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Nano semi c^* generalized closed sets in nano topological spaces

A.Pushpalatha [1*] and S.Sridevi [2]

Abstract

The aim of this paper is to introduce and investigate the nano semi c^* generalized closed set in nano topological spaces. Its basic properties are also analyzed and also investigate its relation with already existing well known sets.

Keywords

Nsc^*g-closed set, Nc^*g-closed set.

AMS Subject Classification

54B05,54C05.

[1] *Assistant Professor, Department of Mathematics, Government Arts College, Udumalpet, Tamil Nadu, India.*

[2] *Assistant Professor, Department of Mathematics, Vidyasagar College of Arts and Science, Udumalpet, Tamil Nadu, India.*

*__Corresponding author__: [1] velu _ pushpa@yahoo.co.in, [2] sriyukana22@gmail.com

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Contents

1. Introduction

M. Lellis Thivagar and Carmel Richard [6] introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He has also defined nano closed sets, nano interior and nano closure of a set. In 1970, Levine [7] first considered the concept of generalized closed sets were defined and investigated. Hatir et al [5] introduced t-sets and α-sets. A.Pushpalatha and R.Nithyakala introduced a concept of scg-closed, sc^*g-closed sets. In this paper we have introduced a new class of set called nano semi c^* generalized closed set and obtain some of its properties.

2. Preliminaries

This section is to recall some basic definitions and properties of topological space and nano topological spaces which are useful in this study.

Definition 2.1. *A subset A of a topological space (x,τ) is called*

a) generalized closed (g-closed) [7] if $cl(A) \subseteq U$, whenever $A \subseteq U$, and U is an open set in X.

b) semi-generalized closed (sg-closed) [1] if $scl(A) \subseteq U$, whenever $A \subseteq U$, and U is a semi open set in X.

c) α^*-set [7] if $int(A) = int(cl(int(A)))$.

d) c^α-set [12] if $A = G \cap F$, where G is g-open and F is a α^*-set.

e) semi c^*-generalized closed (sc^*g-closed) [11] if $scl(A) \subseteq U$, whenever $A \subseteq U$, and U is a c^*-set in X.

Definition 2.2. *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,*

(i)The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.

$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ *where $R(x)$ denotes the equivalence class determined by $x \in U$.*

(ii)The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$

(iii)The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X

with respect to R and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.3. *If* (U,R) *is an approximation space and* $X, Y \subseteq U$, *then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.4. *Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$. *Then* $\tau_R(X)$ *satisfies the following axioms*
(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ *is in* $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ *is in* $\tau_R(X)$.
Then $\tau_R(X)$ *is a topology on U called the nano topology on U with respect to X. We call* $(U, \tau_R(X))$ *as nano topological space. The elements of* $\tau_R(X)$ *are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.*

Remark 2.5. *If* $\tau_R(X)$ *is the nano topology on U with respect to X, then the set* $B = \{U, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.6. *If* $(U, \tau_R(X))$ *is a nano topological space with respect to X where* $X \subseteq U$ *and if* $A \subseteq U$, *then*
(i)The nano interior of A is defined as the union of all nano-open subsets of A is contained in A and is denoted by $Nint(A)$. *That is,* $Nint(A)$ *is the largest nano-open subset of A.*
(ii)The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by $Ncl(A)$). *That is,* $Ncl(A)$) *is the smallest nano-closed set containing A.*

Definition 2.7. *If* $(U, \tau_R(X))$ *be a nano topological space and* $A \subseteq U$ *then A is said to be*
i) nano pre open set [6] if $A \subseteq Nint(Ncl(A))$.
ii) nano semi open set [6] if $A \subseteq Ncl(Nint(A))$.
iii) nano α*-open set [6] if* $A \subseteq Nint(Ncl(Nint(A)))$.
iv) nano regular open set [6] if $A = Nint(Ncl(A))$.

