Malaya Journal of Matematik, Vol. S, No. 1, 389-393, 2019

https://doi.org/10.26637/MJM0S01/0069

Multiplicative operations of Pythagorean fuzzy matrices

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Abstract

In this paper, we define two new multiplicative operations $\otimes_{1P}$ and $\otimes_{2P}$ on Pythagorean fuzzy matrix and investigate their algebraic properties. The properties of PFMs are also obtained in the case where these new operations are combined with the well known operations of Pythagorean fuzzy matrices.

Keywords

Intuitionistic Fuzzy Matrix(IFM), Pythagorean Fuzzy Set(PFS), Pythagorean Fuzzy Matrix(PFM).

AMS Subject Classification

94D05, 15B15, 15B99.

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Article History: Received **24** November **2018**; Accepted **09** April **2019**

Contents

1. Introduction

The concept of intuitionistic fuzzy matrix(IFM) was introduced by Pal[3] and simultaneously by Im[2] to generalize the concept of Thomason's[11] fuzzy matrix. Each element in an IFM is expressed by an ordered pair $\left\langle a_{ij}, a'_{ij}\right\rangle$ with $a_{ij}, a'_{ij} \in [0,1]$. The sum $a_{ij} + a'_{ij}$ of each ordered pair is less than or equal to 1. Since the appearance of IFM in 2001, several researchers [7,8,10] have importantly contributed for the development of IFM theory and its applications. In particular, Muthuraji et.al[6] introduced a new composition operator and studied the algebraic properties. Also they obtained a decomposition of an IFM. Pal[4] defined some basic operations and relations of IFMs including max-min, min-max, complement, algebraic sum, algebraic product etc. and proved equality between IFMs. Mondal and Pal[5] studied the similarity relations, together with invertibility conditions and eigenvalues of intuitionistic fuzzy matrices(IFMs). Emam and Fndh[1] defined some kinds of IFMs, the max-min and min-max composition of IFMs. Also they derived several importent results by these compositions and construct an idempotent intuitionistic fuzzy matrix from any given one through the min-max composition. Venkatesan and Sriram[12,13] defined Multiplicative operations of IFMs namely X_1, X_2, X_3 and X_4 and investigated their algebraic properties.

Yager[14,15] introduced Pythagorean fuzzy set(PFS) characterized by a membership degree and a nonmembership degree satisfying the condition that the square sum of its membership degree and nonmembership degree is equal to or less than 1, has much stronger ability than IFS to model such uncertain information in MCDM problems. Also the MCDM problem can be represented as PFM. Zhang and Xu[16] defined some novel operational laws of PFS and discuss its desirable properties.

The motivation of introducing PFSs is that in the real-life decision process, the sum of the support degree and the against degree to which an alternative satisfying a criterion provided by the decision maker may be bigger than 1 but their square sum is equal to or less than 1. Silambarasan and Sriram[9] introduced Pythagorean fuzzy matrix(PFM) and the operations algebraic sum and algebraic product of Pythagorean Fuzzy Matrices. Also they investigated its algebraic properties.

The remainder of this paper is organized as follows. In

Section 2, some basic definitions of IFM and its multiplicative operations are briefly reviewed. In Section 3, by using PFS theory, we define two new multiplicative operations $\otimes_{1P}$ and $\otimes_{2P}$ on PFM, and investigate their algebraic properties. In Section 4, the operator complement obey the De Morgan's laws for the operations $\otimes_{1P}$ and $\otimes_{2P}$. In Section 5, the absorption and distributive properties in the case where the operations $\otimes_{1P}, \otimes_{2P}$, max-min and min-max compositions are combined each other.

2. Preliminaries

Several operators such as max-min, min-max and multiplicative operations are defined using t-norm and t-conorm. In this section, we give some basic definitions of PFS and IFM that are necessary for this paper.

Definition 2.1. ([15])*: Let a set X be a universe of discourse. A PFS P is an object having the form,*
$P = (\langle x, P(\mu_P(x), \nu_P(x))/(x \in X)\rangle)$.
Where the function $\mu_P : X \to [0,1]$ *defines the degree of membership and* $\nu_P : X \to [0,1]$ *defines the degree of nonmembership of the element* $x \in X$ *to P,respectively, and for every* $x \in X$*, it holds that* $(\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$.

