$(T,S)$-Intuitionistic fuzzy ideals in near-rings

P. Murugadas$^1$* and V. Vetrivel$^2$

Abstract
In this paper, the concept of intuitionistic fuzzy ideals in near-rings with respect to a $t$-norm and $t$-co-norm has been studied and some of their properties are investigated. Using $(T,S)$-intuitionistic fuzzy ideals, characterizations of quotient near-rings are established.

Keywords
Near-ring, subnear-ring (ideal), fuzzy ideals, intuitionistic fuzzy ideals, $(T,S)$-intuitionistic fuzzy ideals.

AMS Subject Classification
03E72, 16Y30.

1 Department of Mathematics, Govt. Arts and Science College, Karur-639 005, India  
2 Department of Mathematics, Annamalai University, Annamalainagar- 608 002, India.
*Corresponding author: 1 bodi_muruga@yahoo.com; 2 vetrivelmath@gmail.com  
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1. Introduction
The concept of a fuzzy set was formulated by Zadeh in [15], since then, the theory of fuzzy sets developed by Zadeh and others has evoked tremendous interest among researchers working in different branches of mathematics. In 1971, Rosenfeld [12] extended the concept of fuzzy set theory to group theory and defined fuzzy group and derived some properties. Fuzzy semigroups were introduced by Kuroki [9] as a generalization of classical semigroups. Many classes of semigroups were studied by Kuroki using fuzzy ideals in [10]. Mordeson et. al. [11] gave a systematic exposition of fuzzy semigroups, where one can find theoretical results on fuzzy semigroups and their use in fuzzy coding, fuzzy finite state machines and fuzzy languages. In 1991, Abou-Zaid [1] introduced the notion of fuzzy subnear-rings and ideals in near-rings. Atanassov introduced intuitionistic fuzzy sets which constitute a generalization of the notion of fuzzy sets [2, 3]. The degree of membership of an element is given in a fuzzy set, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. Biswas [4] introduced the notion of intuitionistic fuzzy subgroup of a group by using the notion of intuitionistic fuzzy sets. Kim and Jun [7] introduced the concept of intuitionistic fuzzy ideals of semigroups and in [8], Kim and Lee studied intuitionistic fuzzy bi-ideals of semigroups. Kim and Lee [6] gave the concept of intuitionistic $(T,S)$-normed fuzzy ideals of $*$-rings. Zhan Jianming and Ma Xueling [16], also discussed the various properties on intuitionistic fuzzy ideals of near-rings. Also Cho et al in [5] the notion of a normal intuitionistic fuzzy $N$-subgroup in a near-ring is introduced and related properties are investigated. In this paper we introduce the notion of intuitionistic fuzzy subnear-ring (ideal) of a near-ring with respect to $tnorm T$ and $tconorm S$. Then we characterize all of them based on special kind of level sets $U(A;[t,s])$ and $L(A;[r,s])$, which is a generalization of classic level subsets. At the following the behaviour of these structures under homomorphisms is investigated. In particular, by the help of the congruence relations on near-rings, we construct $(T,S)$-intuitionistic fuzzy subnear-ring (ideal) on near-ring of quotient.

2. Preliminaries
In this section, we review some elementary aspects that are necessary for this paper.

Definition 2.1. An algebra $(N,+,\cdot)$ is said to be a near-ring if it satisfies the following conditions:
(i) $(N,+)\ is\ a\ (not\ necessarily\ abelian)\ group.$
Definition 2.2. A subset $I$ of a near-ring $N$ is said to be a subnear-ring if $(I, +, ·)$ is also a near-ring.

Proposition 2.3. A subset $I$ of a near-ring $N$ is a subnear-ring of $N$ if and only if $x - y, xy \in I$ for all $x, y \in I$.

Definition 2.4. A mapping $f : N_1 \rightarrow N_2$ is called a near-ring homomorphism if $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $c, y \in N_1$.

Definition 2.5. An ideal $I$ of a near-ring $(R, +, ·)$ is a subset of $N$ such that
- $(a). I, +)$ is a normal subgroup of $(N, +)$,
- $(b). NI \subseteq I$,
- $(c). (r + i)s - rs \in I$ for all $i \in I$ and $r, s \in N$.