Definition 2.8. *[17] A subset* $M_x \subset U$ *is called a nano semi pre-neighbourhood (*$N\beta$*-nhd) of a point* $x \in U$ *iff there exists a* $A \in N\beta O(U, X)$ *such that* $x \in A \subset M_x$ *and a point x is called* $N\beta$*-nhd point of the set A.*

Definition 2.9. *A subset H of a nano topological space* $(U, \tau_R(X))$ *is called; (i) nano generalized closed set (Ng-closed) [2] if* $Ncl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano open set in* $(U, \tau_R(X))$.
(ii) nano semi generalized closed set (Nsg-closed) [3] if $Nscl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano semi - open set in* $(U, \tau_R(X))$.
(iii) nano generalized semi closed set (Ngs-closed) [3] if $Nscl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano open set in* $(U, \tau_R(X))$.
(iv) nano generalized Alpha closed set(Ngα*-closed) [9] if* $N\alpha cl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano* α*-open set in* $(U, \tau_R(X))$.
(v) nano Alpha generalized closed set (Nα*g-closed) [9] if* $N\alpha cl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano open set in* $(U, \tau_R(X))$.
(vi) nano regular genera;ized closed set (Nrg-closed) [20] if $Ncl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano regular open set in* $(U, \tau_R(X))$.
(vii) nano generalized pre closed set (Ngp-closed) [4] if $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano open set in* $(U, \tau_R(X))$.
(viii) nano pre generalized closed set (Npg-closed) [4] if $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano pre - open set in* $(U, \tau_R(X))$.
(ix) nano generalized pre regular closed set (Ngpr-closed) [10] if $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano regular open set in* $(U, \tau_R(X))$.
(x) nano weakly generalized closed set (Nwg-closed) [8] if $Ncl(Nint(H)) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano open set in* $(U, \tau_R(X))$.
(xi) nano generalized semi pre closed set (Ngsp-closed) [18] if $Nspcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano - open set in* $(U, \tau_R(X))$.
(xii) nano semi pre generalized closed set (Nspg-closed) [18] if $Nspcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano semi - open set in* $(U, \tau_R(X))$.
(xiii) nano generalized semi generalized closed set (Ngsg-closed) [21] if $Ncl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano sg-open set in* $(U, \tau_R(X))$.
(xiv) nano generalized**-closed set (Ng***-closed) [13] if* $Ncl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano g-open set in* $(U, \tau_R(X))$.
(xv) strongly nano generalized**-closed set (strongly Ng***-closed) [16]if* $Ncl(Nint(H)) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano g-open set in* $(U, \tau_R(X))$.
(xvi) nano generalized* *semi-closed set (Ng***s-closed) [14]* $Nscl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano g-open set in* $(U, \tau_R(X))$.
(xvii) nano generalized**pre-closed set (Ng** *p-closed) [15] if* $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and G is nano g-open set in* $(U, \tau_R(X))$.

(xviii) nano regular generalized*- closed set (Nrg^*-closed) [19] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano regular-open set in $(U, \tau_R(X))$.
The complement of the above mentioned nano closed sets are their respective nano open sets.

3. Nano semi c^* generalized-closed sets

In this section, we define and study the forms of nano semi c^* generalized closed sets in nano topological space. Also the above existing sets are compare with Nsc^*g–closed set.

Definition 3.1. *A subset A of $(U, \tau_R(X))$ is called Nc^*-set if $A = G \cap F$, where G is Ng–open and F is a $N\alpha$-set.*

Definition 3.2. *A subset A of $(U, \tau_R(X))$ is called a nano semi c^* generalized closed set if $Nscl(A) \subseteq G$, whenever $A \subseteq G$, and G is Nc^*-set in $(U, \tau_R(X))$.*

The complement of Nsc^*g-closed set is Nsc^*g–open set in $(U, \tau_R(X))$.

