionistic fuzzy set in N and let $t \in [0,1]$. Then the sets $U(A : t,s) = \{\{x \in N : \mu_A(x) \ge t\}$ and $\{\nu_A(x) \le s\}$ with $t+s \le 1\}$ is called level set of A.

3. (T,S)-Intuitionistic Fuzzy ideals in Near-rings

In this section, we define (T,S) Intuitionistic Fuzzy ideals of near-rings and prove some basic properties of these ideals.

Definition 3.1. An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in N is called intuitionistic fuzzy subnearring with respect to a t-norm and co-t-norm, shortly (T,S)- intuitionistic fuzzy subnear-ring, of N if:
$(TF1)$ for all $x,y \in N, \mu_A(x-y) \geq T(\mu_A(x),\mu_A(y))$ and $\nu_A(x-y) \leq S(\nu_A(x),\nu_A(y))$
$(TF2)$ for all $x,y \in N, \mu_A(xy) \geq T(\mu_A(x),\mu_A(y))$ and $\nu_A(xy) \leq S(\nu_A(x),\nu_A(y))$

Definition 3.2. An (T,S)-intuitionistic fuzzy subnearring $A = (\mu_A, \nu_A)$ in N is called (T,S)- intuitionistic fuzzy ideal of N if:
$(TF3)$ for all $x,y \in N, \mu_A(y+x-y) \geq T(\mu_A(x))$ and $\nu_A(y+x-y) \leq S(\nu_A(x))$
$(TF4)$ for all $x,y \in N, \mu_A(xy) \geq T(\mu_A(y))$ and $\nu_A(xy) \leq S(\nu_A(y))$
$(TF5)$ for all $x,y \in N, \mu_A((x+z)y-xy) \geq T(\mu_A(z))$ and $\nu_A((x+z)y-xy) \leq T(\nu_A(z))$.

Note that $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy left ideal of R if it satisfies $(TF1),(TF2),(TF3)$ and $(TF4)$, and $A = (\mu_A, \nu_A)$ is an (T,S)-intuitionistic fuzzy right ideal of N if it satisfies $(TF1),(TF2),(TF3)$ and $(TF5)$. $A = (\mu_A, \nu_A)$ is called (T,S)- intuitionistic fuzzy ideal of N if it is both left and right (T,S)-intuitionistic fuzzy ideal of N.

Example 3.3. Consider a nearring $N = \{a,b,c,d\}$ with the following Cayley's tables:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

We define an intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ by $\mu_A(a) > \mu_A(b) > \mu_A(d) = \mu_A(c)$ and $\nu_A(a) < \nu_A(b) < \nu_A(d)$ $= \nu_A(c)$. Let $T : [0,1]\times[0,1] \to [0,1]$ be a function defined by $T(x,y) = max(x+y-1,0)$ which is a t-norm for all $x,y \in [0,1]$ and let $S : [0,1]\times[0,1] \to [0,1]$ be a function defined by $S(x,y) = min(x+y,1)$ which is a t-conorm for all $x,y \in [0,1]$. By routine calculations, it is easy to check that A is a (T,S)- intuitionistic fuzzy ideal of N.

The following propositions are obvious.

Proposition 3.4. Every intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in a near-ring N is a (T,S)- intuitionistic fuzzy ideal of N if and only if the level set
$U(A;\alpha,\beta) = \{x \in N / \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$
is an ideal of N when it is nonempty.

Proposition 3.5. Every (T,S)-intuitionistic fuzzy ideal of a near-ring N is a (T,S)-intuitionistic fuzzy subnearring of N. Converse of this proposition may not be true in general as seen in the following example.