Definition 2.2. ([3])*: An Intuitionistic fuzzy matrix(IFM) is a matrix of pairs* $A = \left(\left\langle a_{ij}, a'_{ij}\right\rangle\right)$ *of non negative real numbers satisfying* $0 \leq a_{ij} + a'_{ij} \leq 1$ *for all i, j.*

Definition 2.3. ([6])*: For any two IFMs A and B of same size, we have*
(i) *The max-min composition of A and B is defined by*
$A \vee B = \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})\right\rangle\right)$.
(ii)*The min-max composition of A and B is defined by*
$A \wedge B = \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})\right\rangle\right)$.
(iii) *The complement of a IFM A is* $A^C = \left(\left\langle a'_{ij}, a_{ij}\right\rangle\right)$.

Definition 2.4. ([1])*: For any two IFMs A and B of same size,* $A \geq B$ *iff* $a_{ij} \geq b_{ij}$ *and* $a'_{ij} \leq b'_{ij}$ *for all i, j.*

Definition 2.5. ([1])*: The* $m \times n$ *zero IFM O is an IFM all of whose entries are* $\langle 0, 1\rangle$.
The $m \times n$ *universal IFM J is an IFM all of whose entries are* $\langle 1, 0\rangle$.

Definition 2.6. ([12])*: For any two IFMs A and B of same size, then we define*
$(i) AX_1B = \left(\left\langle \max(a_{ij}, b_{ij}), a'_{ij}b'_{ij}\right\rangle\right)$.
$(ii) AX_2B = \left(\left\langle a_{ij}b_{ij}, \max(a'_{ij}, b'_{ij})\right\rangle\right)$.
Where '.' is the ordinary multiplication.

3. Main Results

In this section, Pythagorean fuzzy matrix(PFM) is a new extension of IFM that arise in the condition $a_{ij} + a'_{ij} > 1$ but satisfying the condition $a_{ij}^2 + a'^2_{ij} \leq 1$. We define two new multiplicative operations $\otimes_{1P}$ and $\otimes_{2P}$ on PFM and investigate their algebraic properties.

Definition 3.1. ([9]) *: An Pythagorean fuzzy matrix(PFM) is a matrix of pairs* $A = \left(\left\langle a_{ij}, a'_{ij}\right\rangle\right)$ *of non negative real numbers* $a_{ij}, a'_{ij} \in [0,1]$ *satisfying the condition* $0 \leq a_{ij}^2 + a'^2_{ij} \leq 1$ *for all i, j.*

Remark 3.2. *The main difference between IFM and PFM is their different constraint conditions. According to their definitions, we know that the constraint condition of IFM is* $a_{ij} + a'_{ij} \leq 1$*, where as the constraint condition of PFM is* $a_{ij}^2 + a'^2_{ij} \leq 1$*. Because the fact that for any* $a, b \in [0,1]$ *if* $a + b \leq 1$ *then* $a^2 + b^2 \leq 1$*, if one is an IFM, then it must also be a PFM, but not all PFM are the IFMs. This is illustrated by the following example.*

Example 3.3. $A = \begin{bmatrix} \langle 0.8, 0.5\rangle & \langle 0.2, 0.3\rangle \\ \langle 0.1, 0.4\rangle & \langle 0.4, 0.3\rangle \end{bmatrix}$, *it is not an IFM, but it is a PFM.*

Similar to the operations of IFMs[13] in the following, we introducing the multiplicative operations of PFMs.

Definition 3.4. *For any two PFMs A and B of same size, then we define*
$(i) A \otimes_{1P} B = \left(\left\langle \sqrt{\max(a_{ij}^2, b_{ij}^2)}, a'_{ij}b'_{ij}\right\rangle\right)$.
$(ii) A \otimes_{2P} B = \left(\left\langle a_{ij}b_{ij}, \sqrt{\max(a'^2_{ij}, b'^2_{ij})}\right\rangle\right)$.
Where '.' is the ordinary multiplication.