Note that $I$ is a left ideal of $N$ if $I$ satisfies $(a)$ and $(b)$, and $I$ is a right ideal of $N$ if $I$ satisfies $(a)$ and $(c)$. If $I$ is both left and right ideal, $I$ is called an ideal of $N$.

Definition 2.6. A quotient near-ring (also called a residue-class near-ring) is a near-ring that is the quotient of a near-ring and one of its ideals $I$, denoted $N/I$. If $I$ is an ideal of a near-ring $N$ and $a \in N$, then a coset of $I$ is a set of the form $a + I = \{a + s/s \in I\}$. The set of all cosets is denoted by $N/I$.

Theorem 2.7. If $I$ is an ideal of a near-ring $N$, the set $N/I$ is a near-ring under the operations $(a + I) + (b + I) = (a + b) + I$ and $(a + I)(b + I) = (ab) + I$.

Definition 2.8. A mapping $\mu : X \rightarrow [0, 1]$, where $X$ is an arbitrary nonempty set and is called a fuzzy set in $X$.

Definition 2.9. Let $X$ be a nonempty fixed set. An intuitionistic fuzzy set $A$ in $X$ is an object having the form $A = \{< x, \mu_A, \nu_A | x \in X \}$, where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degree of membership and degree of non-membership of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Notation: For the sake of simplicity, we shall use the symbol $A = < \mu_A, \nu_A >$ for the IFS.

Definition 2.10. A fuzzy subset $\mu$ in a near-ring $N$ is said to be fuzzy subnear-ring of $N$ if it satisfies the following conditions:
- $(F1)$ for all $x, y \in N$, $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,
- $(F2)$ for all $x, y \in N$, $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2.11. A fuzzy subnear-ring $\mu$ of $N$ is said to be fuzzy ideal if it satisfies the following conditions:
- $(F3)$ for all $x, y \in N$, $\mu(y - x) \geq \mu(x)$,
- $(F4)$ for all $x, y \in N$, $\mu(xy) \geq \mu(y)$,
- $(F5)$ for all $x, y \in N$, $\mu((x + z)y - xy) \geq \mu(z)$.

A fuzzy subnear-ring satisfying $(F3)$ and $(F4)$ is called fuzzy left ideal and it is a right ideal if it satisfy $(F3)$ and $(F5)$.

Definition 2.12. An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of a near-ring $N$ is called an intuitionistic fuzzy subnear-ring of $N$ if
- $(i)$ $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
- $(ii)$ $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$, for all $x, y \in N$.

Definition 2.13. An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in a near-ring $N$ is called an intuitionistic fuzzy ideal of $N$ if
- $(i)$ $\mu_A(x - y) \geq \mu_A(x) \land \mu_A(y)$ and $\lambda_A(x - y) \leq \lambda_A(x) \lor \lambda_A(y)$,
- $(ii)$ $\mu_A(xy) \geq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(x)$,
- $(iii)$ $\mu_A(rx) \geq \mu_A(x)$ and $\lambda_A(rx) \leq \lambda_A(x)$,
- $(iv)$ $\mu((x + i)y - xy) \geq \mu_A(i)$ and $\lambda((x + i)y - xy) \leq \lambda_A(i)$ for all $x, y, i \in N$.

If $A = (\mu_A, \lambda_A)$ satisfies $(i)$, $(ii)$ and $(iii)$ then $A$ is called an intuitionistic fuzzy left ideal of $N$ and if $A = (\mu_A, \lambda_A)$ satisfies $(i)$, $(ii)$ and $(iv)$ then $A$ is called an intuitionistic fuzzy right ideal of $N$.

Lemma 2.14. If $\mu$ is an intuitionistic fuzzy ideal of $N$, then $\mu(0) \geq \mu(x)$ and $\nu(0) \leq \nu(x)$ for all $x \in N$.

Definition 2.15. A t-norm is a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies the following conditions for all $x, y, z \in [0, 1]$:
- $(T1) T(x, 1) = x$,
- $(T2) T(x, y) = T(y, x)$,
- $(T3) T(x, T(y, z)) = T(T(x, y), z)$,
- $(T4) T(x, y) \leq T(x, z)$, whenever $y \leq z$.