Example 3.6. Let $N = \{a,b,c,d\}$ be a set with binary operations as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Then $(N,+,\cdot)$ is a near-ring. We define an intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ and $\nu_A : N \to [0,1]$ by $\mu_A(a) > \mu_A(b) > \mu_A(d) = \mu_A(c)$ and $\nu_A(a) < \nu_A(b) \leq \nu_A(c) = \nu_A(d)$. Let $T : [0,1]\times[0,1] \to [0,1]$ be a function defined by $T(x,y) = max(x+y-1,0)$ which is a t-norm for all $x,y \in [0,1]$ and let $S : [0,1]\times[0,1] \to [0,1]$ be a function defined by $S(x,y) = min(x+y,1)$ which is a t-conorm for all $x,y \in [0,1]$. By routine calculations, it is easy to check that A is a (T,S)-intuitionistic fuzzy subnear-ring of N. It is clear that A is also (T,S)- intuitionistic fuzzy left ideal of N, but $A = (\mu_A, \nu_A)$ is not (T,S) intuitionistic fuzzy right ideal of N. Since $\mu_A(c+b)d-cd) = \mu_A(d) \leq \mu_A(b)$.

Definition 3.7. Let N_1 and N_2 be two near-rings and f a function of N_1 into N_2. If B is a fuzzy set in N_2, then the image of A under f is the fuzzy set in N_1 defined by

$$f(A)(x) = \begin{cases} (\sup\limits_{x \in f^{-1}(y)} \mu_A(x), \inf\limits_{x \in f^{-1}(y)} \nu_A(x)) \\ if\ f^{-1}(y) \neq \phi, \\ 0, otherwise \end{cases}$$

for each $y \in N_2$.

Theorem 3.8. Let $f : N_1 \to N_2$ be an onto homomorphism of near-rings. If $A = (\mu_A, \nu_A)$ is a (T,S)- intuitionistic fuzzy ideal in N_1, then $f(A)$ is a (T,S)- intuitionistic fuzzy ideal in N_2.

Proof. Let $y_1, y_2 \in R_2$. Then
$\{x/x \in f^{-1}(y_1-y_2)\} \supseteq \{x_1-x_2/x_1 \in f^{-1}(y_1),$
$x_2 \in f^{-1}(y_2)\},$
and hence
$f(A)(y_1-y_2)$
$= (\sup\{\mu_A(x)/f^{-1}(y_1-y_2)\},$
$\inf\{\nu_A(x)/f^{-1}(y_1-y_2)\}$
$\geq \sup\{T(\mu_A(x_1),\mu_A(x_2))/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\},$
$\inf\{S(\nu_A(x_1),\nu_A(x_2))/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$
$= \sup\{T(\mu_A(x_1)/x_1 \in f^{-1}(y_1)\},$
$\sup\{\mu_A(x_2)/x_2 \in f^{-1}(y_2)\}), \inf\{S(\nu_A(x_1)/x_1 \in f^{-1}(y_1)\},$
$\sup\{\nu_A(x_2)/x_2 \in f^{-1}(y_2)\})$
$= T(f(\mu_A)(y_1), f(\mu_A(y_2))), S(f(\nu_A)(y_1), f(\nu_A(y_2)))$
$= (T,S)\{f(\mu_A,\nu_A)(y_1),(y_2)\}$
and since

