Malaya Journal of Matematik, Vol. S, No. 1, 355-360, 2019
https://doi.org/10.26637/MJM0S01/0064

On N$g\hat{\alpha}$-Continuous functions in nano topological spaces

V.Rajendran [1], P.Sathishmohan [2] and M.Malarvizhi [3]

Abstract
The aim of this paper is to introduce N$g\hat{\alpha}$-continuous functions in nano topological spaces and also we introduce and study the relation between N$g\hat{\alpha}$- irresolute functions and N$g\hat{\alpha}$-continuous functions and N$g\hat{\alpha}$-closed functions.

Keywords
N$g\hat{\alpha}$-continuous, N$g\hat{\alpha}$- irresolute funcions, N$g\hat{\alpha}$-closed functions.

AMS Subject Classification

54B05.

[1,2] *Assitant Professor, Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamil Nadu,India.*
[3] *Research Scholar, Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamil Nadu,India.*
***Corresponding author**: [1] rajendrankasc@gmail.com
Article History: Received **18** February **2019**; Accepted **12** May **2019**
©2019 MJM.

Contents

1. Introduction

In 1970 In general, continuous function is one of the main concepts in Topology. In 1991, Balachandran [1] et.al., introduced and studied the notions of generalized continuous functions. The concept of Nano topology was introduced by Lellis Thivagar[6] which was defined in terms of approximations and boundary regions of a subset of a universe using an equivalence relation on it and he also defined Nano continuous functions, Nano open maps, Nano closed maps and Nano homeomorphisms and their representations in terms of Nano interior and Nano closure. In this paper we introduce N$g\hat{\alpha}$-continuous functions and N$g\hat{\alpha}$- irresolute function and N$g\hat{\alpha}$-open and closed functions and discuss some of their properties.
The structure of this manuscript is as follows:
In section 2, we recall some existing definitions and lemmas which are more important to prove our main results.
In section 3, we induct and study some theorems which satisfies the conditions of N$g\hat{\alpha}$-continuous functions.
In section 4, we introduce and examine some theorems which satisfies the conditions of N$g\hat{\alpha}$- irresolute funcions.
In section 5, we introduced and study some theorems which satisfies the conditions of N$g\hat{\alpha}$-open and closed functions.

2. Preliminaries

In this section, we recall some basic definitions and results in nano topological spaces are given, which are useful to prove the main results

Definition 2.1. *[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indispensability relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,*
(i)The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.
$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ *where $R(x)$ denotes the equivalence class determined by $x \in U$.*
(ii)The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$
(iii)The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X)$ - $L_R(X)$.

Definition 2.2. *[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms*
(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.

Remark 2.3. *[6] If $\tau_R(X)$ is the nano topology on U with respect to X , then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.4. *[6] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then*
(i)The nano interior of A is defined as the union of all nano-open subsets of A is contained in A and is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest nano-open subset of A.
(ii)The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by $Ncl(A)$). That is, $Ncl(A)$) is the smallest nano-closed set containing A.

Definition 2.5. *[?] A function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is called*

- *nano continuous if $f^{-1}(V)$ is nano open in (X,τ) for every nano open set V in (Y,σ)*
- *nano semi-continuous if $f^{-1}(V)$ is nano semi-open in (X,τ) fo every nano open set V in (Y,σ)*
- *nano pre-continuous if $f^{-1}(V)$ is nano pre-open in (X,τ) for every nano open set V in (Y,σ)*
- *nano α-continuous if $f^{-1}(V)$ is nano α open in (X,τ) for every nano open set V in (Y,σ)*
- *nano regular-continuous if $f^{-1}(V)$ is nano regular-open in (X,τ) for every nano open set V in (Y,σ)*

Definition 2.6. *[?] If (U,τ_R(X)) is a nano toplogical space and if A$\subseteq$ U. Then A is said to be*

1. *Ng-closed [1] if Ncl(A)$\subseteq$V whenever A$\subseteq$V and V is nano-open in U.*
2. *Ngα-closed [4] if Nαcl(A)$\subseteq$V whenever A$\subseteq$V and V is nanoα-open in U.*
3. *Nαg-closed [4] if Nαcl(A)$\subseteq$V whenever A$\subseteq$V and V is nano-open in U.*
4. *Ngp-closed [3] if Npcl(A)$\subseteq$V whenever A$\subseteq$V and V is nano-open in U.*
5. *Ngpr-closed [8] if Npcl(A)$\subseteq$V whenever A$\subseteq$V and V is nano regular open in U.*
6. *Ngsp-closed [3] if Nspcl(A)$\subseteq$V whenever A$\subseteq$V and V is nano-open in U.*
7. *N$\hat{g}$-closed [3] if Ncl(A)$\subseteq$V whenever A$\subseteq$V and V is nano semi-open in U.*

3. Ng$\hat{}\alpha$-continuous functions

In this section we define Ng$\hat{\alpha}$-continuous functions and discuss some of their properties.

