Malaya Journal of Matematik, Vol. S, No. 1, 375-384, 2019

https://doi.org/10.26637/MJM0S01/0067

Decompositions of *NAB*-continuity and nano weak *AB*-continuity

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Abstract

The primary intention of this article is to introduce the class of *NAB*-set as the set that are the intersection of a nano-open and a nano semi-regular set. Also we define a class of nano weak *AB*-set as the intersection of a nano-open and a nano semi pre-regular set and examined some of their related attributes and theorems. Several classes of well-known nano topological spaces are characterized via the new concept. By using these sets, a new decomposition of nano-continuity is provided.

Keywords

NAB-set, nano weak *AB*-set, nano *NDB*-set, nano locally indiscrete, nano locally-dense, nano semi-connected. *NAB*-continuous, nano strongly irresolute-continuous and nano weak *AB*-continuous.

AMS Subject Classification

54B05.

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Article History: Received **12** February **2019** ; Accepted **09** May **2019**

Contents

1 Introduction 375

2 Preliminaries 376

3 Nano *NDB* and *NAB*-sets 377

4 Characterization of some peculiar nano topological spaces 378

5 Nano weak *AB*-sets 381

6 Decomposition of *NAB*, nano strongly irresolute and nano weak *AB*-continuous functions 382

References 384

1. Introduction

A "space" will always mean a topological space. Broadly speaking topology is the study of space and continuity. The concept of continuity plays a very major role in general topology and they are now the research topics of many topologists worldwide. In 1986 and in 1989, Jingcheng Tong in [5, 6] introduced two new classes of sets namely *A*-sets and *B*-sets and using them we obtained a new decompositions of nano-continuity. The concepts of *A*-sets, locally closed sets and *B*-sets play an important role when continuous functions are decomposed.

In 2002 [14] Z. Pawlak discussed the applications of rough set theory with an example. Based on this the theory of nano topology [7] proposed by Lellis Thivagar and Richard is an extension of set theory for the study of intelligent systems characterized by in sufficient and incomplete information. The elements of a nano topological space are called the nano-open sets. Jayalakshmi and Janaki in[4] defined the notion of *NA*-set & *NB*-set in nano topological spaces. Extensive research on decomposition were done in recent years. Several new decompositions of nano continuous and related mappings were recently obtained in [22]

Lellis Thivagar in [9] studied the notions of expansion of nano-open sets and obtain decomposition of nano- continuity in nano topological spaces. Since the advent of these notions several research papers with interesting results in different respects came to existence. In this paper, the connection of *NAB*-sets to other classes of generalized nano-open sets is investigated as well as several characterizations of nano topological spaces via *NAB*-sets are given. Also we introduce the notion of a new classes of subsets called nano weak *AB*-sets which lies between the class of *NAB*-sets and the class of *NC*-sets. A new decomposition of nano *AB*-continuity and a decomposition of nano weak *AB*-continuity is produced at the last section.

2. Preliminaries

In this section, we recall some requisite ideas definitions and basic results of nano topology which will be used throughout the paper.

Definition 2.1. *[7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:*

- *U and $\phi \in \tau_R(X)$.*
- *The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.*
- *The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.*

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ is a nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.

Definition 2.2. *[7, 16] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*

- *nano semi-open if $A \subseteq Ncl(Nint(A))$.*
- *nano pre-open if $A \subseteq Nint(Ncl(A))$.*
- *nano α-open if $A \subseteq Nint(Ncl(Nint(A)))$.*
- *nano semi pre-open if $A \subseteq Ncl(Nint(Ncl(A)))$.*
- *Nr-open if $A = Nint(Ncl(A))$.*

Definition 2.3. *[4] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*

- *Nt-set if $Nint(A) = Nint(Ncl(A))$.*
- *NA-set if $A = U \cap F$ where U is nano-open and F is nano regular-closed.*
- *NB-set if $A = U \cap F$ Where U is nano-open and F is Nt-set.*

Definition 2.4. *[8] A subset A of a nano topological space $(U, \tau_R(X))$ is said to be nano-dense if $Ncl(A) = U$.*

Definition 2.5. *[20] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be NC-set if $A = G \cap F$ where G is Ng-open and F is a Nt-set in U.*

Definition 2.6. *[18] A subset H of a space $(U, \tau_R(X))$ is said to be nano semi-regular if H is nano semi-open and a nano t-set.*

Definition 2.7. *[24] A subset U is called nano-submaximal if each nano-dense subset of U is called nano-open.*

Definition 2.8. *[23] A nano topological space $(U, \tau_R(X))$ is said to be nano-hyperconnected if every nano open-set is nano dense.*

Definition 2.9. *[11] Let $(U, \tau_R(X))$ be a nano topological space and Let $A \subseteq U$, then A is called nano nowhere dense if $Nint(Ncl(A)) = \phi$*

Definition 2.10. *[7] A nano topological space $(U, \tau_R(X))$ is said to be nano extremally disconnected, if the nano-closure of each nano-open set is nano-open.*

Definition 2.11. *[2] Let (U, $\tau_R(X)$) be a nano topological space and $A \subseteq U$. Then A is said to be a nano locally-closed (briefly, NLC) set, if $A = U \cap F$, where U is nano-open and F is a nano-closed in U.*

Definition 2.12. *[12] A space $(U, \tau_R(X))$ is called locally-indiscrete if every $NO(U,X)$ is $NC(U,X)$.*

Definition 2.13. *[17] A subset H of a space $(U, \tau_R(X))$ is called a nano t_α-set if $Nint(H) = Ncl(Nint(Ncl(H)))$.*

Definition 2.14. *[15] A subset H of a space $(U, \tau_R(X))$ is called nano β-regular if H is a nano β-open and a nano t_α-set.*

Definition 2.15. *[3] A subset H of a space $(U, \tau_R(X)$ is called a nano α^*-set if $Nint(Ncl(Nint(H))) = Nint(H)$.*