$\{x/x \in f^{-1}(y_1y_2)\} \supseteq \{x_1x_2/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$,
$f(A)(y_1y_2)$
$= \sup\{T(\mu_A(x)/f^{-1}(y_1y_2))\}, \inf\{S(\nu_A(x)/f^{-1}(y_1y_2))\}$
$\geq \sup\{T(\mu_A(x_1), \mu_A(x_2)/x_1 \in f^{-1}(y_1),$
$x_2 \in f^{-1}(y_2)\})$,
$\inf\{S(\nu_A(x_1), \nu_A(x_2)/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\})$
$\sup\{\mu_A(x_2/x_2) \in f^{-1}(y_2)\})$,
$S(\inf\{\nu_A(x_1)/x_1 \in f^{-1}(y_1)\}$,
$\inf\{\nu_A(x_2/x_2) \in f^{-1}(y_2)\})$
$= T(f(\mu_A)(y_1), f(\mu_A)(y_2)), S(f(\nu_A)(y_1), f(\nu_A)(y_2))$.
$= (T,S)\{f(A)(y_1), f(A)(y_2)\}$.
This shows that $f(\mu_A, \nu_A)$ is an (T,S)-intuitionistic fuzzy subnear-ring in N_2. Let $y_1, y_2, y_3 \in N_2$. Then
$f(A)(y_1+y_2-y_1) = \sup\{\mu_A(x)/f^{-1}(y_1+y_2-y_1)\}$,
$\inf\{\nu_A(x)/f^{-1}(y_1+y_2-y_1)\}$
$\geq \sup\{T(\mu_A(x_1), \mu_A(x_2)/x_1 \in f^{-1}(y_1),$
$x_2 \in f^{-1}(y_2)\})$,
$\geq \sup\{T(\mu_A(x_1+x_2-x_1)/x_1 \in f^{-1}(y_1),$
$x_2 \in f^{-1}(y_2))\}$,
$\inf\{s(\nu_A(x_1+x_2-x_1)/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2))\}$
$\geq \sup\{T(\mu_A(x_1)/x_1 \in f^{-1}(y_1))\}$,
$\inf S(\nu_A(x_1)/x_1 \in f^{-1}(y_1))$
$= (T,S)f(A)(y_1)$,
$f(A)(y_1y_2)$
$= \sup\{T(\mu_A(x)/f^{-1}(y_1y_2))\}$,
$\inf\{S(\nu_A(x)/f^{-1}(y_1y_2))\}$
$\geq \sup\{T(\mu_A(x_1x_2)/x_1 \in f^{-1}(y_1),$
$x_2 \in f^{-1}(y_2))\}$,
$\inf\{S(\nu_A(x_1x_2)/x_1 \in f^{-1}(y_1),$
$x_2 \in f^{-1}(y_2))\}$
$\geq \sup\{T(\mu_A(x_2/x_2 \in f^{-1}(y_2))\}$,
$\inf\{S(\nu_A(x_2/x_2 \in f^{-1}(y_2))\}$
$= (T,S)f(A)(y_2)$,
$f(A)((y_1+y_2)y_3 - y_1y_3) = \sup\{T(\mu_A\ (x)/f^{-1}((y_1+y_2)y_3$-$y_1y_3))\}$,
$\inf\{S(\nu_A\ (x)/f^{-1}((y_1+y_2)y_3y_1y_3)\}$
$\geq \sup\{T(\mu_A((x_1+x_2)x_3 - x_1x_3)/$
$x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2), x_3 \in f^{-1}(y_3)\}$
$\geq \sup\{T(\mu_A(x_3)/x_3 \in f^{-1}(y_3))\}$,
$\inf\{S(\nu_A(x_3)/x_3 \in f^{-1}(y_3))\}$
$= (T,S)f(A)(y_3)$.
Hence $f(A)$ is a (T,S)-intuitionistic fuzzy ideal of near-ring in N_2. The following propositionosition is trivial. □

Proposition 3.9. If μ_A and ν_A are (T,S)-intuitionistic fuzzy ideals of a near-ring N, then the function $\mu \wedge \nu : N \to [0,1]$ defined by

$$(\mu \wedge \nu)(x) = (T\{\mu(x), \nu(x)\}, S\{\mu(x), \nu(x)\})$$

for all $x \in N$ is an (T,S)-intuitionistic fuzzy ideal of N.

Definition 3.10. An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of a near-ring N is said to be normal if $\mu_A(0) = 1$, and $\nu_A(0) = 0$.

Theorem 3.11. Let $A = (\mu_A, \nu_A)$ be an (T,S)-intuitionistic fuzzy ideal of a near-ring R and let $A^* = (\mu_A^*, \nu_A^*)$ be an intuitionistic fuzzy set defined by $\mu_A^*(x) = \mu_A(x) + 1 - \mu_A(0)$ and $\nu_A^*(x) = \nu_A(x) - \nu_A(0)$ for all $x \in N$. Then A^* is a normal (T,S)-intuitionistic fuzzy ideal of N containing A.