Definition 3.1. *Let (U,τ_R(X)) and (V,τ_R(Y)) be two nano topological spaces. Then a mapping f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is called Ng$\hat{}$ α-continuous if the inverse image of every nano closed set in (V,τ_R(Y)) is Ng$\hat{\alpha}$-closed in (U,τ_R(X)).*

Theorem 3.2. *: A function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{\alpha}$-continuous if and only if the inverse image of every nano closed set in (V,τ_R(Y)) is Ng$\hat{\alpha}$-closed in (U,τ_R(X)).*

Proof. Suppose the function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{\alpha}$-continuous. Let G be a nano closed set in (V,τ_R(Y)). Then Y-G is nano open set in (V,τ_R(Y)). Since f is Ng$\hat{\alpha}$-continuous, $f^{-1}(Y-G)$ is Ng$\hat{\alpha}$-open set in (U,τ_R(X)). But $f^{-1}(Y-G)$= X- $f^{-1}(G)$ is Ng$\hat{\alpha}$-open set in (U,τ_R(X)).So $f^{-1}(G)$ is Ng$\hat{\alpha}$-closed in (U,τ_R(X)).

Conversely, suppose that the inverse image of every nano closed set in (V,τ_R(Y)) is Ng$\hat{\alpha}$-closed in (U,τ_R(X)). Let G be a nano open set in (V,τ_R(Y)). Then Y-G is nano closed set in (V,τ_R(Y)).By hypothesis $f^{-1}(Y-G)$ is Ng$\hat{\alpha}$-closed in (U,τ_R(X)). But $f^{-1}(Y-G)$= X- $f^{-1}(G)$ is Ng$\hat{\alpha}$-closed in (U,τ_R(X)).Therefore $f^{-1}(G)$ is Ng$\hat{\alpha}$-open in (U,τ_R(X)). Hence f is Ng$\hat{\alpha}$-continuous function. □

Theorem 3.3. *: Let (U,τ_R(X)) and (V,τ_R(Y)) are any two nano topological spaces. If f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is nano continuous function, then f is Ng$\hat{}$ α-continuous, but not conversely.*

Proof. Let F be any nano closed set in (V,τ_R(Y)). Since f is continuous, $f^{-1}(F)$ is nano closed in (U,τ_R(X)). Since every nano closed set is Ng$\hat{\alpha}$-closed. Therefore $f^{-1}(F)$ is Ng$\hat{\alpha}$-closed in (U,τ_R(X)). Hence f is Ng$\hat{\alpha}$-continuous.

The converse of the above theorem need not be true, as proved by the following example. □

Example 3.4. *Let U = {a,b,c,d}, X = {a,b} and U/R = {{a},{b,c},{d}}. Then τ_R(X)={U,ϕ,{a},{a,b,c},{b,c}}, and τ_R^C(X)={U,ϕ,{b,c,d},{d},{a,d}}. Also V = {w,x,y,z}, Y =*

$\{w,x\}$ and V/R = $\{\{w,z\},\{x\},\{y\}\}$. Then $\tau_R(Y)$=$\{V,\phi,\{x\},\{w,x,z\},\{w,z\}\}$, and $\tau_R^C(X)$=$\{V,\phi,\{w,y,z\},\{y\},\{x,y\}\}$. Define f:(U,$\tau_R$(X)) $\rightarrow$ (V,τ_R(Y)) as f(a) = w, f(b) = x, f(c) = y, f(d) = z which is nano Ng$\hat{\alpha}$-continuous. But for the nano closed set $\{w,y,z\}$ in (V,τ_R(Y)). its inverse $f^{-1}(w,y,z) = \{a,c,d\}$is not nano closed set in (U,τ_R(X)). Hence f is not nano continuous.

Theorem 3.5. *: Let (U,τ_R(X)) and (V,τ_R(Y)) are any two nano topological spaces. If f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is nano α-continuous function, then f is Ng$\hat{}$ α-continuous, but not conversely.*

Proof. Let F be any nano closed set in (V,τ_R(Y)). Since f is nano α-continuous , then $f^{-1}(F)$ is nano α-closed set in (U,τ_R(X)). Since every nano α-closed set is Ng$\hat{\alpha}$-closed. Therefore $f^{-1}(F)$ is Ng$\hat{\alpha}$-closed in (U,τ_R(X)). Hence f is Ng$\hat{\alpha}$-continuous.