Example 3.3. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\}, \{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\}, \{b,d\}\}$ is a nano topology on U with respect to X and $\tau_{R^c}(X) = \{\phi,U,\{c\},\{b,c,d\},\{a,c\}\}$. A subset H of a nano topological space $(U, \tau_R(X))$ is called:*
(1) nano pre–closed : $\{\phi,U,\{b\},\{c\},\{d\},\{a,c\},\{b,c\},\{c,d\}, \{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(2) nano α-closed : $\{\phi,U,\{c\},\{a,c\},\{b,c,d\}\}$
(3) nano regular–closed: $\{\phi,U,\{a,c\},\{b,c,d\}\}$
(4) nano g-closed : $\{\phi,U,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\}, \{a,c,d\},\{b,c,d\}\}$
(5) nano sg-closed:$\{\phi,U,\{a\},\{b\},\{c\},\{d\},\{a,c\},\{b,c\}, \{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(6) nano gs-closed :$\{\phi,U,\{a\},\{b\},\{c\},\{d\},\{a,c\},\{b,c\}, \{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(7) nano gα-closed : $\{\phi,U,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\}, \{a,c,d\},\{b,c,d\}\}$
(8) nano αg-closed: $\{\phi,U,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\}, \{a,c,d\},\{b,c,d\}\}$
(9) nano rg-closed : $\{\phi,U,\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\}, \{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$
(10) nano wg-closed: $\{\phi,U,\{b\},\{c\},\{d\},\{a,c\},\{b,c\}, \{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(11) nano gp-closed: $\{\phi,U,\{b\},\{c\},\{d\},\{a,c\},\{b,c\}, \{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(12) nano pg-closed: $\{\phi,U,\{b\},\{c\},\{d\},\{a,c\},\{b,c\}, \{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(13) nano gpr-closed: $\{\phi,U,\{b\},\{c\},\{d\},\{a,b\},\{a,c\}, \{a,d\},\{b,c\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$
(14) nano gsp-closed : $\{\phi,U,\{a\},\{b\},\{c\},\{d\},\{a,b\}, \{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(15) nano spg-closed : $\{\phi,U,\{a\},\{b\},\{c\},\{d\},\{a,b\}, \{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(16) nano gsg-closed : $\{\phi,U,\{c\},\{a,c\},\{b,c,d\}\}$
(17) nano g^*- closed: $\{\phi,U,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\}, \{a,c,d\},\{b,c,d\}\}$
(18) nano strongly g^*- closed: $\{\phi,U,\{b\},\{c\},\{d\},\{a,c\}, \{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(19) nano g^*s closed : $\{\phi,U,\{a\},\{c\},\{a,c\},\{b,c\},\{b,d\},\{c,d\}, \{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(20) nano g^*p closed : $\{\phi,U,\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{b,c\}, \{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
(21) nano rg^*- closed : $\{\phi,U,\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\}, \{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$
(22) nano c^*-set : $P(A)$
(23) nano sc^*g closed: $\{\phi,U,\{a\},\{c\},\{a,c\},\{b,d\},\{b,c,d\}\}$

Theorem 3.4. *Every nano closed set is a Nsc^*g-closed set in $NTS(U, \tau_R(X))$ but not conversely.*

Proof. Assume that A is a nano closed set in $(U, \tau_R(X))$. Let G be a nano c^*-set such that $A \subseteq G$. Since A is nano closed, $Ncl(A) = A$, $Ncl(A) \subseteq G$. But $Nscl(A) \subseteq Ncl(A) \subseteq G$. Therefore $Nscl(A) \subseteq G$ where G is Nc^*-set. Hence A is a Nsc^*g-closed set. □

Example 3.5. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\}, \{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\}, \{b,d\}\}$ is a nano topology on U with respect to X and $\tau_{R^c}(X) = \{\phi,U,\{c\},\{b,c,d\},\{a,c\}\}$. In example 3.2, the sets $\{a\}$, $\{b,d\}$ are Nsc^*g-closed sets but $\{a\}$, $\{b,d\}$ are not a nano closed sets in $(U, \tau_R(X))$.*

Theorem 3.6. *Every nano α-closed set is a Nsc^*g-closed set in $NTS(U, \tau_R(X))$ but not conversely.*

Proof. Assume that A is a nano closed set in $(U, \tau_R(X))$. Let G be a nano c^*-set such that $A \subseteq G$. Since A is nano α-closed, $N\alpha cl(A) = A$,
$N\alpha cl(A) \subseteq G$. But $Nscl(A) \subseteq N\alpha cl(A)$, $N\alpha cl(A) \subseteq G$. Therefore $Nscl(A) \subseteq G$ where G is Nc^*-set. Hence A is a Nsc^*g-closed set. □

Example 3.7. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\}, \{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\}, \{b,d\}\}$ is a nano topology on U with respect to X and $\tau_{R^c}(X) = \{\phi,U,\{c\},\{b,c,d\},\{a,c\}\}$. In example 3.2, the sets $\{a\}$, $\{c\}$, $\{a,c\}$, $\{b,d\}$, $\{b,c,d\}$ are Nsc^*g-closed sets but $\{a\}$, $\{b,d\}$ are not a nano α-closed sets in $(U, \tau_R(X))$.*