Proof. *For any* $x, y \in N$,
$(\mu_A^*(x-y), \nu_A^*(x-y)) = (\mu_A(x-y) + 1 - \mu_A(0), \nu_A(x), \nu_A(0))$
$\geq T\{(\mu_A(x), \mu_A(y)) + 1 - \mu_A(0)\}$,
$S\{(\nu_A(x), \nu_A(y)) - \nu_A(0)\}$
$= T(\mu_A(x) + 1 - \mu_A(0), \mu_A(y) + 1 - \mu_A(0))$,
$S(\nu_A(x) - \nu_A(0), \nu_A(y) - \nu_A(0))$
$= T(\mu_A^*(x), \mu_A^*(y)), S(\nu_A^*(x), \nu_A^*(y))$
$= (T,S)\{(\mu_A^*(x), \mu_A^*(y), (\nu_A^*(x), \nu_A^*(y)\}$
$(\mu_A^*(xy), \nu_A^*(xy)) = (\mu_A(xy) + 1 - \mu_A(0), \nu_A(x) - \nu_A(0)$
$\geq T(\mu_A(x), \mu_A(y)) + 1 - \mu_A(0), S(\nu_A(x), \nu_A(y)) - \nu_A(0)$
$= T(\mu_A(x) + 1 - \mu_A(0), \mu_A(y) + 1 - \mu_A(0))$,
$S(\nu_A(x) - \nu_A(0), \nu_A(y) - \nu_A(0))$
$= (T,S)\{(\mu_A^*(x), \mu_A^*(y)), (\nu_A^*(x), \nu_A^*(y))\}$
Hence from the obtained results it is clear that A^* *is a* (T,S)-*intuitionistic fuzzy subnear-ring of* N. □

Theorem 3.12. If $A = (\mu_A, \nu_A)$ is an (T,S)-intuitionistic fuzzy ideal of a near-ring N, then for all $x \in N$,
$A(x) = \Big\{\sup\{t \in [0,1] | \mu_A(x) \geq t\}, \inf\{s \in [0,1] | \nu_A(x) \leq s\}$.
such that $t + s \leq 1$.

Proof. *Let* $p = \sup\{t \in [0,1] | x \in U(A;t,s)\}$
and $q = \inf\{t \in [0,1] | x \in U(A;t,s)\}$.
Let $\varepsilon \geq 0$. *Then* $p - \varepsilon \leq t$ *and* $q + \varepsilon \geq s$, *for some* $t, s \in [0,1]$ *with* $t + s \leq 1$.
Since ε *is arbitrary* $(p,q) \leq \langle \mu_A(x), \nu_A(x) \rangle = A(x)$. *Let* $A(x) = (v_1, v_1')$, *then* $x \in U(A : v_1, v_1')$ *and so*
$v_1 \in \{t \in [0,1] | x \in U(A : t, s)\}$
and $v_1' \in \{s \in [0,1] | x \in U(A : t, s)\}$
Then $A(x) = (v_1, v_1') \leq (p,q)$. *Hence* $A(x) = (p,q)$.
This completes the proof. □

4. The quotient near-rings via intuitionistic fuzzy ideals

Theorem 4.1. Let I be an ideal of a near-ring N. If A is a (T,S)-intuitionistic fuzzy ideal of N, then the fuzzy set $\overline{A}$ of N/I defined by

$$\overline{A}(a+I) = \left(\sup_{x \in I}\{\mu_A(a+x)\}, \inf_{x \in I}\{\nu_A(a+x)\}\right)$$

is a (T,S)-intuitionistic fuzzy ideal of the quotient near-ring N/I of N with respect to I.