A simple example of such defined t-norm is a function $T(x, y) = \min(x, y)$. In general case, $T(x, y) \leq \min(x, y)$ and $T(x, 0) = 0$ for all $x, y \in [0, 1]$.

Definition 2.16. A mapping $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a t-co-norm if the following conditions are satisfied:
- $(S1) S(x, 0) = x$, for all $x \in [0, 1]$,
- $(S2) S(x, y) = S(y, x)$ for all $x, y \in [0, 1]$,
- $(S3) S(x, S(y, z)) = S(S(x, y), z)$, for all $x, y, z \in [0, 1]$,
- $(S4) if x \leq p and y \leq q$, then $S(x, y) \leq S(p, q)$, for all $x, y, p, q \in [0, 1]$.

Proposition 2.17. Let $S$ be a t-co-norm. Then the following conditions are satisfied:
- $(1) S(x, 1) = 1$, for all $x \in [0, 1]$,
- $(2) S(x, y) \geq S(A, y)$ for all $x, y \in [0, 1]$.

Definition 2.18. Let $A = < \mu_A, \nu_A >$ be an intuitionistic fuzzy set in $N$ and let $t \in [0, 1]$. Then the sets $U(A : t, s) = \{x \in N : \mu_A(x) \geq r\}$ and $V_A(x) \leq s$ with $t + s \leq 1$ is called level set of $A$.

3. $(T, S)$-Intuitionistic Fuzzy ideals in Near-rings

In this section, we define $(T, S)$ Intuitionistic Fuzzy ideals of near-rings and prove some basic properties of these ideals.
**Definition 3.1.** An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in $N$ is called intuitionistic fuzzy subnear-ring with respect to a $t$-norm and co-$t$-norm, shortly $(T,S)$-intuitionistic fuzzy subnear-ring, of $N$ if:

$(TF1)$ for all $x, y \in N$, $\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))$ and $\nu_A(x - y) \leq S(\nu_A(x), \nu_A(y))$

$(TF2)$ for all $x, y \in N$, $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$ and $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$

**Definition 3.2.** An $(T,S)$-intuitionistic fuzzy subnear-ring $A = (\mu_A, \nu_A)$ in $N$ is called $(T,S)$-intuitionistic fuzzy ideal of $N$ if:

$(TF3)$ for all $x, y \in N$, $\mu_A(y + x - y) \geq T(\mu_A(x))$ and $\nu_A(y + x - y) \leq S(\nu_A(x))$

$(TF4)$ for all $x, y \in N$, $\mu_A(xy) \geq T(\mu_A(y))$ and $\nu_A(xy) \leq S(\nu_A(y))$

$(TF5)$ for all $x, y \in N$, $\mu_A((x + z)y - xy) \geq T(\mu_A(z))$ and $\nu_A((x + z)y - xy) \leq T(\nu_A(z))$.

Note that $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy left ideal of $R$ if it satisfies $(TF1)$, $(TF2)$, $(TF3)$ and $(TF4)$, and $A = (\mu_A, \nu_A)$ is an $(T,S)$-intuitionistic fuzzy right ideal of $N$ if it satisfies $(TF1)$, $(TF2)$, $(TF3)$ and $(TF5)$. $A = (\mu_A, \nu_A)$ is called $(T,S)$-intuitionistic fuzzy ideal of $N$ if it is both left and right $(T,S)$-intuitionistic fuzzy ideal of $N$.

**Example 3.3.** Consider a near-ring $N = \{a, b, c, d\}$ with the following Cayley’s tables:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>b</td>
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<td>a</td>
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</tr>
<tr>
<td>c</td>
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<td>b</td>
<td>a</td>
</tr>
<tr>
<td>d</td>
<td>d</td>
<td>c</td>
<td>a</td>
<td>b</td>
</tr>
</tbody>
</table>

We define an intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ by $\mu_A(a) > \mu_A(b) > \mu_A(d) = \mu_A(c)$ and $\nu_A(a) < \nu_A(b) < \nu_A(d) = \nu_A(c)$. Let $T : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by $T(x, y) = \max(x + y - 1, 0)$ which is a $t$-norm for all $x, y \in [0, 1]$ and let $S : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by $S(x, y) = \min(x + y, 1)$ which is a $t$-conorm for all $x, y \in [0, 1]$. By routine calculations, it is easy to check that $A$ is a $(T,S)$-intuitionistic fuzzy ideal of $N$.