The converse of the above theorem need not be true, as proved by the following example. □

Example 3.6. *From the example 3.4, the subset A = $\{a,c,d\}$ is not nano α-closed set in (U,τ_R(X)). Hence f is not nano α-continuous.*

Theorem 3.7. *: Let (U,τ_R(X)) and (V,τ_R(Y)) are any two nano topological spaces. If a function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous function, Then f is Ng$\hat{}$ α-continuous and hence Ngp-continuous but not conversely.*

Proof. Let F be any nano closed set in (V,τ_R(Y)). Then $f^{-1}(F)$ is Ng$\hat{\alpha}$-closed set in (U,τ_R(X)).Since f is Ng$\hat{}$ α-continuous . Since every Ng$\hat{}$ α-closed set is Nαg-closed. Therefore $f^{-1}(F)$ is Nαg-closed in (U,τ_R(X)). Hence f is Nαg-continuous and by the definition, f is Ngp-continuous.

The converse of the above theorem need not be true, as proved by the following example. □

Example 3.8. *Let U = $\{a,b,c,d\}$, X = $\{a,c\}$ and U/R = $\{\{a\},\{b,c,d\}\}$. Then $\tau_R(X)$=$\{U,\phi,\{a\},\{b,c,d\}\}$ and $\tau_R^C(X)$= $\{U,\phi,\{b,c,d\},\{a\}\}$. Also V = $\{w,x,y,z\}$, Y = $\{w,x\}$ and V/R = $\{\{w\},\{y\},\{x,z\}\}$. Then $\tau_R(Y)$=$\{V,\phi,\{w\},\{w,x,z\},\{x,z\}\}$, and $\tau_R^C(Y)$=$\{V,\phi,\{x,y,z\},\{y\},\{w,y\}\}$. Define f:(U,$\tau_R$(X)) $\rightarrow$ (V,τ_R(Y)) as f(a) = w, f(b) = x, f(c) = y, f(d) = z which is nano Ng$\hat{\alpha}$-continuous. But for the nano closed set $\{y\}$ in (V,τ_R(Y)). its inverse $f^{-1}(y) = \{c\}$is not Nαg-continuous and Ngp-continuous. Hence f is not Nαg-continuous and Ngp-continuous.*

From the theorem 3.7, the class of Ng$\hat{}$ α-continuous function contained in Nαg-continuous and Ngp-continuous functions. Therefore the class of Ng$\hat{}$ α-continuous functions lies between Nα-continuous and Nαg-continuous functions.

Theorem 3.9. *If a function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous function, Then f(Ng$\hat{}$ αcl(A)) $\subseteq$ Ncl(f(A)), for every subset A of (U,τ_R(X)).*

Proof. Suppose f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous function. Let A be any subset of (U,τ_R(X)).Then Ncl(f(A)) is nano closed in (V,τ_R(Y)). As f is Ng$\hat{}$ α-continuous, $f^{-1}(Ncl(f(A)))$ is Ng$\hat{}$ α-closed set in (U,τ_R(X)) and it contains A. That is A $\subseteq f^{-1}(f(A)) \subseteq f^{1}(Ncl(f(A)))$. But Ng$\hat{}$ αcl(A) is the intersection of all Ng$\hat{}$ α-closed sets containing A. Therefore Ng$\hat{}$ αcl(A) $\subseteq f^{-1}(Ncl(f(A)))$ and hence f(Ng$\hat{}$ αcl(A)) $\subseteq$ Ncl(f(A)). □

Theorem 3.10. *Every Ng$\hat{}$ α-continuous function is Ngpr-continuous.*

Proof. Let a function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) be Ng$\hat{}$ α-continuous function. Let G be any nano closed set in (V,τ_R(y)). Then $f^{-1}(G)$ is Ng$\hat{\alpha}$-closed. Since every Ng$\hat{\alpha}$-closed set is Ngpr-closed. Therefore $f^{-1}(G)$ is Ngpr-closed in (U,τ_R(X)). Hence f is Ngpr-continuous.

The converse of the above theorem need not be true, as proved by the following example. □

Example 3.11. *Let U = $\{a,b,c,d\}$, X = $\{a,c\}$ and U/R = $\{\{a\},\{b,c,d\}\}$. Then $\tau_R(X)$=$\{U,\phi,\{a\},\{b,c,d\}\}$ and $\tau_R^C(X)$= $\{U,\phi,\{b,c,d\},\{a\}\}$. Also V = $\{w,x,y,z\}$, Y = $\{w,x\}$ and V/R = $\{\{w\},\{y\},\{x,z\}\}$. Then $\tau_R(Y)$=$\{V,\phi,\{w\},\{w,x,z\},\{x,z\}\}$, and $\tau_R^C(X)$=$\{V,\phi,\{x,y,z\},\{y\},\{w,y\}\}$. Define f:(U,$\tau_R$(X)) $\rightarrow$ (V,τ_R(Y)) as f(a) = w, f(b) = x, f(c) = y, f(d) = z which is nano Ng$\hat{\alpha}$-continuous. But for the nano closed set $\{w,y\}$ in (V,τ_R(Y)). Its inverse $f^{-1}(w,y) = \{a,c\}$ is not $Ngpr-continuous$. Hence f is not $Ngpr-continuous$.*