Proof. *Let* $a, b \in N$ *be such that* $a + I = b + I$. *Then* $b = a + y$ *for some* $y \in I$. *Thus*

$$
\begin{aligned}
\overline{A}(b+I) &= \{\sup_{x\in I}(\mu_A(b+x)),\\
&\inf_{x\in I}(\nu_A(b+x))\}\\
&= \{\sup_{x\in I}(\mu_A(a+y+x)), \inf_{x\in I}(\nu_A(a+y+x))\}\\
&= \{\sup_{x+y=z\in I}(\mu_A(a+z)), \inf_{x\in I}(\nu_A(a+z))\}\\
&= \{\overline{A}(a+I)\}
\end{aligned}
$$

This shows that $\overline{A}$ is well defined. Let $x+I, y+I \in N/I$, then
$\overline{A}((x+I)-(y+I)) = \overline{A}((x-y)+I)$
$= \{\sup_{z\in I}(\mu_A((x-y)+z)),$
$\inf_{z\in I}(\nu_A((x-y)+z))\}$
$= \{\sup_{z=u-v\in I}(\mu_A((x-y)+(u-v))),$
$\inf_{z=u-v\in I}(\nu_A((x-y)+(u-v)))\}$
$\geq \{\sup_{u,v\in I} T(\mu_A(x+u)), \mu_A(y+v)\},$
$\inf_{u,v\in I} S\{\nu_A(x+u), \nu_A(y+v)\}$
$= T\{\sup_{u\in I}\{\mu_A(x+u)\}, \sup_{v\in I}\{\mu_A(y+v))\}\},$
$S\{\inf_{u\in I}\nu_A(x+u), \inf_{v\in I}\nu_A(y+v))\}$
$= (T,S)\{\overline{A}(x+I), \overline{A}(y+I)\}$
$\overline{A}(x+I)(y+I)) = \overline{A}((xy)+I)$
$= \{\sup_{z\in I}\{\mu_A(xy)+z)\},$
$\inf_{z\in I}\{\nu(xy)+z))\}$
$= \{\sup_{z=uv\in I}\{\mu_A((xy)+(uv))\},$
$\inf_{z=uv\in I}\{\nu_A((xy)+(uv))\}$
$\geq \{\sup_{u,v\in I} T\{\mu_A(x+u), \mu_A(y+v)\},$
$\inf_{u,v\in I} S\{\nu_A(x+u), \nu_A(y+v)\}\}$
$= T\{\sup_{u\in I}\{\mu_A(x+u)\}, \sup_{v\in I}\{\mu_A(y+v)\},$
$S\{\inf_{u\in I}\{\nu_A(x+u)\}, \inf_{v\in I}\{\nu(y+v)\}\}$
$= T\{\overline{A}(x+I), \overline{A}(y+I)\},$
$S\{\overline{A}(x+I), \overline{A}(y+I)\}$
$= (T,S)\{\overline{A}(x+I), \overline{A}(y+I)\}$ *Thus $\overline{A}$ is a (T,S)-intuitionistic fuzzy ideal of N/I.* □

Theorem 4.2. Let I be an ideal of a near-ring N. Then there is a one-to-one correspondence between the set of (T,S) intuitionistic fuzzy ideals A of N such that $A(0) = A(s)$ for all $s \in I$ and the set of all (T,S) intuitionistic fuzzy ideals $\overline{A}$ of N/I.

Proof. Let A be a (T,S)-intuitionistic fuzzy ideal of R. Using the previous theorem, we prove that $\overline{A}$ defined by

$$\overline{A}(a+I) = \sup\{(\mu_A(a+x))\}, \inf\{(\nu_A(a+x))\}$$

is a (T,S)-intuitionistic fuzzy ideal of N/I. Since $A(0) = A(s)$ for all $s \in I, \Rightarrow (1,0) = (\mu_A(s), \nu_A(s))$

$$
\begin{aligned}
A(a+s) &\geq \{T(\mu_A(a), \mu_A(s)),\\
S(\nu_A(a), \nu_A(s))\} &= \left(\{\mu_A(a), \nu_A(a)\}\right) = A(a).
\end{aligned}
$$

Again,

$$
\begin{aligned}
A(a) &= (\mu_A(a+s-s), \nu_A(a+s-s))\\
&\geq \{T(\mu_A(a+s), \mu_A(s)),\\
&\quad S(\nu_A(a+s), \nu_A(s))\}\\
&= A(a+s).
\end{aligned}
$$