Definition 3.5. *: For any two PFMs A and B of same size, we have the basic operations are*
(i) *The max-min composition of A and B is defined by*
$A \vee B = \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})\right\rangle\right)$.
(ii) *The min-max composition of A and B is defined by*
$A \wedge B = \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})\right\rangle\right)$.
(iii) *The complement of a PFM A is* $A^C = \left(\left\langle a'_{ij}, a_{ij}\right\rangle\right)$.

Definition 3.6. *For any two PFMs A and B of same size,* $A \geq B$ *if and only if* $a_{ij} \geq b_{ij}$ *and* $a'_{ij} \leq b'_{ij}$ *for all i, j.*

Definition 3.7. *The* $m \times n$ *zero PFM O is an PFM all of whose entries are* $\langle 0, 1\rangle$.
The $m \times n$ *universal PFM J is an PFM all of whose entries are* $\langle 1, 0\rangle$.

Proposition 3.8. *If A and B are any two PFMs of same size, we have*
$A \otimes_{2P} B \leq A \otimes_{1P} B$.

Proof. Since $a_{ij}b_{ij} \leq \sqrt{\min(a_{ij}^2, b_{ij}^2)} \leq \sqrt{\max(a_{ij}^2, b_{ij}^2)}$ and $\sqrt{\max(a'^2_{ij}, b'^2_{ij})} \geq a'_{ij}b'_{ij}$.
Hence, $A \otimes_{2P} B \leq A \otimes_{1P} B$. □

The following properties are obvious.

Proposition 3.9. *Let A, B and C be any three PFMs of same size, we have*
$(i) A \otimes_{1P} B = B \otimes_{1P} A$.
$(ii) A \otimes_{2P} B = B \otimes_{2P} A$.

Proposition 3.10. *For any three PFMs A, B and C of same size, we have*
(i)*Nullity:* $A \otimes_{1P} J = J, A \otimes_{2P} O = O$.
(ii)*Identity:* $A \otimes_{1P} O = A, A \otimes_{2P} J = A$.
(iii)*Distributivity:* $A \otimes_{1P} (B \otimes_{2P} C) \neq (A \otimes_{1P} B) \otimes_{2P} (A \otimes_{1P} C)$ *and* $A \otimes_{2P} (B \otimes_{1P} C) \neq (A \otimes_{2P} B) \otimes_{1P} (A \otimes_{2P} C)$.
(iv)*Absorption:* $A \otimes_{1P} (A \otimes_{2P} B) \neq A$ *and* $A \otimes_{2P} (A \otimes_{1P} B) \neq A$.

Proof. The proof of (i) and (ii) are straightforward.
(iii) $A \otimes_{1P} (BX_2C)$
$= \left(\left\langle \sqrt{\max(a_{ij}^2, b_{ij}^2 c_{ij}^2)}, a'_{ij}\sqrt{\max(b_{ij}'^2, c_{ij}'^2)} \right\rangle\right)$.
$(A \otimes_{1P} B) \otimes_{2P} (A \otimes_{1P} C) =$
$(\sqrt{\max(a_{ij}^2, b_{ij}^2)}\sqrt{\max(a_{ij}^2, c_{ij}^2)}, \sqrt{\max(a_{ij}'^2 b_{ij}'^2, a_{ij}'^2 c_{ij}'^2)})$
$= (\sqrt{\max(a_{ij}^2, b_{ij}^2)}\sqrt{\max(a_{ij}^2, c_{ij}^2)}, a'_{ij}\sqrt{\max(b_{ij}'^2, c_{ij}'^2)})$
Since $\sqrt{\max(a_{ij}^2, b_{ij}^2)}\sqrt{\max(a_{ij}^2, c_{ij}^2)} \leq \sqrt{\max(a_{ij}^2, b_{ij}^2 c_{ij}^2)}$, for all i, j.
Hence, $A \otimes_{1P} (B \otimes_{2P} C) \neq (A \otimes_{1P} B) \otimes_{2P} (A \otimes_{1P} C)$.
Similarly we can prove the other one.
(iv)$A \otimes_{1P} (A \otimes_{2P} B)$

$$
\begin{aligned}
&= \left(\left\langle \sqrt{\max(a_{ij}^2, a_{ij}^2 b_{ij}^2)}, a'_{ij}\sqrt{\max(a_{ij}'^2, b_{ij}'^2)} \right\rangle\right) \\
&= \left(\left\langle a_{ij}^2, a'_{ij}\sqrt{\max(a_{ij}'^2, b_{ij}'^2)} \right\rangle\right) \\
&\neq A.
\end{aligned}
$$

Hence, $A \otimes_{1P} (A \otimes_{2P} B) \neq A$.
Similarly we can prove the other one. □

4. Results on complement of PFM

In this section, the operation complements obey the de morgan's laws for $\otimes_{1P}$ and $\otimes_{2P}$ operations. This is established in the following properties.

