New equalities connected with intuitionistic fuzzy matrices

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Abstract
This paper introduces a new implication of intuitionistic fuzzy matrix and studies its algebraic properties. Also investigates the equalities of intuitionistic fuzzy matrix in the case where this new implication is combined with the well-known operations for intuitionistic fuzzy matrices.

Keywords
Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Matrix (IFM), Intuitionistic Fuzzy Implication (IFI).

AMS Subject Classification
94D05, 15B15, 15B99.

1. Introduction
In [1], he was introduced the concept of IFS as a generalization of fuzzy sets introduced by Zadeh [19]. From the look, IFS is studied by many researchers and applies in many areas such as decision-making, clustering Analysis etc., Using the theory of fuzzy set, FMs as a generalization of matrices over the two elements Boolean algebra was studied by Kim and Roush [7]. Using the IFS theory, Im et al. [5] defined the idea of IFM as a generalization of the fuzzy matrix (FM).

In [12], they have established the set of all IFMs form a semiring with respect to the min-max composition of the IFM. Shyamal and Pal [13,14], they have introduced two binary operations of FMs and established some of these properties and operations has been extended to IFMs and studied its algebraic properties by Sriram and Boobalan [3]. In [4], they are defined some types of IFMs, the min-max and max-min composition of IFMs. Introduced a new composition operation and studied the algebraic properties also got a decomposition of an IFM by them [8]. In [9,10], they are studied representation and decomposition of an IFM using some (\(\alpha, \alpha'\)) cuts and reduction of an IFM to FM with some algebraic properties. Atanasov [2], he defined five new intuitionistic fuzzy operations on IFSs and taking multiplication and studied their algebraic properties. Verma and Sharma [15,16,17] developed some new results on IFSs and some new equalities connected with IFSs. In [18], we defined multiplicative operations of IFMs and investigated their algebraic properties.

The paper is organized as follows:
In Section 2 we shall briefly review IFMs and their operations.

In Section 3 Atanassov [1,2] defined a standard intuitionistic fuzzy implication for IFSs. We extend this implication to IFM and proved its algebraic properties. In Section 4 we formulate and prove some new elementary, but interesting equalities involving implication with other set operations on IFMs.

2. Preliminaries
This section briefly review IFMs and their operations.

Definition 2.1. A fuzzy matrix is a matrix \(A = (a_{ij})\), where \(a_{ij} \in [0, 1]\).

Definition 2.2. ([6]): An IFM is a matrix of pairs
A = \left( a_{ij}, a'_{ij} \right) \) of a non negative real numbers satisfying 
\( 0 \leq a_{ij} + a'_{ij} \leq 1 \) for all \( i, j \).

**Definition 2.3.** ([4]): For the IFMs \( A \) and \( B \) of same size, we have

(i) The max-min composition of \( A \) and \( B \) is defined by

\[ A \lor B = \left( \max (a_{ij}, b_{ij}), \min (a'_{ij}, b'_{ij}) \right) \]  

(ii) The min-max composition of \( A \) and \( B \) is defined by

\[ A \land B = \left( \min (a_{ij}, b_{ij}), \max (a'_{ij}, b'_{ij}) \right) \]  

(iii) The complement of a IFM \( A \) is \( A^c = \left( a'_{ij}, a_{ij} \right) \).

**Definition 2.4.** ([4]): For the IFMs \( A \) and \( B \) of same size, \( A \geq B \) if and only if \( a_{ij} \geq b_{ij} \) and \( a'_{ij} \leq b'_{ij} \) for all \( i, j \).

**Definition 2.5.** ([4]): Let \( O \) be an \( m \times n \) zero IFM that have all entries are \((0,1)\). Let \( J \) be an \( m \times n \) universal IFM \( J \) is an IFM that have all entries are \((1,0)\).

**Definition 2.6.** ([18]): For the IFMs \( A \) and \( B \) of same size, then we have

(i) \( A \rightarrow X_1 B = \left( \max (a_{ij}, b_{ij}), a'_{ij}, b'_{ij} \right) \).

(ii) \( A \rightarrow X_2 B = \left( a_{ij} b_{ij}, \max (a'_{ij}, b'_{ij}) \right) \).

Where \( \rightarrow \) is the ordinary multiplication.