Definition 2.16. *[8] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. Then the mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano-continuous on U, if the inverse image of every nano-open set in V is nano-open in U.*

Definition 2.17. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is*

- *nano A-continuous [20], if $f^{-1}(A)$ is a nano A-set in U for every nano-open set A in V*
- *nano B-continuous [20], if $f^{-1}(A)$ is a nano B-set in U for every nano-open set A in V*
- *nano C-continuous [20], if $f^{-1}(A)$ is a nano C-set in U for every nano-open set A in V.*
- *nano locally closed continuous function [shortly, NLC-continuous] [2], if $f^{-1}(B)$ is a nano locally closed set of $(U, \tau_R(X))$ for each nano-open set B of $(V, \tau_{R'}(Y))$*
- *nano ic-continuous [20], if $f^{-1}(A)$ is a nano ic-set in U for every nano-open set A in V.*
- *nano semi-continuous [19], $f^{-1}(A)$ is nano semi-open on U for every nano-open set in V*
- *nano pre-continuous [19], $f^{-1}(A)$ is nano pre-open on U for every nano-open set in V*

- *nano α-continuous[8], if $f^{-1}(A)$ is nano α-open in U for every nano-open set A in V.*
- *nano β-continuous (or nano semi pre-continuous)[13], if $f^{-1}(A)$ is nano β-open set in U for every nano-open set A in V.*

Remark 2.18. *[7] Let U be a non-empty finite universe and $X \subseteq U$.Then the following statements hold:*

1. *If $L_R(X) = \phi$ and $U_R(X) = U$, then $\tau_R(X)) = \{U, \phi\}$ is the indiscrete nano topology on U.*
2. *if $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \phi$ and $U_R(X) = U$, then $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ is the discrete nano topology on U*

Remark 2.19. *Throughout this paper, nano nowhere-dense boundary and nano regular-closed is denoted by nano NDB-set and NRC respectively.*

3. Nano NDB and NAB-sets

Definition 3.1. *A subset A of a space $(U, \tau_R(X))$ is called a NAB-set if $A = U \cap V$, where U is nano-open and V is nano semi-regular. The collection of all NAB-sets in U will be denoted by $NAB(U)$.*

Definition 3.2. *Let $(U, \tau_R(X))$ be a nano topological space and let $A \subseteq U$, then A is called a nano NDB-set if A has nano nowhere-dense boundary.*

Theorem 3.3. *Every NB-set is a nano NDB-set.*

Proof. : It is trivial to see that the intersection of two nano NDB-sets is a nano NDB-set. Since a NB-set is the intersection of a nano (semi-) open and a Nt-set, it is enough to show that every nano semi-open and every Nt- set is a nano NDB-set. If A is nano semi-open, then for some nano-open U we have $U \subset A \subset Ncl(U)$. Since $NFr(A) = Ncl(A) \cap Ncl(U - A) = Ncl(A) \cap Ncl(U - A) \subset Ncl(U) \cap Ncl(U - A) = NFr(U)$, clearly $NFr(A)$ is nano nowhere-dense being a subset of the nano nowhere-dense set $NFr(U)$. In fact it is obvious that every nano-open set has nano nowhere-dense boundary. Thus every nano semi-open (and hence every Nt)-set is a nano NDB-set. □

Remark 3.4. *The converse is not true. For consider the space $U = \{a, b, c\}$ with the only non-trivial nano-open set $\{a\}$. The subset $\{a, b\}$ is a nano NDB-set but not a NB-set.*

In [4] Jeyalakshmi defined the notion of Nt-sets in nano topological spaces. The following result shows that the defined property coincides with the class of nano semi-closed sets.

Theorem 3.5. *For a subset A of a nano topological space U the following are equivalent:*

1. *A is a Nt-set.*
2. *A is nano semi-closed.*
3. *A is a nano semi-pre closed and NB-set.*
4. *A is a nano semi-pre closed and NDB-set*

Proof. : $(1) \Rightarrow (2)$ proof follows from Theorem 3.5[4].
$(2) \Rightarrow (3)$. Every nano semi-closed set is trivially nano semi-pre closed. Since $A = P \cap Q$, where Q is Nt-set and P is nano-open, then A is a NB-set.
$(3) \Rightarrow (4)$. Theorem 3.3.
$(4) \Rightarrow (1)$. Since A is a nano NDB-set, then $B = U - A$ is also a nano NDB-set.
It is easy to see that from the identity

$$\begin{aligned} Nint(NFr(B)) = {} & Nint(Ncl(B)) \cap Nint(Ncl(U - B)) \\ = {} & Nint(Ncl(B)) \cap (U - Ncl(Nint(B)) \\ = {} & Nint(Ncl(B)) - Ncl(Nint(B)) \end{aligned}$$

It follows that $Nint(Ncl(B)) \subset Ncl(Nint(B))$. Since B is nano semi-pre open, $B \subseteq Ncl(Nint(Ncl(B)))$. Thus $B \subset Ncl(Nint(B))$ or equivalently $Ncl(B) = Ncl(Nint(B))$. Since $B = U - A$, then $Nint(Ncl(A)) = Nint(A)$. Thus A is Nt-set. □

Theorem 3.6. *For a subset A of a nano topological space $(U, \tau_R(X))$, the following are equivalent:*

1. *A is a NA-set.*
2. *A is nano semi-open and nano-locally closed.*
3. *A is nano semi pre-open and nano-locally closed.*

Proof. : The equivalence of conditions $(1) \Rightarrow (2)$ proof follows from Theorem 3.11[4].
$(2) \Rightarrow (3)$ is trivial.
$(3) \Rightarrow (1)$ Since A is nano locally-closed, $A = G \cap Ncl(B)$, where G is a nano-open set. Since A is nano semi-pre open and since trivially $Ncl(Nint(A)) \subseteq Ncl(A)$, where $Ncl(A)$ is nano-regular closed. Thus A is the intersection of a nano-open and a nano regular- closed set, i.e., it is a NA-set. □