**Example 3.6.** Let $N = \{a, b, c, d\}$ be a set with binary operations as follows:

<table>
<thead>
<tr>
<th>+</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<tbody>
<tr>
<td>a</td>
<td>a</td>
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<td>a</td>
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<tr>
<td>b</td>
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<td>b</td>
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<td>c</td>
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<td>d</td>
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<td>d</td>
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<td>d</td>
</tr>
</tbody>
</table>

Then $(N, +, \cdot)$ is a near-ring. We define an intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ and $\nu_A : N \to [0, 1]$ by $\mu_A(a) > \mu_A(b) > \mu_A(d) = \mu_A(c)$ and $\nu_A(a) < \nu_A(b) < \nu_A(d) = \nu_A(c)$. Let $T : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by $T(x, y) = \max(x + y - 1, 0)$ which is a $t$-norm for all $x, y \in [0, 1]$ and let $S : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by $S(x, y) = \min(x + y, 1)$ which is a $t$-conorm for all $x, y \in [0, 1]$. By routine calculations, it is easy to check that $A = (\mu_A, \nu_A)$ is not $(T,S)$ intuitionistic fuzzy ideal of $N$. Since $\mu_A(c + b - d - cd) = \mu_A(d) \leq \mu_A(b)$.

**Definition 3.7.** Let $N_1$ and $N_2$ be two near-rings and $f$ a function of $N_1$ into $N_2$. If $B$ is a fuzzy set in $N_2$, then the image of $A$ under $f$ is the fuzzy set in $N_1$ defined by

$$f(A)(x) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ \inf_{x \in f^{-1}(y)} \nu_A(x), & \text{if } f^{-1}(y) = \emptyset, \end{cases}$$

for each $y \in N_2$.

**Theorem 3.8.** Let $f : N_1 \to N_2$ be an onto homomorphism of near-rings. If $A = (\mu_A, \nu_A)$ is a $(T,S)$-intuitionistic fuzzy ideal in $N_1$, then $f(A)$ is a $(T,S)$-intuitionistic fuzzy ideal in $N_2$.

**Proof.** Let $y_1, y_2 \in R_2$. Then

$$\{x/x \in f^{-1}(y_1 - y_2)\} \supseteq \{x_1 - x_2/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\},$$

and hence

$$f(A)(y_1 - y_2) = \{\sup_{x \in f^{-1}(y_1 - y_2)} \mu_A(x), \inf_{x \in f^{-1}(y_1 - y_2)} \nu_A(x)\} \supseteq \{\sup_{x \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)} T(\mu_A(x), \mu_A(x_2)), \inf_{x \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)} S(\nu_A(x_1), \nu_A(x_2))\} = \{\sup_{x \in f^{-1}(y_1)} \mu_A(x), \sup_{x \in f^{-1}(y_1)} \nu_A(x)\} = \{T(f(\mu_A)(y_1), f(\mu_A)(y_2)), S(f(\nu_A)(y_1), f(\nu_A)(y_2))\} = (T,S)\{f(\mu_A, \nu_A)(y_1), (y_2)\}$$

and since
{y/x \in f^{-1}(y_1 y_2)} \supseteq \{x_1 x_2 / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}, \\
\inf\{T(\mu_A(x))/f^{-1}(y_1 y_2)\} \supseteq \{x_1 x_2 / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}, \\
\sup\{\mu_A(x_1), \mu_A(x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}, \\
\inf\{S(\nu_A(x))/f^{-1}(y_1 y_2)\} \supseteq \{x_1 x_2 / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}, \\
\sup\{\nu_A(x_1), \nu_A(x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}.
\]

This shows that \( f(\mu_A, \nu_A) \) is an \((T, S)\)-intuitionistic fuzzy subnear-ring in \( N_2 \). Let \( y_1, y_2, y_3 \in N_2 \). Then

\[
f(A)(y_1 + y_2 - y_1) = \sup\{\mu_A(x)/f^{-1}(y_1 y_2)\}, \\
\inf\{\nu_A(x)/f^{-1}(y_1 y_2)\} = \sup\{\mu_A(x_1), \mu_A(x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}, \\
\sup\{\nu_A(x_1), \nu_A(x_2) / x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}.
\]

Hence from the obtained results it is clear that \( A^* \) is a \((T, S)\)-intuitionistic fuzzy subnear-ring of \( N \).