Theorem 3.12. *Every Ng$\hat{}$ α-continuous function is Nαgr-continuous.*

Proof. Let G be a nano closed set in (V,τ_R(y)). Then $f^{-1}(G)$ is Ng$\hat{\alpha}$-closed in (U,τ_R(X)). Since f is Ng$\hat{\alpha}$-continuous. Since every Ng$\hat{\alpha}$-closed set is Nαgr-closed. Therefore $f^{-1}(G)$ is Nαgr-closed set in (U,τ_R(X)). Hence f is Nαgr-continuous.

The converse of the above theorem need not be true, as proved by the following example. □

Example 3.13. *From by the above theorem 3.12 $f^{-1}(w,y)$ = $\{a,c\}$ is not Nαgr-continuous but not Ng$\hat{\alpha}$-continuous.*

Theorem 3.14. *If a function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{\alpha}$-continuous and if A is Ng$\hat{\alpha}$-open (or Ng$\hat{\alpha}$-closed) subset of (V,τ_R(Y)). Then $f^{-1}(A)$ is Ng$\hat{\alpha}$-open (or Ng$\hat{\alpha}$-closed) in (U,τ_R(X)).*

Proof. Let A be a Ng$\hat{\alpha}$-open set in (V,τ_R(Y)) and G be any Ng$\hat{}$-closed set in (U,τ_R(X)). Such that G $\subseteq f^{-1}(A)$. Then f(G) $\subseteq$ A. By hypothesis f(G) is Ng$\hat{}$-closed set and A is Ng$\hat{\alpha}$-open set in (V,τ_R(Y)). Therefore f(G) $\subseteq$ Nαint(A). Since a subset A of (U,τ_R(X)) is Ng$\hat{\alpha}$-open iff F $\subseteq$ Nαint(A) whenever F is Ng-closed and F $\subseteq$ A and so G $\subseteq f^{-1}(N\alpha$int(A)). *Since f is* Ng$\hat{\alpha}-$*continuous*, $N\alpha$int(A) is Nα-open in (V,τ_R(Y)). So Nαint(A) is nano open in (V,τ_R(Y)). Therefore $f^{-1}(N\alpha$int(A)) *is* Ng$\hat{}\alpha-$ *open set in* $(U,\tau_R$(X)). Thus G $\subseteq$ Nαint($f^{-1}(N\alpha$int(A))$\subseteq$ Nint(f$^{-1}(A)$), that is G $\subseteq$ Nint(f$^{-1}(A)$), So a subset A of (U,τ_R(X)) is Ng$\hat{}\alpha$-open iff F $\subseteq$ Nαint(A) whenever F is Ng-closed and F $\subseteq$ A. Hence $f^{-1}(A)$ is Ng$\hat{}\alpha$-open set in (U,τ_R(X)).

□

Remark 3.15. *The concepts of Ng$\hat{}$ α-continuous and Ng$\hat{}$-continuous independent of each other as seen from the following examples.*

Example 3.16. *Let U = $\{a,b,c,d\}$, X = $\{a,b\}$ and U/R = $\{\{a,d\},\{b\},\{c\}\}$. Then τ_R(X)=$\{U,\phi,\{b\},\{a,b,d\},\{a,d\}\}$, and τ_R^C(X)=$\{U,\phi,\{a,c,d\},\{c\},\{b,c\}\}$. Also V = $\{w,x,y,z\}$, Y = $\{w,x\}$ and V/R = $\{\{w\},\{y,z\},\{x,z\}\}$. Then τ_R(Y)=$\{V,\phi,\{w\}$, $\{w,x,y\},\{x,y\}\}$, and τ_R^C(X)=$\{V,\phi,\{x,y,z\},\{z\},\{w,z\}\}$. Let f : (U,$\tau_R$(X)) $\rightarrow$ (V,τ_R(Y)) be an identity function. Then f is Ng$\hat{}$ α-continuous but not Ng$\hat{}$-continuous. Since the closed set $\{w,z\}$ in (V,τ_R(Y)). its inverse $f^{-1}(w,z)=\{a,d\}$ is not Ng$-$closed set in $(U,\tau_R$(X)).*

Remark 3.17. *The following examples shows that Ng$\hat{}$ α-continuous is independent of Ng-closed, Ngα-closed, Ns-closed and Nβ-closed sets.*