Theorem 3.7. *For a subset A of a nano topological space $(U, \tau_R(X))$, the following are equivalent:*

1. *A is nano-open.*
2. *A is nano pre-open and nano-locally closed.*

Proof. : $(1) \Rightarrow (2)$ and $(2) \Rightarrow (1)$ follows from the Theorem 3.11[4]. □

Theorem 3.8. *For a subset A of a nano topological space $(U, \tau_R(X))$, the following are equivalent:*

1. *A is nano-open.*
2. *A is nano pre-open and NB-set.*

3. *A is nano pre-open and NA-set.*

Proof. : (1) $\Rightarrow$ (2) and (3) $\Rightarrow$ (1) follows from the Theorem 3.6[4]
(2) $\Rightarrow$ (3) is obvious. (Since, Every NA-set is NB-set)[4] □

Since nano regular-closed set are nano semi-regular and since nano semi-regular set is Nt-set, then the following implications shows that the nano AB-set is properly lies between nano A-set and nano B-set.

$$NA\text{-set} \Rightarrow NAB\text{-set} \Rightarrow NB\text{-set}$$

None of them of course is reversible as the following examples shows:

Example 3.9. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a,b\},\{c\},\{d\}\}$ and $X = \{a,c\}$. Then $\tau_R(X) = \{U,\phi,\{c\},\{a,b\},\{a,b,c\}\}$. Let $A = \{c,d\}$. The subset $\{c,d\}$ is NAB-set but not NA-set.*

Example 3.10. *Let $U = \{a,b,c\}$ with $U/R = \{\{a\},\{b,c\}\}$ and $X = \{a\}$. Then $\tau_R(X) = \{U,\phi,\{a\}\}$. In $(U,\tau_R(X))$, the subset $A = \{c\}$. It is easily observed that A is a NB-set but not a NAB-set.*

Clearly every nano-open and every nano semi-regular set is a NAB-set. But the NAB-set is neither nano-open nor nano semi-regular.
The following examples shows that NAB-set but neither nano-open nor nano semi-regular.

Example 3.11. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$. Let $A = \{a,c\}$. The subset $\{a,c\}$ is NAB-set but not nano-open set.*

Example 3.12. *In Example 3.11, Let $A = \{a,b,d\}$. The subset $\{a,b,d\}$ is NAB-set but not nano semi-regular.*

Moreover, since the intersection of a nano-open set and a nano semi-regular set is always nano semi-open, then the following implication is clear:

$$NAB\text{-set} \Rightarrow \text{nano semi-open set}$$

However, if one considers the space from Example 3.10 above, it becomes clear that not all nano semi-open sets are NAB-sets. The set $\{a,b\}$ is nano semi-open but not a NAB-set.

Next the relation between NB-sets and NAB-sets is shown but first consider the following, probably known lemma.

Lemma 3.13. *The nano semi-closure of every $N\beta$-open set is nano semi-regular.*

Recall that in [1] If $(U,\tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then
i)The nano semi-closure of A is defined as the intersection of all nano semi-closed sets containing A and is denoted by $Nscl(A)$. $Nscl(A)$ is the smallest nano semi-closed set containing A and $Nscl(A) \subseteq A$.

From [21] we will define the following family:
In the nano topological space $(U,\tau_R(X))$
, $NA_5 = B(X) = \{M \cap N : M \in \tau_R(X), Nint(Ncl(A)) \subset N\}$

Theorem 3.14. *Let S be a subset of $(U,\tau_R(X))$, $S \in NA_5$ iff there exists a nano-open set B such that $S = B \cap Nscl(S)$*

Proof. : Let $S \in NA_5$. Then there exist a nano-open set B and a nano semi-closed set N such that $S = M \cap N$.
We have $S \subset M$, $S \subset N$, $S \subset Nscl(N)$, $S \subset M \cap Nsc(S) \subset B \cap Nscl(S) \subset M \cap N = S$.
Hence $S = M \cap Nscl(S)$.
The converse is obvious since $Nscl(S)$ is nano semi-closed. □

Theorem 3.15. *For a subset A of a space $(U,\tau_R(X))$, the following are equivalent:*

1. *A is a NAB-set.*
2. *A is nano semi-open and a NB-set.*
3. *A is nano semi pre-open and a NB-set*

Proof. : (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.
(3) $\Rightarrow$ (1) Since A is a NB-set, then from the notion of Theorem 3.14, there exists a nano-open sets U such that $A = U \cap Nscl(A)$, where $Nscl(A)$ denotes the nano semi-closure of A in U. By lemma 3.13, $Nscl(A)$ is nano semi-regular, Since by (3) A is nano semi pre-open. Thus A is a NAB-set. □

4. Characterization of some peculiar nano topological spaces

Theorem 4.1. *For a nano topological space $(U,\tau_R(X))$ the following are equivalent:*

1. *U is nano-submaximal.*
2. *Every subset of U is a NB-set.*
3. *Every nano dense subset of U is a NB-set.*

Proof. : (1) $\Rightarrow$ (2). Let $A \subset U$. Since every subspace of a nano-submaximal space is nano-submaximal, then $Ncl(A)$ is nano-submaximal. Since A is nano-dense in $Ncl(A)$, A is nano-open in $Ncl(A)$. Thus $A = P \cap Ncl(A)$, where P is nano-open in $(U,\tau_R(X))$ and $Ncl(A)$ is nano-closed in U. Thus A is nano-locally closed and hence a NB-set, since every nano-closed set is Nt-set.
(2) $\Rightarrow$ (3) is trivial.
(3) $\Rightarrow$ (1). Let $A \subset (U,\tau_R(X))$ be nano-dense. By (3) $A = P \cap B$, where P is nano-open and B is Nt-set. Since $A \subset B$, then B is nano-dense. Thus $Nint(B) = Nint(Ncl(B)) = Nint(U) = U$ and hence $B = U$. Thus $A = P$ is nano-open and so $(U,\tau_R(X))$ is nano-submaximal. □