Proposition 4.1. *For the PFMs A and B of same size, we have*
$(i)(A \otimes_{1P} B)^C = A^C \otimes_{2P} B^C$.
$(ii)(A \otimes_{2P} B)^C = A^C \otimes_{1P} B^C$.
$(iii)(A \otimes_{1P} B)^C \leq A^C \otimes_{1P} B^C$.
$(iv)(A \otimes_{2P} B)^C \geq A^C \otimes_{2P} B^C$.

Proof. $(i)(A \otimes_{1P} B)^C$

$$
\begin{aligned}
&= \left(\left\langle a'_{ij} b'_{ij}, \sqrt{\max(a_{ij}^2, b_{ij}^2)} \right\rangle\right) \\
&= A^C \otimes_{2P} B^C.
\end{aligned}
$$

$(ii)(A \otimes_{2P} B)^C$

$$
\begin{aligned}
&= \left(\left\langle \sqrt{\max(a_{ij}'^2, b_{ij}'^2)}, a_{ij} b_{ij} \right\rangle\right) \\
&= A^C \otimes_{1P} B^C.
\end{aligned}
$$

(iii)Since $a'_{ij} b'_{ij} \leq \sqrt{\max(a_{ij}'^2, b_{ij}'^2)}$ and $\sqrt{\max(a_{ij}^2, b_{ij}^2)} \geq a_{ij} b_{ij}$, for all i, j.
Hence, $(A \otimes_{1P} B)^C \leq A^C \otimes_{1P} B^C$.
The proof (iv) is similar to that of (iii). □

Proposition 4.2. *For any PFM A, we have*
$(i)(A \otimes_{1P} A^C)^C = A \otimes_{2P} A^C$.
$(ii)(A \otimes_{2P} A^C)^C = A \otimes_{1P} A^C$.

Proof. $A \otimes_{1P} A^C = \left(\left\langle \sqrt{\max(a_{ij}^2, a_{ij}'^2)}, a'_{ij} a_{ij} \right\rangle\right)$ and $A \otimes_{2P} A^c = \left(\left\langle a_{ij} a'_{ij}, \sqrt{\max(a_{ij}'^2, a_{ij}^2)} \right\rangle\right)$.
$(i)(A \otimes_{1P} A^C)^C$

$$
\begin{aligned}
&= \left(\left\langle a'_{ij} a_{ij}, \sqrt{\max(a_{ij}^2, a_{ij}'^2)} \right\rangle\right) \\
&= \left(\left\langle a_{ij} a'_{ij}, \sqrt{\max(a_{ij}'^2, a_{ij}^2)} \right\rangle\right) \\
&= A \otimes_{2P} A^C.
\end{aligned}
$$

Hence, $(A \otimes_{1P} A^C)^C = A \otimes_{2P} A^C$.
$(ii)(A \otimes_{2P} A^C)^C$

$$
\begin{aligned}
&= \left(\left\langle \sqrt{\max(a_{ij}'^2, a_{ij}^2)}, a_{ij} a'_{ij} \right\rangle\right) \\
&= \left(\left\langle \sqrt{\max(a_{ij}^2, a_{ij}'^2)} \right\rangle\right) \\
&= A \otimes_{1P} A^C.
\end{aligned}
$$

Hence, $(A \otimes_{2P} A^C)^C = A \otimes_{1P} A^C$. □

5. The results for $\otimes_{1P}$ and $\otimes_{2P}$ are combined with the min-max and max-min PFMs

We will discuss absorption property in the case where the operations $\wedge$, $\vee$, $\otimes_{1P}$ and $\otimes_{2P}$ are combined each other.