**Theorem 3.12.** If \( A \) is an \((T, S)\)-intuitionistic fuzzy ideal of a near-ring \( N \), then for all \( x \in N \),

\[
A(x) = \left\{ \sup\{t \in [0, 1]|\mu_A(x) \geq t\}, \inf\{s \in [0, 1]|\nu_A(x) \leq s\} \right\}.
\]

such that \( t + s \leq 1 \).

**Proof.** Let \( p = \sup\{t \in [0, 1]|x \in U(A; t, s)\} \), \( q = \inf\{t \in [0, 1]|x \in U(A; t, s)\} \).

Let \( e > 0 \). Then \( p - e \leq t \) and \( q + e \geq s \), so for some \( t, s \in [0, 1] \)

with \( t + s \leq 1 \).

Since \( e \) is arbitrary \( (p, q) \subseteq (A(x), U(A; x, y)) = A(x) \).

Let \( A(x) = (y_1, y_2, y_3) \), then \( x \in U(A : y_1, y_2) \) and so

\[
y_1 = \{t \in [0, 1]|x \in U(A : t, s)\} \quad \text{and} \quad y_2 = \{s \in [0, 1]|x \in U(A : t, s)\}.
\]

Then \( A(x) = (y_1, y_2, y_3) \subseteq (p, q) \).

Hence \( A(x) = (p, q) \).

This completes the proof.

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**4. The quotient near-rings via intuitionistic fuzzy ideals**

**Theorem 4.1.** Let \( I \) be an ideal of a near-ring \( N \). If \( A \) is a \((T, S)\)-intuitionistic fuzzy ideal of \( N \), then the fuzzy set \( \bar{A} \) of \( N/I \) defined by

\[
\bar{A}(a + I) = \left( \sup\{\mu_A(a + x)\}, \inf\{\nu_A(a + x)\} \right)
\]

is a \((T, S)\)-intuitionistic fuzzy ideal of the quotient near-ring \( N/I \) of \( N \) with respect to \( I \).

**Proof.** Let \( a, b \in N \) be such that \( a + I = b + I \). Then \( b = a + y \) for some \( y \in I \). Thus
\( \mathcal{A}(b + I) = \{ \sup_{x \in I} (\mu_A(b + x)), \inf_{x \in I} (\nu_A(b + x)) \} \)

This shows that \( \mathcal{A} \) is well defined. Let \( x + I, y + I \in N/I \), then
\[
\mathcal{A}((x + I) - (y + I)) = \mathcal{A}((x - y) + I)
\]
\[
= \{ \sup_{z \in I} (\mu_A((x - y) + z)), \inf_{z \in I} (\nu_A((x - y) + z)) \}
\]
\[
= \{ \inf_{z = u + v \in I} (\mu_A((x - y) + (u - v))), \sup_{z = u + v \in I} (\nu_A((x - y) + (u - v))) \}
\]
\[
\geq \{ \sup_{u \in I} T(\mu_A(x + u), \mu_A(y + v)), \inf_{u \in I} S(\nu_A(x + u), \nu_A(y + v)) \}
\]
\[
= T(\{ \sup_{u \in I} (\mu_A(x + u)), \inf_{u \in I} (\nu_A(x + u)) \}, \{ \inf_{u \in I} (\nu_A(y + v)), \sup_{u \in I} (\mu_A(y + v)) \})
\]
\[
= (T, S)[\mathcal{A}(x + I), \mathcal{A}(y + I)] = \mathcal{A}((xy) + I)
\]
\[
= \mathcal{A}(x + I) \mathcal{A}(y + I) = \mathcal{A}((x + I)(y + I)) = \mathcal{A}(x + I) \mathcal{A}(y) + \mathcal{A}((x + I)(y + I))
\]
\[
= T(\mathcal{A}(x), \mathcal{A}(y)), S(\mathcal{A}(x), \mathcal{A}(y)) \}
\]
\[
\mathcal{A}(xy) = \mathcal{A}((xy) + I) = \mathcal{A}((x + I) \cdot (y + I))
\]
\[
\geq \{ T(\mu_A(x + I), \mu_A(y + I)), S(\nu_A(x + I), \nu_A(y + I)) \}
\]
\[
= T(\mathcal{A}(x), \mathcal{A}(y)), S(\mathcal{A}(x), \mathcal{A}(y)) \}
\]
\[
\mathcal{A}(xy) = \mathcal{A}((xy) + I) = \mathcal{A}(x)(y) + \mathcal{A}((x + I)(y + I))
\]