### 3. Main Results

Atanassov [1,2] defined a standard intuitionistic fuzzy implication for IFs. Verma and Sharma[15,16,17] proved the results associated with this implication for IFs. We extend this implication of IFs to IFMs and proved its algebraic properties.

**Definition 3.1.** For the IFMs \( A \) and \( B \) of same size, then we define \( A \rightarrow _1 B = \left( \max (a'_{ij}, b_{ij}), \min (a_{ij}, b'_{ij}) \right) \).

This implication is constructed in such way that they produces an IFM

\[ 0 \leq \max (a'_{ij}, b_{ij}) + \min (a_{ij}, b'_{ij}) \leq \max (a_{ij}, b_{ij}) + \min (1 - a'_{ij}, 1 - b_{ij}) \leq \max (a'_{ij}, b_{ij}) + 1 - \max (a_{ij}, b_{ij}) \leq 1. \]

**Proposition 3.2.** For the zero IFM \( O \) and universal IFM \( J \) of same size, we have

(i) \( O \rightarrow _1 O = J \).

(ii) \( O \rightarrow _1 J = J \).

(iii) \( J \rightarrow _1 O = O \).

(iv) \( J \rightarrow _1 J = J \).

Proof. (i) \( O \rightarrow _1 O \)

\[ = \left( \max (1,0), \min (0,1) \right) \]

\[ = \left( 1,0 \right) \]

\[ = J. \]

Hence, \( O \rightarrow _1 O = J. \)

The proof (ii), (iii) and (iv) are similar to that of (i).

**Proposition 3.3.** For the IFMs \( A \) and \( B \) of same size, we have

(i) \( A \rightarrow _1 A^C = A^C \).

(ii) \( A \rightarrow _1 B = B^C \rightarrow _1 A^C \).

Proof. (i) \( A \rightarrow _1 A^C \)

\[ = \left( \max (a'_{ij}, a_{ij}), \min (a_{ij}, a'_{ij}) \right) \]

\[ = \left( a'_{ij}, a_{ij} \right) \]

\[ = A^C. \]

Hence, \( A \rightarrow _1 A^C = A^C. \)

(ii) \( A \rightarrow _1 B \)

\[ = \left( \max (a'_{ij}, b_{ij}), \min (a_{ij}, b'_{ij}) \right) \]

\[ = B^C \rightarrow _1 A^C. \]

Hence, \( A \rightarrow _1 B = B^C \rightarrow _1 A^C. \)

**Proposition 3.4.** For any IFM \( A \), we have

(i) \( A \rightarrow _1 J = J \).

(ii) \( J \rightarrow _1 A = A \).

(iii) \( O \rightarrow _1 A = A \).

(iv) \( A \rightarrow _1 O = A^C \).

Proof. Let \( A = \left( a_{ij}, a'_{ij} \right) \) be an IFM. Then

(i) \( A \rightarrow _1 J \)

\[ = \left( \max (a'_{ij}, 1), \min (a_{ij}, 0) \right) \]

\[ = (1,0) \]

\[ = J. \]

Hence, \( A \rightarrow _1 J = J. \)

The proof (ii),(iii) and (iv) are similar to that of (i).

### 4. New equalities connected with intuitionistic fuzzy matrices

In this section, we formulate and prove some new elementary, but interesting equalities involving implication with other set operations on IFMs.

**Proposition 4.1.** For the IFMs \( A \) and \( B \) of same size, we have

(i) \( (A^C \rightarrow _1 B)X_1 (A \rightarrow _1 B^C)^C = AX_1 B. \)

(ii) \( (A^C \rightarrow _1 B)X_2 (A \rightarrow _1 B^C)^C = AX_2 B. \)

(iii) \( (A \rightarrow _1 B^C)X_1 (B \rightarrow _1 A) = AX_1 B^C. \)

(iv) \( (A \rightarrow _1 B)^C X_2 (B \rightarrow _1 A) = AX_2 B^C. \)

(v) \( (AX_1 B)^C \rightarrow _1 (AX_2 B) = AX_3 B. \)

(vi) \( (AX_2 B)^C \rightarrow _1 (AX_1 B) = AX_4 B. \)

(vii) \( (AX_1 B)^C \rightarrow _1 (A \oplus B) = AX_5 B. \)

(viii) \( (AX_2 B)^C \rightarrow _1 (A \oplus B) = A \oplus B. \)

(ix) \( (A \oplus B)^C \rightarrow _1 (AX_1 B) = AX_6 B. \)