Example 3.18. *Let U = $\{a,b,c,d\}$, X = $\{a,c\}$ and U/R = $\{\{a\},\{b,c,d\}\}$. Then τ_R(X)=$\{U,\phi,\{a\},\{b,c,d\}\}$, and τ_R^C(X)= $\{U,\phi,\{b,c,d\},\{a\}\}$. Also V = $\{w,x,y,z\}$, Y = $\{w,x\}$ and V/R = $\{\{w\},\{x,y\},\{z\}\}$. Then τ_R(Y)=$\{V,\phi,\{w\},\{w,x,y\},\{x,y\}\}$, and τ_R^C(X)=$\{V,\phi,\{x,y,z\},\{z\},\{w,z\}\}$. Let f:(U,$\tau_R$(X)) $\rightarrow$ (V,τ_R(Y)) be an identity function. Then f is Ng-continuous, Ngα-continuous, Ns-continuous and Nβ-continuous but not Ng$\hat{}\alpha$-continuous. Since the closed set $\{z\}$ in (V,τ_R(Y)). its inverse $f^{-1}(z)=\{d\}$ is not Ng$\hat{}\alpha-$continuous.*

Theorem 3.19. *Let f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}\alpha$-continuous, then f(Ng$\hat{}\alpha$cl(A)) is not necessarily equal to Ncl(f(A)) where A $\subseteq$ U.*

Proof. Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}}. Let X = {a,b} $\subseteq$U. Then τ_R(X)={U,ϕ,{a},{a,b,d},{b,d}} which are nano open sets. Ng$\hat{}\alpha$-closed sets are {U,ϕ,{c},{a,c},{b,c}, {c,d},{a,b,c},{a,c,d},{b,c,d}}. Ng$\hat{}\alpha$-open sets are {U,ϕ, {a,b,d},{b,d},{a,d},{a,b},{d},{b},{a}}. Let V = {w,x,y,z} with V/R = {{w},{x,y},{z}}. Let Y = {x,z} $\subseteq$ V. Then τ_R(Y)={V,ϕ,{x},{y,z},{x,y,z}} which are nano open sets. Ng$\hat{}\alpha$-closed sets are {V,ϕ,{y,z,w},{x,z,w},{x,y,w},{x,y,z}, {y,z},{x,w},{x,y},{x,z},{x},{y},{z}}. Ng$\hat{}\alpha$-open sets are {V,ϕ,{x},{y},{z},{w},{x,w},{x,z},{z,w},{y,w},{y,z,w}, {x,z,w},{x,y,w}}. Define f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) as f(a) = y, f(b) = x, f(c) = w, f(d) = z. Then f$^{-1}(V)=U$, $f^{-1}(\phi)$ = ϕ, f$^{-1}(\{y,z\})=\{a,d\}$, $f^{-1}(\{x\})=\{b\}$, $f^{-1}(\{x,y,z\})=$ $\{a,b,d\}$. *Thus the inverse image of every nano open set in V is* Ng$\hat{}\alpha-$*open in U. Hence* $f:(U,\tau_R$(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}\alpha$-continuous on U. Let A = {b,d} $\subseteq$ U. Now Ng$\hat{}\alpha$-cl(A) = {b,d} and hence f(Ng$\hat{}\alpha$cl(A)) = f({b,d}) = {x,z}. Now Ncl(f(A)) = Ncl(f({b,d})) = Ncl({x,z}) = {x,y}. That is, the equality does not hold in the above theorem when f is nano continuous and thus f(Ng$\hat{}\alpha$cl(A)) $\neq$ Ncl(f(A)) even though f is Ng$\hat{}\alpha$-continuous. □

Theorem 3.20. *A function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous function if and only if Ng$\hat{}$ αcl($f^{-1}(B)$)) $\subseteq$ $f^{-1}(Ncl(B))$ for every subset B of (V,τ_R(Y)).*