Thus \( A(a + s) = A(a) \) for all \( s \in I \), that is, \( \mathcal{A}(a + I) = A(a) \).
Hence, the correspondence \( A \rightarrow \mathcal{A} \) is one-to-one. Let \( A \) be a \((T, S)\) intuitionistic fuzzy ideal of \( N/I \) and define an intuitionistic fuzzy set \( A \) in \( N \) by \( A(a) = \mathcal{A}(a + I) \) for all \( a \in I \). For \( x, y \in N \), we have \( A(x - y) = \mathcal{A}(x + I) = \mathcal{A}((x + I) - (y + I)) \)
\[
\geq \{ T(\mu_A(x + I), \mu_A(y + I)), S(\nu_A(x + I), \nu_A(y + I)) \}
\]
\[
= T(\mathcal{A}(x), \mathcal{A}(y)), S(\mathcal{A}(x), \mathcal{A}(y)) \}
\]
\[
\mathcal{A}(xy) = \mathcal{A}(x)(y) + \mathcal{A}((x + I)(y + I))
\]

Theorem 4.3. Let \( T \) and \( S \) be a \( t \)-norm and \( t \)-conorm respectively and let \( I \) be an ideal of a near-ring \( N \). Then for all \( \gamma, \gamma' \in [0, 1]^2 \), there exists a \((T, S)\)-intuitionistic fuzzy ideal \( A \) of \( N \) such that \( A(0) = (\gamma, \gamma') \) and \( U(A; (\gamma, \gamma')) = I \).

Proof. Let \( A : N \rightarrow [0, 1] \times [0, 1] \) be an intuitionistic fuzzy subset of \( R \) defined by
\[
A(x) = \begin{cases} 
(\gamma, \gamma') & \text{if } x \in I, \\
0 & \text{Otherwise}
\end{cases}
\]
where \( (\gamma, \gamma') \) is a fixed number in \([0, 1]^2\). Then clearly,
\( U(A; (\gamma, \gamma')) = I \). Let \( x, y \in N \), then a routine calculation shows that \( A \) is a \((T, S)\)-intuitionistic fuzzy ideal of \( N \).

The following theorem is evident. □

Theorem 4.4. Let \( A \) be a \((T, S)\)-intuitionistic fuzzy ideal of a near-ring \( N \) and let \( A(0) = (\gamma, \gamma') \). Then the intuitionistic fuzzy subset \( A^* \) of the quotient near-ring \( N/U(A; (\gamma, \gamma')) \) defined by \( A^*(x + U(A; (\gamma, \gamma'))) = A(x) \) for all \( x \in N \) is a \((T, S)\)-intuitionistic fuzzy ideal of \( N/U(A; (\gamma, \gamma')) \).

Proof. (Continued from previous page...).
Proof. Define a \((T,S)\)-intuitionistic fuzzy ideal \(B\) of \(N\) by \(B(x) = A(x + I)\) for all \(x \in N\). It is easy to see that \(B\) is a \((T,S)\)-intuitionistic fuzzy ideal of \(N\). Next, we prove that \(U(B; (\gamma, \gamma')) = I\). Let \(x \in U(B; (\gamma, \gamma'))\) \(\iff B(x) = B(0) \iff A(x + I) = A(x) \iff x \in I\). Hence, \(U(B; (\gamma, \gamma')) = I\). Finally, we prove that \(B^* = A\). Since, \(B^* (x + I) = B^* (x + U(B; \gamma, \gamma')) = B(x) = A(x + I)\) hence, \(B^* = A\). This completes the proof.

5. Conclusion

In this article, \((T,S)\)-intuitionistic fuzzy ideals has been studied and as an extension one can study \((T,S)\)-intuitionistic fuzzy bi-ideals, quasi-ideals etc.

References