Theorem 4.2. *If $(U, \tau_R(X))$ is a nano-submaximal space, then $NAB(U) = N\beta O(U)$.*

Proof. : Since U is nano-submaximal, then by above Theorem 4.1, Every ($N\beta$-open) subset of U is a NB-set. Thus by Theorem 3.15, every $N\beta$-open subset of U is a NAB-set. On the other hand, every NAB-set is $N\beta$-open. □

Theorem 4.3. *Let S be a subset of a nano topological space $(U, \tau_R(X))$. Then S is a NA-set if and only if S is nano semi-open and nano locally closed.*

Proof. : Let $S \in A(U, \tau_R(X))$, so $S = U \cap F$ where $U \in \tau_R(X)$ and $F \in NRC(U, \tau_R(X))$. Clearly S is nano locally -closed. Now $Nint(S) = U \cap Nint(F)$, so that $S = U \cap Ncl(Nint(F)) \subset Ncl(U \cap Nint(F)) = Ncl(Nint(S))$, and hence S is nano semi-open.

Conversely, let S be nano semi-open and nano locally-closed, so that $S \subset Ncl(Nint(S))$ and $S = U \cap Ncl(S)$ where U is nano-open. Then $Ncl(S) = Ncl(Nint(S))$ and so is nano regular-closed. Hence S is a NA-set. □

The class of nano locally-closed sets is also properly placed between the classes of NA and NB-sets but the concepts of NAB-sets and nano locally closed sets are independent from each other:

If first, every nano locally-closed set is a NAB-set, then it would be nano semi-open as well. But nano locally closed, nano semi-open sets are NA-sets from previous Theorem; however not all nano locally-closed sets are NA-sets. Second, if every NAB-set would be nano locally-closed, then again it must be a NA-set but as shown above not all NAB-sets are NA-sets.

Theorem 4.4. *For a subset A of a nano topological space $(U, \tau_R(X))$, the following are equivalent:*

1. *A is nano semi-regular.*
2. *A is nano semi-closed and a NAB-set.*
3. *A is nano semi pre-closed and a NAB-set.*

Proof. : (1) ⇒ (2) and (2) ⇒ (3) are obvious.
(3) ⇒ (1) Since A is nano semi pre-closed and a NB-set, then by Theorem 3.5, A is nano semi-closed . On the other hand A is nano semi-open, since it is a NAB-set. Thus A is nano semi-regular, being both nano semi-open and Nt-set. □

Recall that a subset A of a space $(U, \tau_R(X))$ is called nano interior-closed (= Nic-set)[20], if $Nint(A)$ is nano-closed in A.

Definition 4.5. *If $A \subseteq Nint(Ncl(A))$, then A is called nano locally-dense(= nano pre-open).*

Theorem 4.6. *For a subset A of a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *A is nano-open.*
2. *A is a NAB-set and A is either nano locally-dense or an Nic-set.*

Proof. : (1) ⇒ (2) is obvious.
(2) ⇒ (1) If A is nano locally-dense, then since A is also a NB-set, it follows from Theorem 3.15 in [21] that A is nano-open. If A is a Nic-set, then in the notion of Theorem 3.20 from [22], A is again nano-open, since A is also nano semi-open. □

Theorem 4.7. *A nano topological space $(U, \tau_R(X))$ is nano extremally disconnected iff $NSO(U, \tau_R(X)) \subset NPO(U, \tau_R(X))$.*

Proof. : Let $A \in NSO(U, \tau_R(X))$. Then there exists a U such that $U \subset A \subset Ncl(U)$. Since U is nano extremally disconnected, $Ncl(U) \in \tau_R(X))$ so that $U \subset A \subset Nint(Ncl(U))$. This shows that $A \in \tau^{\alpha}{}_R(X) \subset NPO(U, \tau_R(X))$. Conversely, let $A \in NRC(U, \tau_R(X))$. Then $A \in NSO(U, \tau_R(X))$ and by hypothesis, $A \in NPO(U, \tau_R(X))$. Therefore, $A \subset Nint(Ncl(A))$ and since A is nano-closed, $A \in \tau_R(X))$. This shows that $(U, \tau_R(X))$ is nano extremally disconnected . □

Theorem 4.8. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *$(U, \tau_R(X))$ is nano extremally disconnected.*
2. *$\tau_R(X) = NAB(U)$.*
3. *Every NAB-set is nano-open.*

Proof. : (1) ⇒ (2) Let $A \in NAB(U)$. Clearly A is nano semi-open. By Theorem 4.7, it follows that A is nano pre-open, since U is nano extremally disconnected. Moreover A is a NB-set and since it is nano pre-open, it follows from Theorem 3.15 in [21] that $A \in \tau_R(X)$. Hence $NAB(U) \subseteq \tau_R(X)$. On the other hand it is obvious that $\tau_R(X) \subseteq NAB(U)$.
(2) ⇒ (3) is obvious.
(3) ⇒ (1) Let $A \subseteq U$ be nano regular-closed. Thus A is a NAB-set. By (3) A is nano-open. So, U is nano extremally disconnected . □

Theorem 4.9. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is nano submaximal.*
2. *Every nano locally dense set is a NAB-set.*
3. *Every nano-dense set is a NAB-set*

Proof. : (1) ⇒ (2) Let $A \subseteq U$ be nano locally-dense (= nano pre-open). By (1), A is nano-open, since in nano submaximal spaces every nano locally-dense set is nano-open. Hence A is a NAB-set.
(2) ⇒ (3) every nano-dense set is nano locally-dense.
(3) ⇒ (1) Let $A \subseteq U$ be nano-dense. By (3), A is a NAB-set. Hence A is both nano pre-open and a NB-set. From Theorem