Theorem 3.8. *Every nano regular-closed set is a Nsc^*g-closed set in $NTS(U, \tau_R(X))$ but not conversely.*

Proof. Assume that A is a nano regular-closed set in $(U, \tau_R(X))$. Let G be a Nc^*-set such that $A \subseteq G$. Since A is nano regular-closed, $Nrcl(A) = A$, $Nrcl(A) \subseteq G$. But $Nscl(A) \subseteq Nrcl(A) \subseteq G$. Therefore $Nscl(A) \subseteq G$ where G is Nc^*-set. Hence A is a Nsc^*g-closed set. □

Example 3.9. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\}, \{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\}, \{b,d\}\}$ is a nano topology on U with respect to X and $\tau_{R^c}(X) = \{\phi,U,\{c\},\{b,c,d\},\{a,c\}\}$. In example 3.2, the sets $\{a\}$, $\{c\}$,*

$\{b,d\}$ are Nsc^*g-closed sets. But not nano regular-closed sets.

Theorem 3.10. *Every Ngsg-closed set is a Nsc^*g-closed set in $NTS(U,\tau_R(X))$ but not conversely.*

Proof. Assume that A is a $Ngsg$-closed set in $(U,\tau_R(X))$. Let $A \subseteq V$, $Nscl(A) \subseteq V$, where V is Nc^*-set. Since A is $Ngsg$-closed, $Ncl(A) = A$. But $Nscl(A) \subseteq Ncl(A) \subseteq V$, $Nscl(A) \subseteq V$ where V is Nc^*-set. Therefore A is a Nsc^*g-closed set. □

Example 3.11. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In example 3.2, the sets $\{a\}$, $\{b,d\}$ are Nsc^*g-closed sets. Let $A = \{a\}$ be a Nsc^*g-closed and Nsg-open set in $(U,\tau_R(X))$. But $Ncl(A) = \{a,c\} \nsubseteq \{a\}$. Therefore $\{a\}$ is not a Ngsg-closed set.*

Theorem 3.12. *The intersection of two Nsc^*g–closed sets is a Nsc^*g–closed set in $NTS(U,\tau_R(X))$.*

Proof. Let A and B be Nsc^*g–closed sets in $(U,\tau_R(X))$. Let V be a Nsc^*g–closed set in $(U,\tau_R(X))$ such that $A\cap B \subseteq V$. Then $A \subseteq V$ and $B \subseteq V$. Since A and B are Nsc^*g-closed, $Nscl(A) \subseteq V$ and $Nscl(B) \subseteq V$. Hence $Nscl(A\cap B) \subseteq Nscl(A)\cap Nscl(B) \subseteq V$. Therefore $A\cap B$ is Nsc^*g–closed set in $(U,\tau_R(X))$. □

Remark 3.13. *The union of two Nsc^*g–closed sets need not be a Nsc^*g–closed set as seen from the following example.*

Example 3.14. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In Example 3.2the sets $\{a\}$ and $\{b,d\}$ are Nsc^*g-closed sets but their union $\{a,b,d\}$ is not a Nsc^*g-closed set.*

Theorem 3.15. *Every Nsc^*g-closed set is Nsg-closed in a $NTS(U,\tau_R(X))$, but not conversely.*

Proof. Assume that A is a Nsc^*g-closed set in $(U,\tau_R(X))$. Let $A \subseteq V$, where V is Nc^*-set. Then V is written as $G\cap F$, where G is nano g-open and F is nano α^*-set. Since A is Nsc^*g-closed and every nano open set is Ng-open, therefore $Nscl(A) \subseteq G$ where G is nano open. Hece A is nano gs- closed set in $(U,\tau_R(X))$. □

Example 3.16. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In example 3.2, the sets $\{b\}$, $\{d\}$, $\{b,c\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,c,d\}$ are Nsg-closed sets. Let $A = \{b\}$ be a Ngs-closed and Nc^* set in $(U,\tau_R(X))$. But $Nscl(A) = \{b,d\} \nsubseteq \{b\}$. Therefore $\{b\}$ is not a Nsc^*g-closed set.*