Proof. Let B $\subseteq$ V and f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) be Ng$\hat{}$ α-continuous. Then Ncl(B) is nano closed in (V,τ_R(Y)) and hence $f^{-1}(Ncl(B))$ is Ng$\hat{}$ α-closed in (U,τ_R(X)). Therefore, Ng$\hat{}$ αcl($f^{-1}(Ncl(B))$) = $f^{-1}(Ncl(B))$. Since B $\subseteq$ Ncl(B), then $f^{-1}(B)\subseteq f^{-1}(Ncl(B)$. ie.,Ng$\hat{}$ αcl($f^{-1}(B)$) $\subseteq$ Ng$\hat{}$ αcl ($f^{-1}(Ncl(B))$) = $f^{-1}(Ncl(B)$. Hence Ng$\hat{}$ αcl($f^{-1}(B)$) $\subseteq$ $f^{-1}(Ncl(B))$. Conversely, Let Ng$\hat{}$ αcl($f^{-1}(B)$)) $\subseteq f^{-1}(Ncl$ $(B))$ for every subset B $\subseteq$ V. Now let B be nano closed in (V,τ_R(Y)) then Ncl(B) = B. Given Ng$\hat{}$ αcl($f^{-1}(Ncl(B))$) = $f^{-1}(Ncl(B))$. Hence Ng$\hat{}$ αcl($f^{-1}(B)$) = $f^{-1}(B)$. But $f^{-1}(B)$ $\subseteq$ Ng$\hat{}$ αcl($f^{-1}(B)$) and hence Ng$\hat{}$ αcl($f^{-1}(B)$) = $f^{-1}(B)$. Thus $f^{-1}(B)$ is Ng$\hat{}$ α-closed set in (U,τ_R(X)) for every nano closed set B in (V,τ_R(Y)). Hence f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous. □

Theorem 3.21. *A function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous function if and only if $f^{-1}(Nint(B))$ $\subseteq$ Ng$\hat{}$ αint($f^{-1}(B)$) for every subset B of (V,τ_R(Y)).*

Proof. Let B $\subseteq$ V and f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) be Ng$\hat{}$ α-continuous. Then Ncl(B) is nano closed in (V,τ_R(Y)). Now $f^{-1}(Nint(B))$ is Ng$\hat{}$ α-open set in (U,τ_R(X)). ie.,Ng$\hat{}$ αint(f^{-1} $(Nint(B))$). Also, for B $\subseteq$ V, Nint(B) $\subseteq$ B always.Then $f^{-1}(Nint(B))\subseteq f^{-1}(B)$. Therefore, Ng$\hat{}$ αint($f^{-1}(Nint(B))$) $\subseteq$ Ng$\hat{}$ αint($f^{-1}(B)$). ie.,$f^{-1}(Nint(B))\subseteq$ Ng$\hat{}$ αint($f^{-1}(B)$).

Conversely, Let $f^{-1}(Nint(B))$ $\subseteq$ Ng$\hat{}$ αint($f^{-1}(B)$) for every subset B of (V,τ_R(Y)). Let B be nano open in (V,τ_R(Y)) and hence Nint(B) = B. Given $f^{-1}(Nint(B))\subseteq$ Ng$\hat{}$ αint(f^{-1} $(Nint(B))$). ie.,$f^{-1}(B)\subseteq$ Ng$\hat{}$ αint($f^{-1}(B)$). Also Ng$\hat{}$ αint ($f^{-1}(B)$) $\subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ = Ng$\hat{}$ αint($f^{-1}(B)$) which implies that $f^{-1}(B)$ is Ng$\hat{}$ α-open in (U,τ_R(X)) for every nano open set B of (V,τ_R(Y)). Therefore f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}$ α-continuous. □

Example 3.22. *In example 3.19, Let us define f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) as f(a) = y, f(b) = x, f(c) = z, f(d) = w. Here f is Ngα-continuous. Since the inverse image of every nano open se in (V,τ_R(Y)) is Ngα-open in (U,τ_R(X)). Let B = $\{y\}\subset$ V. Then Ncl(B) = $\{y,z,w\}$. Hence $f^{-1}(Ncl(B))=f^{-1}(\{y,z,w\}))$ = $\{a,c,d\}$. Also $f^{-1}(B)$ = $\{a\}$. Hence Ng$\hat{}$ αcl($f^{-1}(B)$) = Ng$\hat{}$ αcl(a) = $\{a\}$. Thus Ng$\hat{}$ αcl($f^{-1}(B)$) $\neq f^{-1}(Ncl(B))$. Also when A = $\{y,z,w\}\subseteq$ V, $f^{-1}(Nint(A))=f^{-1}(\{y,z\})=\{a,c\}$. But Ng$\hat{}$ αint($f^{-1}(A)$) = Ng$\hat{}$ αint($\{a,c,d\}$) = $\{a,c,d\}$. That is*

$f^{-1}(Nint(A)) \neq Ng\hat{\alpha}int(f^{-1}(A))$. Thus the equality does not hold in the above theorem when f is nano continuous.

4. Ng$\hat{\alpha}$-irresolute functions

Analogous to irresolute maps in nano topological spaces we introduce the class of Ng$\hat{\alpha}$-irresolute functions which is included in the class of Ng$\hat{\alpha}$-continuous functions. in this section we investigate basic properties of Ng$\hat{\alpha}$-irresolute functions.