Thus $A(a+s) = A(a)$ for all $s \in I$, that is, $\overline{A}(a+I) = A(a)$. Hence, the correspondence $A \to \overline{A}$ is one-to-one. Let A be a (T,S) intuitionistic fuzzy ideal of N/I and define an intuitionistic fuzzy set A in N by $A(a) = \overline{A}(a+I)$ for all $a \in I$. For $x,y \in N$, we have $A(x-y) = \overline{A}((x-y)+I) = \overline{A}((x+I)-(y+I))$
$\geq \{T\{\mu_A(x+I), \mu_A(y+I)\},$
$S\{\nu_A(x+I), \nu_A(y+I)\}\}$
$= T\{\mu_A(x), \mu_A(y)\},$
$S\{\nu(x), \nu(y)\}$
$A(xy) = \overline{A}((xy)+I) = \overline{A}((x+I)\cdot(y+I))$
$\geq \{T\{\mu_A(x+I), \mu_A(y+I)\},$
$S\{\nu(x+I), \nu(y+I)\}\}$
$= \{T\{\mu_A(x), \mu_A(y)\},$
$S\{\nu(x), \nu(y)\}\}$ *Thus A is a (T,S) intuitionistic fuzzy ideal of N. Note that $A(z) = \overline{A}(z+I) = \overline{A}(I)$ for all $z \in I$, which shows that $A(z) = A(0)$ for all $z \in I$. This ends the proof.* □

Theorem 4.3. Let T and S be a t-norm and t-conorm respectively and let I be an ideal of a near-ring N. Then for all $(\gamma,\gamma') \in [0,1]^2$, there exists a (T,S)-intuitionistic fuzzy ideal A of Nsuch that $A(0) = (\gamma,\gamma')$ and $U(A;(\gamma,\gamma')) = I$.

Proof. Let $A : N \to [0,1] \times [0,1]$ be an intuitionistic fuzzy subset of R defined by

$$A(x) = \begin{cases} (\gamma,\gamma') & \textit{if } x \in I,\\ 0 & \textit{Otherwise} \end{cases}$$

where (γ,γ') is a fixed number in $[0,1]^2$. Then clearly, $U(A;(\gamma,\gamma')) = I$. Let $x,y \in N$, then a routine calculation shows that A is a (T,S)-intuitionistic fuzzy ideal of N.
The following theorem is evident. □

Theorem 4.4. Let A be a (T,S)-intuitionistic fuzzy ideal of a near-ring N and let $A(0) = (\gamma,\gamma')$. Then the intuitionistic fuzzy subset A^* of the quotient near-ring $N/U(A;(\gamma,\gamma')$ defined by $A^*(x+U(A;(\gamma,\gamma'))) = A(x)$ for all $x \in N$ is a (T,S)-intuitionistic fuzzy ideal of $N/U(A;(\gamma,\gamma'))$.

Theorem 4.5. Let I be a ideal of a near-ring N and A be an intuitionistic fuzzy ideal of N/I such that $A(x+I) = A(x)$ only if $x \in I$. Then there exists a (T,S)-intuitionistic fuzzy ideal A^* of N such that $U(A;(\gamma,\gamma')) = I, (\gamma,\gamma') = A(0)$ and $A = A^*$.

Proof. *Define a (T,S)-intuitionistic fuzzy ideal B of N by $B(x) = A(x+I)$ for all $x \in N$. It is easy to see that B is a (T,S)-intuitionistic fuzzy ideal of N. Next, we prove that $U(B;(\gamma,\gamma')) = I$. Let $x \in U(B;(\gamma,\gamma')) = I$.*
$\Leftrightarrow B(x) = B(0) \Leftrightarrow A(x+I) = A(x) \Leftrightarrow x \in I$.
Hence, $U(B;(\gamma,\gamma')) = I$. Finally, we prove that $B^* = A$,
Since, $B^*(x+I) = B^*(x+U(B;\gamma,\gamma')) = B(x) = A(x+I)$
Hence, $B^* = A$. This completes the proof. □

5. Conclusion

In this article, (T,S)-intuitionistic fuzzy ideals has been studied and as an extension one can study (T,S)-intuitionistic fuzzy bi-ideals, quasi-ideals etc.

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