Theorem 3.17. *Every Nsc^*g-closed set is Ngs-closed in a $NTS(U,\tau_R(X))$, but not conversely.*

Proof. Assume that A is a Nsc^*g-closed set in $(U,\tau_R(X))$. Let $A \subseteq V$, where V is Nc^*-set. Since A is Nsc^*g-closed and every nano open set is Nc^*-open, therefore $Nscl(A) \subseteq G$ where G is nano open. Hece A is Ngs- closed set in $(U,\tau_R(X))$. □

Example 3.18. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In example 3.2, the sets $\{b\}$, $\{d\}$, $\{b,c\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,c,d\}$ are Ngs-closed sets. Let $A = \{b\}$ be a Nsg-closed and Nc^*-set in $(U,\tau_R(X))$. But $Nscl(A) = \{b,d\} \nsubseteq \{b\}$. Therefore $\{b\}$ is not a Nsc^*g-closed set.*

Theorem 3.19. *Every Nsc^*g-closed set is Ngsp-closed in a $NTS(U,\tau_R(X))$, but not conversely.*

Proof. Assume that A is a Nsc^*g-closed set in $(U,\tau_R(X))$. Let $A \subseteq V$,$Nscl(A) \subseteq V$ where V is Nc^*-set. Since A is Nsc^*g-closed and every nano open set is Nc^*-open, so V is nano-open set. But $Nspcl(A) \subseteq Nscl(A)$, $Nspcl(A) \subseteq V$ where V is nano-open set. Hence A is $Ngsp$-closed set in $(U,\tau_R(X))$. □

Example 3.20. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In Example 3.2, $P(A) = \{a,b,d\}$ are Ngsp-closed sets. Let $A = \{b\}$ be a Ngsp-closed and Nc^*-set in $(U,\tau_R(X))$. But $Nscl(A) = \{b,d\} \nsubseteq \{b\}$. Therefore $\{b\}$ is not a Nsc^*g-closed set.*

Theorem 3.21. *Every Nsc^*g-closed set is Nspg-closed in a $NTS(U,\tau_R(X))$, but not conversely.*

Proof. Assume that A is a Nsc^*g-closed set in $(U,\tau_R(X))$. Let $A \subseteq V$,$Nscl(A) \subseteq V$ where V is Nc^*-set. Since A is Nsc^*g-closed and every nano semi-open set is Nc^*-set, so V is nano semi-open set. But $Nspcl(A) \subseteq Nscl(A)$, $Nspcl(A) \subseteq V$ where V is nano semi-open set. Hence A is $Nspg$-closed set in $(U,\tau_R(X))$. □

Example 3.22. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In Example 3.2, $P(A) = \{a,b,d\}$ are Nspg-closed sets. Let $A = \{d\}$ be a Nspg-closed and Nc^*-set in $(U,\tau_R(X))$. But $Nscl(A) = \{b,d\} \nsubseteq \{d\}$. Therefore $\{d\}$ is not a Nsc^*g-closed set.*

Theorem 3.23. *Every Nsc^*g-closed set is Ng^*s-closed in a $NTS(U,\tau_R(X))$, but not conversely.*

Proof. Assume that A is a Nsc^*g-closed set in $(U,\tau_R(X))$. Let $A \subseteq V$,$Nscl(A) \subseteq V$ where V is Nc^*-set. But every nano g-open set is Nc^*-set, V is Ng-open set. Therefore $Nscl(A) \subseteq V$, $A \subseteq V$ where V is Ng-open set. Hence A is Ng^*s-closed set in $(U,\tau_R(X))$. □

Example 3.24. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$. In example 3.2, the sets $\{a\}$, $\{c\}$, $\{a,c\}$, $\{b,c\}$ $\{b,c\}$, $\{b,d\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,c,d\}$, $\{b,c,d\}$ are Ng^*s-closed sets. Let $A = \{b,c\}$ be a Ng^*s-closed and Nc^*-set in $(U,\tau_R(X))$. But $Nscl(A) = \{b,c,d\} \nsubseteq \{b,c\}$. Therefore $\{b,c\}$ is not a Nsc^*g-closed set.*