3.15 in [21] it follows that A is nano-open. Thus U is nano submaximal. □

Theorem 4.10. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is a nano locally indiscrete.*
2. *Every NB-subset is nano-clopen.*
3. *Every NBsubset is nano-closed.*

Proof. : (1) ⇒ (2). If A is a NB-set, $A = U \cap B$, where U is nano-open and B is Nt-set. By (1) U is nano-clopen. On the other hand $Ncl(B)$ is nano-open by (1) and thus $NintNcl(B) \subset B \subset Ncl(B)$ implies $B = Nint(B) = Ncl(B)$ and thus $B$is nano-clopen.
Thus A is nano-clopen being the intersection of two nano-clopen sets.
(2) ⇒ (3) is trivial.
(3) ⇒ (1). Every nano-open set is a NB-set from Theorem 3.11 [21], and thus by (3) nano-closed. □

Theorem 4.11. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is a nano locally indiscrete.*
2. *Every NAB-set is nano-clopen.*
3. *Every NAB-set is nano pre-closed.*

Proof. : (1) ⇒ (2) Let $A \subseteq U$ be a NAB-set. By (1) and from Previous Theorem, A is nano-clopen, since it is a NB-set.
(2) ⇒ (3) Every nano-clopen set is nano pre-closed.
(3) ⇒ (1) Let $A \subseteq U$ be nano-open. Then A is a NAB-set and by (3) it is nano pre-closed. Since every nano pre-closed (nano semi-)open set is (nano regular-)closed, then U is a nano locally-indiscrete. □

Theorem 4.12. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is indiscrete nano topology.*
2. *The only NB-sets in the U are the trivial ones.*
3. *The only NA-sets in the U are the trivial ones.*

Proof. : (1) ⇒ (2). If A is a NB-set, then $A = P \cap B$, where P is nano-open and B is Nt-set. If $A \neq \phi$, then $P \neq \phi$ and by (1) $P = U$. Thus $A = B$ and so $Nint(A) = Nint(Ncl(A)) = Nint(U) = U$. Hence $A = U$.
(2) ⇒ (3). Every NA-set is a NB-set.
(3) ⇒ (1). Since by Theorem 3.8, every nano-open set is a NA-set, by (3) the only nano-open sets in U are the trivial ones, i.e., U is indiscrete nano topology. □

Theorem 4.13. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is indiscrete nano topology.*
2. *$NAB(U) = \{\phi, U\}$.*

Proof. : The theorem follows from Previous Theorem, since the class of NAB-sets is (properly) placed between the classes of NA and NB-sets. □

Theorem 4.14. *A NA-set is nano semi-open.*

Proof. : Let $S = U \cap C$ be a NA-set, where U is nano-open and $C = Ncl(Nint(C))$. Since $S = U \cap C$, we have $Nint(S) \supset U \cap Nint(C)$. It is easily seen that $Nint(S) \subset S \subset C$, hence $Nint(S) = Nint(Nint(S)) \subset Nint(C)$. But $Nint(S) \subset S \subset U$, hence $Nint(S) \subset U \cap Nint(C)$. Therefore $Nint(S) = U \cap Nint(C)$. Now we Prove $S \subset Ncl(Nint(S))$. Let $x \in S$ and V be an arbitrary nano-open set containing x. Then $U \cap V$ is also a nano-open set containing x. Since $x \in C = Ncl(Nint(C))$, there is a point $z \in Nint(C)$ such that $z \neq x$ and $z \in U \cap V$. Hence $z \in U \cap Nint(C) = Nint(S)$. Therefore $x \in Ncl(Nint(S))$ and $S \subset Ncl(Nint(S))$. From $Nint(S) \subset S \subset Ncl(Nint(S))$ we know that S is nano semi-open. □

Theorem 4.15. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is discrete nano-topology.*
2. *Every subset of U is a NA-set.*

Proof. : (1) ⇒ (2). By (1) every set $A \subset U$ is nano-open and nano regular-closed. Hence A is a NA-set.
(2) ⇒ (1) By (2) every singleton $\{u\} \in U$ is a NA-set and by Previous Theorem, NA-set is nano semi-open. If $Nint\{u\} = \phi$, then we have the contradiction $\{u\} \subset Ncl(Nint\{u\} = \phi$. Thus $\{u\} = Nint\{u\}$ or equivalently every singleton in U is nano-open. □

Theorem 4.16. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *$A(U, \tau_R(X)) = \tau_R(X))$.*
2. *$A(U, \tau_R(X))$ is a nano topology on U.*
3. *The intersection of two NA-sets is a NA-set.*
4. *$NSOU, \tau_R(X))$ is a nano topology on U.*
5. *$(U, \tau_R(X))$ is nano extremally disconnected.*

Proof. : (1) ⇒ (2) and (2) ⇒ (3) are clear.
(3) ⇒ (4): Let $S_1, S_2 \in NSO(U, \tau_R(X))$. We wish to show $S_1 \cap S_2 \in NSO(U, \tau_R(X))$. Suppose there is a point $x \in S_1 \cap S_2$ such that $x \notin Ncl(Nint(S_1 \cap S_2))$. so there is a nano-open neighbourhood U of x such that $U \cap Nint(S_1) \cap Nint(S_2) = \phi$. Thus $U \cap Ncl(S_1) \cap Nint(S_2) = \phi$, and hence we have $U \cap Nint(Ncl(S_1) \cap Ncl(S_2) = \phi$. Therefore $U \cap Nint(Ncl(S_1) \cap Ncl(S_2) = \phi$, so that $x \notin Ncl(Nint(Ncl(S_1) \cap Ncl(S_2))$.
But, on the other hand we have $Ncl(S_1), Ncl(S_2) \in NRC(U, \tau_R(X))$ so that $Ncl(S_1), Ncl(S_2) \in NSO(U, \tau_R(X))$.