Definition 4.1. *Let $(U,\tau_R(X))$ and $(V,\tau_R(Y))$ be nano topological spaces and a function f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is called Nano $\hat{g}\,\alpha$-irresolute(briefly Ng$\hat{\alpha}$-irresolute) if the inverse image of every Ng$\hat{\alpha}$-closed set in V is Ng$\hat{\alpha}$-closed in U.*

Theorem 4.2. *A function f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute, then f is Ng$\hat{}\,\alpha$-continuous.*

Proof. Let f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute, then the inverse image of every Ng$\hat{\alpha}$-closed set in $(V,\tau_R(Y))$ is Ng$\hat{\alpha}$-closed in $(U,\tau_R(X))$. Let F be nano closed in $(V,\tau_R(Y))$, then F is Ng$\hat{\alpha}$-closed set in $(V,\tau_R(Y))$ and f is Ng$\hat{\alpha}$-irresolute. Hence $f^{-1}(F)$ is Ng$\hat{\alpha}$-closed. Therefore f is Ng$\hat{\alpha}$-continuous. □

Remark 4.3. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.4. *Let $U = \{a,b,c,d\}$, $X = \{a,c\}$ and $U/R = \{\{a\},\{b,c,d\}\}$. Then $\tau_R(X)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$, and $\tau_R^C(X)=\{U,\phi,\{b,c,d\},\{a,c\},\{c\},\}$. Also $V = \{x,y,z,w\}$, $Y = \{x,z\}$ and $V/R = \{\{x,y\},\{z\},\{w\}\}$. Then $\tau_R(Y)=\{V,\phi,\{z\},\{x,y\},\{x,y,z\}\}$, and $\tau_R^C(X)=\{V,\phi,\{x,y,w\},\{z,w\},\{w\}\}$. Define $f : (U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ as $f(a) = z$, $f(b) = y$, $f(c) = x$, $f(d) = w$. Then $f^{-1}(\{z\}) = \{a\}$, $f^{-1}(\{x,y\}) = \{b,c\}$, $f^{-1}(\{x,y,z\}) = \{a,b,c\}$. Thus $\{a\}$, $\{b,c\}$ and $\{a,b,c\}$ are Ng$\hat{\alpha}$-open sets in $(U,\tau_R(X))$. That is the inverse image of every nano open set in $(V,\tau_R(Y))$ is Ng$\hat{\alpha}$-open set in $(U,\tau_R(X))$. Therefore f is Ng$\hat{\alpha}$-continuous. But f is not Ng$\hat{\alpha}$- irresolute, Since $f^{-1}(\{y,z,w\}) = \{a,b,d\}$ which is not Ng$\hat{\alpha}$-closed in $(U,\tau_R(X))$ where as $\{y,z,w\}$ is Ng$\hat{\alpha}$-closed in $(V,\tau_R(Y))$. Thus a Ng$\hat{\alpha}$-continuous function is not Ng$\hat{\alpha}$-irresolute.*

Theorem 4.5. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ are both Ng$\hat{}\,\alpha$-irresolute, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-irresolute.*

Proof. Let A be Ng$\hat{}\,\alpha$-open in W. Then $g^{-1}(A)$ is Ng$\hat{}\,\alpha$-open in V, Since g is Ng$\hat{}\,\alpha$-irresolute and $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ *is $Ng\alpha$*-open in U, Since f is Ng$\hat{}\,\alpha$-irresolute. Hence $g \circ f$ is Ng$\hat{}\,\alpha$-irresolute. □

Theorem 4.6. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*

Theorem 4.7. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-continuous and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
Proof of the theorem 4.6 and 4.7 are obvious.

Theorem 4.8.

1. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano semi-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
2. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano pre-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
3. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano regular-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
4. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano α-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
5. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano g-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
6. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano gs-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
7. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano gr-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*
8. *If f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is Ng$\hat{}\,\alpha$-irresolute and g:$(V,\tau_R(Y)) \rightarrow (W,\tau_R(Z))$ is nano αg-continuous, then $g \circ f$:$(U,\tau_R(X)) \rightarrow (W,\tau_R(Z))$ is Ng$\hat{}\,\alpha$-continuous.*

proof of the above theorems are obvious.

5. Ng$\hat{\alpha}$-open and closed functions

Definition 5.1. *A function f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ is said to be Ng$\hat{}\,\alpha$-open (resp.Ng$\hat{}\,\alpha$-closed) function if the inverse image of every nano open (resp. nano closed) set in $(U,\tau_R(X))$ is Ng$\hat{}\,\alpha$-open (resp.Ng$\hat{}\,\alpha$-closed) in $(V,\tau_R(Y))$.*

Theorem 5.2. *Let f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ be a function. If f is an nano open function, then f is Ng$\hat{}\,\alpha$-open function.*

Proof. Let f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ be a nano open function and S be a nano open set in U. Thus f(S) is nano open and hence f(S) is Ng$\hat{}\,\alpha$-open in V. Thus f is Ng$\hat{}\,\alpha$-open. □