Remark 3.25. *The notions of Ng-closed, Npre-closed, Nrg-closed, Nwg-closed, Ngp-closed, Npg-closed, Ng^*-closed, strongly Ng^*-closed, Ng^*p-closed, Nrg^*-closed sets are independent with Nsc^*g-closed sets as seen from the following examples.*

Example 3.26. *In example 3.2, the sets $\{b,c\},\{c,d\},\{a,b,c\}$, $\{a,c,d\}$ are Ng–closed sets. Let $A=\{b,c\}$ be a Ng–closed and Nc^*-set. But $Nscl(A)=\{b,c,d\}\not\subseteq\{b,c\}$. Therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{b,d\}$ are Nsc^*g–closed sets. Let $A=\{a\}$ be a nano open set. But $Ncl(A)=\{a,c\}\not\subseteq\{a\}$. Therefore A is not a Ng–closed sets. Hence Ng–closed and Nsc^*g–closed sets are independent sets.*

Example 3.27. *In example 3.2, the sets $\{b\},\{d\},\{b,c\}$, $\{c,d\},\{a,b,c\},\{a,c,d\}$ are Npre-closed sets. Let $A=\{b\}$ be a Nc^*-set. But $Nscl(A)=\{b,d\}\not\subseteq\{b\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\}$, $\{b,d\}$ are Nsc^*g-closed sets. Let $A=\{a\}$ be a Nc^*-set. But $Npcl(A)=\{a,c\}\not\subseteq\{a\}$, therefore A is not a Npre–closed set. Hence Npre-closed and Nsc^*g-closed sets are independent sets.*

Example 3.28. *In example 3.2, the sets $\{a,b\},\{a,d\},\{b,c\}$, $\{c,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\}$ are Nrg-closed sets. Let $A=\{a,b\}$ be a Nrg-closed and Nc^*-set. But $Nscl(A)=\{U\}\not\subseteq\{a,b\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{b,d\}$ are Nsc^*g-closed sets. Let $A=\{b,d\}$ be a nano regular open set. But $Ncl(A)=\{b,c,d\}\not\subseteq\{b,d\}$, therefore A is not a Nrg–closed set. Hence Nrg-closed and Nsc^*g-closed sets are independent sets.*

Example 3.29. *In example 3.2, the sets $\{b\},\{d\},\{b,c\}$, $\{c,d\},\{a,b,c\},\{a,c,d\}$ are Nwg-closed sets. Let $A=\{b\}$ be a Nc^*-set. But $Nscl(A)=\{b,d\}\not\subseteq\{b\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\}$, $\{b,d\}$ are Nsc^*g-closed sets. Let $A=\{b,d\}$ be a Nc^*-set. But $Ncl(Nint(A))=\{b,c,d\}\not\subseteq\{b,d\}$, therefore A is not a Nwg–closed set. Hence Nwg-closed and Nsc^*g-closed sets are independent sets.*

Example 3.30. *In example 3.2, the sets $\{c\},\{a,c\},\{b,c\}$, $\{c,d\},\{a,b,c\},\{a,c,d\}$ are Npre-closed sets. Let $A=\{b\}$ be a Nc^*-set. But $Nscl(A)=\{b,d\}\not\subseteq\{b\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\}$, $\{b,d\}$ are Nsc^*g-closed sets. Let $A=\{a\}$ be a Nc^*-set. But $Npcl(A)=\{a,c\}\not\subseteq\{a\}$, therefore A is not a Npre–closed set. Hence Npre-closed and Nsc^*g-closed sets are independent sets.*

Example 3.31. *In example 3.2, the sets $\{a\},\{c\},\{a,c\}$, $\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{b,c,d\}$ are Ng^*p-closed sets. Let $A=\{b,c\}$ be a Nc^*-set. But $Nscl(A)=\{b,c,d\}\not\subseteq\{b,c\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\},\{a,c\}$, $\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{a\}$ be a nano g-open set. But $Npcl(A)=\{a,c\}\not\subseteq\{a\}$, therefore A is not a Ng^*p–closed set. Hence Ng^*p-closed and Nsc^*g-closed sets are independent sets.*