Then $x \in Ncl(S_1) \cap Ncl(S_2)$ implies $x \in Ncl(Nint(Ncl(S_1) \cap Ncl(S_2))$, which is a contradiction.Thus no such point x exists, and so $S_1 \cap S_2 \in NSO(U, \tau_R(X))$.
$(4) \Rightarrow (5)$ and $(5) \Rightarrow (1)$ is obvious in Theorem 3.12[4]. □

Theorem 4.17. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is discrete nano topology.*
2. *Every subset of U is a NAB-set.*
3. *Every singleton is a NAB-set.*

Proof. : $(1) \Rightarrow (2)$ and $(2) \Rightarrow (3)$ are obvious.
$(3) \Rightarrow (1)$ Let $u \in U$. By (3), $\{u\}$ is a NAB-set and hence nano semi-open. Then $\{u\}$ must contain a non-void nano-open subset. Since the only possibility is $\{u\}$ itself, then each singleton is nano-open or equivalently U is discrete nano topology. □

Theorem 4.18. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is nano-hyperconnected.*
2. *Every NAB-set is nano-dense.*

Proof. : $(1) \Rightarrow (2)$ Let $A \subseteq U$ be a NAB-set. Then A is nano semi-open and hence there exist a nano-open subset U such that $U \subseteq A \subseteq Ncl(A)$. By (1), U is nano-dense. Hence its superset A is also nano-dense.
$(2) \Rightarrow (1)$ Every nano-open subset of U is a NAB-set and hence by (2) nano-dense.

□

Definition 4.19. *A space U is called nano semi-connected if U cannot be expressed as the disjoint union of two non-void nano semi-open sets.*

Theorem 4.20. *For a nano topological space $(U, \tau_R(X))$ the following are equivalent:*

1. *U is nano semi-connected.*
2. *U is not the union of two disjoint non-void NAB-sets.*

Proof. : $(1) \Rightarrow (2)$ If U is the union of two disjoint non-void NAB-sets, then U is not nano semi-connected, since NAB-sets are nano semi-open.
$(2) \Rightarrow (1)$ If U is not nano semi-connected, then U has a non-trivial nano semi-open subset A with nano semi-open complement. Since both A and $B = U - A$ are nano semi-regular, then A and B are NAB-sets. So U is the union of two disjoint non-void AB-sets, contradictory to (2). □

5. Nano weak AB-sets

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Definition 5.1. *A subset H of a space $(U, \tau_R(X))$ is called a nano weak AB-set if $S = P \cap Q$, where P is nano-open set and Q is nano β-regular. The collection of all nano weak AB-sets in U will be denoted by nano weak $AB(U)$.*

Remark 5.2. *Every nano semi-regular set is nano semi pre-regular but not conversely.*

Example 5.3. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b\},\{c,d\}\}$ and $X = \{a,c\}$.*
Then $\tau_R(X) = \{U, \phi, \{a\}, \{a,c,d\}, \{c,d\}\}$. Let $A = \{c\}$. The subset $\{c\}$ is nano semi pre-regular in $(U, \tau_R(X))$. But it is not nano semi-regular, not even nano semi-open.

Remark 5.4. *For some subsets defined above, we have the following implications:*

nano semi-regular $\longrightarrow$ NAB-set $\longrightarrow$ NB-set
$\downarrow$ $\downarrow$ $\searrow$
nano semi pre-regular $\rightarrow$ nano weak AB-set $\longrightarrow$ NC-set

Example 5.5. *Let $U = \{a,b,c\}$ with $U/R = \{\{a\},\{b,c\}\}$ and $X = \{a\}$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$. In $(U, \tau_R(X))$, the subset $A = \{c\}$. It is easily observed that A is a NC-set and even a NB-set. But it is not a nano weak AB-set.*

Example 5.6. *In the Example 5.3, the subset $\{c\}$ is a nano weak AB-set but not NB-set. Therefore by Examples 5.3 and 5.5, NB-sets are independent from nano weak AB-sets.*

Remark 5.7. *Every nano semi-pre regular set is a nano weak AB-set but the converse is not true as shown by the following example.*

Example 5.8. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a,b,d\}, \{b,d\}\}$. then $A = \{b,d\}$ is nano weak AB-set but it is not a nano semi pre-regular*

Theorem 5.9. *In a nano topological space $(U, \tau_R(X)$, every nano weak AB-set is $N\beta$-open.*

Proof. : Let H be a nano weak AB-set.Then $H = G \cap B$, where G is nano-open and B is $N\beta$-regular. Hence B is nano β-open. so $H = G \cap B \subseteq G \cap Ncl(Nint(Ncl(B))) \subseteq Ncl(G \cap Nint(Ncl(B))) = Ncl(Nint(G) \cap Nint(Ncl(B))) = Ncl(Nint(G \cap Ncl(B))) \subseteq Ncl(Nint(Ncl(G \cap B))) = Ncl(Nint(Ncl(H)))$. Hence H is Nβ-open. □

Theorem 5.10. *For a subset S of a nano topological space $(U, \tau_R(X)$ the following are equivalent:*

1. *S is nano semi pre-regular.*
2. *S is nano t_α-set and a nano weak AB-set.*

Proof. : (1) ⇒ (2) Proof follows directly since every nano β-regular set is a nano t_α-set by definition and a nano weak AB-set by (2) of Remark 5.7
(2) ⇒ (1) Let H be a t_α-set and a nano weak AB-set. Since H is a nano weak AB-set, by Theorem 5.9, H is nano β-open. Thus H is a nano t_α-set as well as nano β-open. Hence H is nano β-regular. □

The concepts of being a nano t_α-set and being a nano weak AB-set are independent as shown by the following examples.