(x) \( (A \oplus B)^C \rightarrow _1 (AX_2 B) = A \oplus B. \)
Proof. (i) \((A^C \rightarrow B)X_1(A \rightarrow B^C)^C\)
\[
= (\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \rangle) X_1
\]
\[
= (\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \rangle)
\]
\[
= (\langle \max(a'_{ij}, b'_{ij}), \min(a_{ij}, b_{ij}) \rangle, \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}))
\]
\[
= (\langle \max(a_{ij}, b_{ij}), (a'_{ij}, b'_{ij}) \rangle)
\]
\[
= AX_1B.
\]
Hence, \((A^C \rightarrow B)X_1(A \rightarrow B^C)^C = AX_1B\).
The proof (ii),(iii),(iv),(v),(vi),(vii),(viii),(ix) and (x) are similar to that of (i).

\[\Box\]

**Proposition 4.2.** For the IFMs A and B of same size, we have

(i) \(((AX_1B) \rightarrow (AX_2B)) (A \rightarrow B^C)^C = AX_2B\).

(ii) \(((AX_2B) \rightarrow (AX_1B)) (A \rightarrow B^C)^C = AX_2B\).

(iii) \(((AX_1B) \rightarrow (A(B \rightarrow B))^C = A@B\).

(iv) \(((A@B) \rightarrow (AX_1B)) (A \rightarrow B^C)^C = AX_2B\).

(v) \(((AX_1B) \rightarrow (A \rightarrow B^C)^C = AX_1B\).

Proof. (i) \(((AX_1B) \rightarrow (AX_2B)) (A \rightarrow B^C)^C\)
\[
= (\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \rangle) X_1
\]
\[
= (\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \rangle)
\]
\[
= (\langle \max(a'_{ij}, b'_{ij}), \min(a_{ij}, b_{ij}) \rangle, \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}))
\]
\[
= (\langle \max(a_{ij}, b_{ij}), (a'_{ij}, b'_{ij}) \rangle)
\]
\[
= AX_1B.
\]
Hence, \(((AX_1B) \rightarrow (AX_2B)) (A \rightarrow B^C)^C = AX_1B\).
The proof (ii),(iii),(iv),(v),(vi),(vii),(viii),(ix) and (x) are similar to that of (i).

\[\Box\]

**Proposition 4.3.** For the IFMs A and B of same size, we have

(i) \(((A^C \rightarrow B)X_1(A \rightarrow B^C)^C) \vee ((AX_1B) \rightarrow (AX_2B))^C = AX_2B\).

(ii) \(((A^C \rightarrow B)X_1(A \rightarrow B^C)^C) \vee ((AX_1B) \rightarrow (AX_2B))^C = AX_2B\).

(iii) \(((A^C \rightarrow B)X_1(B \rightarrow B^C)^C) \vee ((AX_1B) \rightarrow (AX_2B))^C = AX_2B\).

(iv) \(((A^C \rightarrow B)X_1(B \rightarrow B^C)^C) \vee ((AX_1B) \rightarrow (AX_2B))^C = AX_2B\).

(v) \(((AX_1B) \rightarrow (AX_2B))^C \vee ((AX_2B) \rightarrow (A \rightarrow B^C)^C = AX_2B\).

Proof. (i) \(((A^C \rightarrow B)X_1(A \rightarrow B^C)^C) \vee ((AX_1B) \rightarrow (AX_2B))^C\)
\[
= (\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \rangle) X_1
\]
\[
= (\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \rangle)
\]
\[
= (\langle \max(a'_{ij}, b'_{ij}), \min(a_{ij}, b_{ij}) \rangle, \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}))
\]
\[
= (\langle \max(a_{ij}, b_{ij}), (a'_{ij}, b'_{ij}) \rangle)
\]
\[
= AX_2B.
\]
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$$AX_2B.$$ Hence, $$(((AX_1B)→_1(AX_2B)^C)^C)ν(((AX_2B)→_1(AX_1B)^C)^C) = AX_2B.$$ The proof (ii),(iii),(iv),(v) and (vi) are similar that of (i). □

5. Conclusion

This paper, we have defined an intuitionistic fuzzy implication for intuitionistic fuzzy matrices and proved its algebraic properties. Also elementary equalities involving this implication with well known operations of IFMs are investigated.

References