Example 3.32. *In example 3.2, the sets $\{b\},\{c\},\{d\}$, $\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}$ are Ngp-closed sets. Let $A=\{b,c\}$ be a Nc^*-set. But $Nscl(A)=\{b,c,d\}\not\subseteq\{b,c\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\}$, $\{a,c\},\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{a\}$ be a nano open set. But $Npcl(A)=\{a,c\}\not\subseteq\{a\}$, therefore A is not a Ngp–closed set. Hence Ngp-closed and Nsc^*g-closed sets are independent sets.*

Example 3.33. *In example 3.2, the sets $\{b\},\{c\},\{d\}$, $\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}$ are Npg-closed sets. Let $A=\{b,c\}$ be a Nc^*-set. But $Nscl(A)=\{b,c,d\}\not\subseteq\{b,c\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\}$, $\{a,c\},\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{a\}$ be a nano pre open set. But $Npcl(A)=\{a,c\}\not\subseteq\{a\}$, therefore A is not a Npg–closed set. Hence Npg-closed and Nsc^*g-closed sets are independent sets.*

Example 3.34. *In example 3.2, the sets $\{c\},\{a,c\},\{b,c\}$, $\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}$ are $N\alpha g$-closed sets. Let $A=\{c,d\}$ be a Nc^*-set. But $Nscl(A)=\{b,c,d\}\not\subseteq\{c,d\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\},\{a,c\}$, $\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{a\}$ be a nano r-open set. But $N\alpha cl(A)=\{a,c\}\not\subseteq\{a\}$, therefore A is not a $N\alpha g$–closed set. Hence $N\alpha g$-closed and Nsc^*g-closed sets are independent sets.*

Example 3.35. *In example 3.2, the sets $\{c\},\{a,c\},\{b,c\}$, $\{c,d\},\{a,b,c\},\{b,c,d\}$ are Ng^*-closed sets. Let $A=\{b,c\}$ be a Nc^*-set. But $Nscl(A)=\{b,c,d\}\not\subseteq\{b,c\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\},\{a,c\}$, $\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{b,d\}$ be a Ng-open set. But $Ncl(A)=\{b,c,d\}\not\subseteq\{b,d\}$, therefore $\{b,d\}$ is not a Ng^*–closed set. Hence Ng^*-closed and Nsc^*g-closed sets are independent sets.*

Example 3.36. *In example 3.2, the sets $\{b\},\{c\},\{d\}$, $\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}$, $\{b,c,d\}$ are Ngpr-closed sets. Let $A=\{b\}$ be a Nc^*-set. But $Nscl(A)=\{b,d\}\not\subseteq\{b\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\},\{a,c\}$, $\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{b,d\}$ be a Nr-set. But $Npcl(A)=\{b,c,d\}\not\subseteq\{b,d\}$, therefore A is not a Ngpr–closed set. Hence Ngpr-closed and Nsc^*g-closed sets are independent sets.*

Example 3.37. *In example 3.2, the sets $\{b\},\{c\},\{d\}$, $\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}$ are strongly Ng^*-closed sets. Let $A=\{d\}$ be a Nc^*-set. But $Nscl(A)=\{b,d\}\not\subseteq\{d\}$, therefore A is not a Nsc^*g-closed set. The sets $\{a\},\{c\},\{a,c\}$, $\{b,d\},\{b,c,d\}$ are Nsc^*g-closed sets. Let $A=\{b,d\}$ be a Ng-open set. But $Ncl(Nint(A))=\{b,c,d\}\not\subseteq\{b,d\}$, therefore A is not a strongly Ng^*-closed set. Hence Ng^*-closed and Nsc^*g-closed sets are independent sets.*

Example 3.38. *In example 3.2, the sets $\{c\},\{a,b\},\{a,c\}$, $\{b,c\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$ are Nrg^*-closed sets. Let $A=\{a,b\}$ be a Nc^*-set. But $Nscl(A)=U\not\subseteq\{a,b\}$, therefore A is not a Nsc^*g-closed set. The sets*

$\{a\},\{c\},\{a,c\}$,
$\{b,d\},\{b,c,d\}$ *are* Nsc^*g*- closed sets. Let* $A=\{a\}$ *be a Nr-open set. But* $Ncl((A)=\{a,c\}\nsubseteq\{a\}$*, therefore A is not a* Nrg^**-closed set. Hence* Nrg^**-closed and* Nsc^*g*-closed sets are independent sets.*

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