Theorem 5.3. *Let f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ be a function. If f is an nano closed function, then f is Ng$\hat{}\,\alpha$-closed function.*

Proof. Let f:$(U,\tau_R(X)) \rightarrow (V,\tau_R(Y))$ be a nano closed function and S be a nano closed set in U. Thus f(S) is nano closed and hence f(S) is Ng$\hat{}\,\alpha$-closed in V. Thus f is Ng$\hat{}\,\alpha$-closed. □

Theorem 5.4. *A function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is Ng$\hat{}\alpha$-closed if and only if for each subset B of V and for each nano open set G containing $f^{-1}(B)$ there exists a Ng$\hat{}\alpha$-open set F of V such that $B \subseteq F$ and $f^{-1}(B) \subseteq G$.*

Proof. **Necessity:** Let G be a nano open subset of (U,τ_R(X)) and B be a subset of V such that $f^{-1}(B) \subseteq$ G. Define, F = V-f(U-G). Since, f is Ng$\hat{}\alpha$-closed. Then F is Ng$\hat{}\alpha$-open set containing B such that $f^{-1}(F) \subseteq$ G.

Sufficiency : Let E be a nano closed subset of (U,τ_R(X)). Then $f^{-1}(V - f(E)) \subseteq$ (U-E) and (U-E) ia nano open. By hypothesis, there is a Ng$\hat{}\alpha$-open set F of (V,τ_R(Y)) such that V-f(E) $\subseteq$ F and $f^{-1}(B) \subseteq$ U-E. Therefore E $\subseteq$ U-$f^{-1}(F)$. Hence, V-F $\subseteq$ f(E) $\subseteq$ f(U-$f^{-1}(F)$) $\subseteq$ V-F. Which implies that f(E) = V-F and hence f(E) is Ng$\hat{}\alpha$-closed in (V,τ_R(Y)). Therefore, f is Ng$\hat{}\alpha$-closed function. ☐

Theorem 5.5. *If a function f:(U,τ_R(X)) $\rightarrow$ (V,τ_R(Y)) is nano closed and a function g:(V,τ_R(Y)) $\rightarrow$ (W,τ_R(Z)) is Ng$\hat{}\alpha$-closed then their composition gf :(U,τ_R(X)) $\rightarrow$ (W,τ_R(Z)) is Ng$\hat{}\alpha$-closed.*

Proof. Let H be a nano closed set in U. Then f(H) is nano closed in V and (g$\circ$f)(H) = g(f(H)) is Ng$\hat{}\alpha$-closed, as g is Ng$\hat{}\alpha$-closed. Hence,g$\circ$f is Ng$\hat{}\alpha$-closed. ☐

References

[1] Balachandran K, Sundaram P and Maki H. On Generalized Continuous Maps in Nano Toplogical Spaces, *Mem.Fac.Sci.Kochi Univ.math.*1 (12)(1991), 5 - 13.

[2] Bhuvaneshwari K. Mythili Gnanapriya K. Nano Generalized closed sets, *Inernaional Journal of Scientific and Reasearch Publications*, 2014;4(5):1-3.

[3] Bhuvaneshwari K. Ezhilarasi K. On Nano semi-Generalized Continuous Maps in Nano Toplogical Spaces*International Research Journal of Pure Algebra*5(9),2015;149-155.

[4] Bhuvaneshwari K. Mythili Gnanapriya K. On Nano Generalized Continuous Functions in Nano topological spaces,*IJMA* Vol.6,Issue 6,2229-5046,2015.

[5] Bhuvaneshwari K. Thanga Nachiyar R. On Nano Generalized A-closed Sets and Nano A- Generalized Closed Sets in Nano Topological Spaces, *IJETT*, Volume 13,2014.

[6] Lellis Thivagar.M and Carmel Richard, On Nano Forms of Weakly Open Sets, *International Journal of Mathematics and stat.Inv.*,, Vol.I, No.I, 31-37(2013).

[7] Levine N.Generalized Closed Sets in Topology, *Rend.Cire.Math.Palerino,*1970;19:89-96.

[8] Maheshwari.A and Sheik John.M, Nano Regular Generalized Star b-continuous Functions in Nano topological Spaces,*International Journal of Mathematics and its Applications* Volume 4, Issue 2-D(2016), 49-55.

[9] Shalini S.B and Indirani.K, On Nano Generalized β Continuous Functions and nano Generalized β irresolute Functions in Nano Topological Spaces,*IOSR Journal of Mathematics*,Volume 13,Issue I Ver.VI(Jan-Feb.2017),pp 79-86.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 – 3786
Malaya Journal of Matematik
ISSN(O):2321 – 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