Example 5.11. *Let $U=\{a,b,c\}$ with $U/R=\{\{a\},\{b,c\}\}$ and $X=\{b,c\}$. Then $\tau_R(X)=\{U,\phi,\{b,c\}\}$.*
(1) In $(U,\tau_R(X))$, the subset $A=\{b,c\}$ is nano weak AB-set but not nano t_α-set .
(2) In $(U,\tau_R(X))$, the subset $A=\{a\}$ is a nano t_α-set but not nano nano weak AB-set.

Now we can improve our above result [Theorem 4.8]

Theorem 5.12. *If U is a nano sub-maximal space, then nano weak $AB(U)=NSPO(U)$.*

Proof. : Since U is nano sub-maximal, every nano semi pre-open set of U is a NAB-set [Theorem 3.15] and hence a nano weak AB-set. Conversely, since every nano weak AB-set is nano semi-pre open, this completes the proof. □

Proposition 5.13. *For a subset S is nano-open in a space $(U,\tau_R(X))$ if and only if it is a $N\alpha$-open and a Nc-set.*

Proof. : It is obvious that every nano-open set is a $N\alpha$-open and a NC-set. Let S be a $N\alpha$-open and a NC-set. Since S is a NC-set, there exist $U\in\tau_R(X))$ and $A\in N\alpha^*(U,\tau_R(X))$ such that $S=U\cap A$. Since S is a $N\alpha$-open, by using Lemma 4.3 in [20], we have ,

$$\begin{aligned} S\subset Nint(Ncl(Nint(S))) &= Nint(Ncl(Nint(U\cap A)))\\ &= Nint(Ncl(U))\cap\\ &\quad Nint(Ncl(Nint(A)))\\ &= Nint(Ncl(U))\cap Nint(A)) \end{aligned}$$

and hence $S=U\cap S\subset U\cap[Nint(Ncl(U))\cap Nint(A)]=U\cap Nint(A)\subset S$. Consequently, we obtain $S=U\cap Nint(A)$ and $S\in\tau_R(X)$. □

Theorem 5.14. *For a subset S of a nano topological space U the following are equivalent:*

1. *S is nano-open.*
2. *S is a nano weak AB-set and $N\alpha$-open set.*

Proof. : It is obvious that (1) ⇒ (2). Conversely, let S be a nano weak AB-set and $N\alpha$-open. Since every nano weak AB-set is a NC-set, it follows from previous Proposition, S is nano-open. □

Theorem 5.15. *For a nano topological space $(U,\tau_R(X)$ the following are equivalent:*

1. *U is nano extremally disconnected.*
2. *$\tau_R(X)$ = weak $NAB(U)$.*
3. *Every nano weak AB-set is nano-open.*

Proof. : The Proof is straight forward. □

6. Decomposition of NAB, nano strongly irresolute and nano weak AB-continuous functions

Definition 6.1. *A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is called NAB-continuous, if for each nano-open subset of V of Y, $f^{-1}(A)$ is a nano AB-set of U.*

Definition 6.2. *A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is called nano strongly irresolute-continuous, $f^{-1}(P)$ is a nano semi-regular in U for every subset P in V.*

The following theorems are consequences of results from the beginning of this paper, Theorem 6.3 and 6.4 gives the relations between NAB-continuous functions and other forms of 'generalized continuity'. Note that none of the implications in Theorem 6.3 is reversible. Theorem 6.5 gives a decomposition of NAB-continuity, while Theorem 6.6 follows from above Theorem 3.6. Theorem 6.7 gives a decomposition of continuity dual to NAB-continuity.

Theorem 6.3.
1. *Every NA-continuous function is NAB-continuous.*
2. *Every NAB-continuous function is NB-continuous.*
3. *Every NAB-continuous function is nano semi-continuous.*

Proof. of (1): Let A be a nano-open subset of V. Since f is nano A-continuous, $f^{-1}(A)$ is a nano A-set in U. That is $f^{-1}(A)$ is a nano AB-set in U. Since A is a nano-open subset of V and $f^{-1}(A)$ is a nano AB-set in U. Then f is NAB-continuous.
Proof of (2) and (3) are obvious. □

The converse of the above theorem need not be true which can be shown from the following examples.

Example 6.4. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a,b\},\{c\},\{d\}\}$ and $X=\{a,c\}$.*
Then $\tau_R(X)=\{U,\phi,\{c\},\{a,b,c\},\{a,b\}\}$. Let $V=\{a,b,c,d\}$ with $V/R'=\{\{a\},\{b\},\{c,d\}\}$ and $Y=\{c,d\}$. Then $\tau_{R'}(Y))=\{V,\phi,\{c,d\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the identity map. Then f is NAB-continuous but not NA-continuous.

Example 6.5. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b,c,d\}\}$ and $X=\{b,c\}$.*
Then $\tau_R(X)=\{U,\phi,\{b,c,d\}\}$. Let $V=\{a,b,c\}$ with $V/R'=$

$\{\{a\},\{b,c\}\}$ and $Y=\{a\}$. Then $\tau_{R'}(Y))=\{U,\phi,\{a\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=a, f(b)=f(c)=b$ and $f(d)=c$. Then f is NB-continuous but not NAB-continuous.

Example 6.6. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a,c\},\{b,d\}\}$ and $X=\{a\}$. Then $\tau_R(X)=\{U,\phi,\{a,c\}\}$. Let $V=\{a,b,c,d\}$ with $V/R=\{\{a,c\},\{b\},\{d\}\}$ and $Y=\{a,b,c\}$. Then $\tau_{R'}(Y))=\{U,\phi,\{a,b,c\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the identity map. Then f is nano semi-continuous but not NAB-continuous.*

Theorem 6.7. *Every nano strongly irresolute-continuous function is NAB-continuous.*

Proof. : Let P be a subset of V. Since f is nano strongly irresolute continuous, $f^{-1}(P)$ is a nano semi-regular in U. That is $f^{-1}(P)$ is a nano AB-set in U. Since P is a nano-open subset of V and $f^{-1}(P)$ is a nano AB-set in U. Then f is NAB-continuous. □

The converse of the above theorem need not be true which can be shown from the following example.

Example 6.8. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a,d\},\{b\},\{c\}\}$ and $X=\{a,c\}$. Then $\tau_R(X)=\{U,\phi,\{c\},\{a,c,d\},\{a,d\}\}$. Let $V=\{a,b,c,d\}$ with $V/R=\{\{a,c\},\{b\},\{d\}\}$ and $Y=\{a,b,c\}$. Then $\tau_{R'}(Y))=\{U,\phi,\{a,b,c\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=a, f(b)=d, f(c)=c$ and $f(d)=b$. Then f is NAB-continuous but not nano strongly irresolute continuous-function.*

Theorem 6.9. *For a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ the following conditions are equivalent:*

1. *f is NAB-continuous.*
2. *f is nano semi-continuous and NB-continuous.*
3. *f is $N\beta$-continuous and NB-continuous.*

Proof. : (1) ⇒ (2) Let B be a nano-open set in V. Since f is nano AB-continuous. $f^{-1}(B)$ is a nano AB-set in U. Hence $f^{-1}(B)$ is nano semi-open and nano B-set in U. Therefore f is both nano semi-continuous and NB-continuous.
(2) ⇒ (3) Given that f is nano semi-continuous and NB-continuous. To prove that f is $N\beta$-continuous. Since every nano semi-continuous is $N\beta$-continuous, hence f is $N\beta$-continuous.
Proof of (3) ⇒ (1) is obvious. □

Theorem 6.10. *For a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ the following conditions are equivalent:*

1. *f is NA-continuous*
2. *f is $N\beta$-continuous and NLC-continuous.*

Proof. : (1) ⇒ (2) Let B be a nano-A set in V. Since f is nano A-continuous. $f^{-1}(B)$ is a nano A-set in U. Hence $f^{-1}(B)$ is $N\beta$-open and nano LC-set in U. Therefore f is both $N\beta$-continuous and NLC-continuous.
Proof of (2) ⇒ (1) is obvious. □

Theorem 6.11. *For a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ the following conditions are equivalent:*

1. *f is nano-continuous*
2. *f is NAB-continuous and either nano pre-continuous or Nic-continuous*

Proof. : (1) ⇒ (2) Let B be a nano-open set in V. Since f is nano-continuous. $f^{-1}(B)$ is a nano-open set in U. Hence $f^{-1}(B)$ is nano AB-set and nano pre-open or Nic-set in U. Therefore f is both NAB-continuous and either precontinuous or Nic-continuous.
Proof of (2) ⇒ (1) is obvious. □

Definition 6.12. *A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is called weakly NAB-continuous , if for each nano-open subset of V of Y, $f^{-1}(A)$ is a nano weak AB-set of U.*

As a consequence of Theorem 5.15, we obtain the following decomposition of nano-continuity

Theorem 6.13. *For a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ the following conditions are equivalent:*

1. *f is nano-continuous*
2. *f is nano weak AB-continuous and $N\alpha$-continuous.*

Proof. : (1) ⇒ (2) Let B be a nano-open set in V. Since f is nano-continuous. $f^{-1}(B)$ is a nano-open set in U. Hence $f^{-1}(B)$ is nano weak AB-set and $N\alpha$-open in U. Therefore f is both weak NAB-continuous and $N\alpha$-continuous.
Proof of (2) ⇒ (1) is obvious. □

The following theorem is immediate consequences of Remark 5.4.

Theorem 6.14. *For a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ the following hold:*

1. *Every NAB-continuous function is nano weak AB-continuous.*
2. *Every nano weak AB-continuous function is NC-continuous.*
3. *Every nano weak AB-continuous function is $N\beta$-continuous.*

Proof. (1): Let A be a nano-open subset of V. Since f is nano AB-continuous, $f^{-1}(A)$ is a nano AB-set in U. That is $f^{-1}(A)$ is a weak nano AB-set in U. Since A is a nano-open subset of V and $f^{-1}(A)$ is a nano weak AB-set in U. Then f is weak NAB-continuous.
Proof of (2) and (3) are obvious. □

The converse of the above theorem need not be true which can be shown from the following examples.

Example 6.15. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a,d\},\{b\},\{c\}\}$ and $X=\{a,b\}$. Then $\tau_R(X)=\{U,\phi,\{b\},\{a,d\},\{a,b,d\}\}$. Let $V=\{a,b,c,d\}$ with $V/R=\{\{a,c\},\{b\},\{d\}\}$ and $Y=\{a,b,c\}$. Then $\tau_{R'}(Y))=\{U,\phi,\{a,b,c\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=a, f(b)=d, f(c)=c$ and $f(d)=b$. Then f is NAB-continuous but not NA-continuous.*

Example 6.16. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b\},\{c,d\}\}$ and $X = \{c,d\}$. Then $\tau_R(X) = \{U,\phi,\{c,d\}$. Let $V = \{a,b,c\}$ with $V/R = \{\{a\},\{b,c\}\}$ and $Y = \{a\}$. Define $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ by $f(a) = a, f(b) = f(c) = b$ and $f(d) = c$. Then f is NC-continuous but not nano weak AB-continuous.*

Example 6.17. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,d\},\{c\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}$. Let $V = \{a,b,c,d\}$ with $V/R = \{\{a,b\},\{c\},\{d\}\}$ and $Y = \{a,b\}$. Then $\tau_{R'}(Y)) = \{U,\phi,\{a,b\}\}$ Define $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be the identity map. Then f is $N\beta$-continuous but not nano weak AB-continuous.*

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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