Proposition 5.1. *Let A and B be any two PFMs of same size, we have*
$(i) A \vee (A \otimes_{1P} B) \neq A \otimes_{1P} B$.
$(ii) A \wedge (A \otimes_{1P} B) = A$.

Proof. (i)$A \vee (A \otimes_{1P} B)$

$$
\begin{aligned}
&= \left(\left\langle \max(a_{ij}, \sqrt{\max(a_{ij}^2, b_{ij}^2)}, \min(a'_{ij}, a'_{ij} b'_{ij} \right\rangle\right) \\
&= \left(\left\langle \max(a_{ij}, \sqrt{\max(a_{ij}^2, b_{ij}^2)}), a'_{ij} b'_{ij} \right\rangle\right) \\
&\neq A \otimes_{1P} B.
\end{aligned}
$$

Hence, $A \vee (A \otimes_{1P} B) \neq A \otimes_{1P} B$.
$(ii) A \wedge (A \otimes_{1P} B)$

$$\begin{aligned} &= \left(\left\langle \min(a_{ij}, \sqrt{\max(a_{ij}^2, b_{ij}^2)}), \max(a'_{ij}, a'_{ij} b'_{ij}) \right\rangle\right) \\ &= (\langle a_{ij}, a'_{ij} \rangle) \\ &= A. \end{aligned}$$

Hence, $A \wedge (A \otimes_{1P} B) = A$.
Similarly, we can prove the following property □

Proposition 5.2. *Let A and B be any two IFMs of same size, we have*
$(i) A \wedge (A \otimes_{2P} B) \neq A \otimes_{2P} B$.
$(ii) A \vee (A \otimes_{2P} B) = A$.

Proposition 5.3. *Let A and B be any two PFMs of same size, we have*
$(i) A \otimes_{1P} (A \vee B) \neq A$.
$(ii) A \otimes_{1P} (A \wedge B) \neq A$.

Proof. (i) $A \otimes_{1P} (A \vee B)$

$$\begin{aligned} &= \left(\left\langle \sqrt{\max(a_{ij}^2, \max(a_{ij}^2, b_{ij}^2)}, a'_{ij} \min(a'_{ij}, b'_{ij} \right\rangle\right) \\ &= (\langle a_{ij}, a'_{ij} \min(a'_{ij}, b'_{ij} \rangle) \\ &\neq (\langle a_{ij}, a'_{ij} \rangle). \end{aligned}$$

Hence, $A \otimes_{1P} (A \vee B) \neq A$.
The proof (ii) is similar to that of (i). □

Proposition 5.4. *Let A and B be any two PFMs of same size, we have*
$(i) A \otimes_{2P} (A \vee B) \neq A$.
$(ii) A \otimes_{2P} (A \wedge B) \neq A$.

Proof. (i) $A \otimes_{2P} (A \vee B)$

$$\begin{aligned} &= \left(\left\langle a_{ij} \max(a_{ij}, b_{ij}), \sqrt{\max(a_{ij}'^2, \min(a_{ij}'^2, b_{ij}'^2)} \right\rangle\right) \\ &= (\langle a_{ij} \max(a_{ij}, b_{ij}), a'_{ij} \rangle) \\ &\neq (\langle a_{ij}, a'_{ij} \rangle). \end{aligned}$$

Hence, $A \otimes_{2P} (A \vee B) \neq A$.
The proof (ii) is similar to that of (i). □

Next we will discuss the distribution, in the case where the Operations $\wedge$, $\vee$, $\otimes_{1P}$ and $\otimes_{2P}$ are combined each other.

Proposition 5.5. *Let A, B and C be any three PFMs of same size, we have*
$(i) A \otimes_{1P} (B \vee C) = (A \otimes_{1P} B) \vee (A \otimes_{1P} C)$.
$(ii) A \otimes_{1P} (B \wedge C) = (A \otimes_{1P} B) \wedge (A \otimes_{1P} C)$.

Proof. (i)$A \otimes_{1P} (B \vee C)$
$= \left(\left\langle \sqrt{\max(a_{ij}^2, \max(b_{ij}^2, c_{ij}^2))}, a'_{ij} \min(b'_{ij}, c'_{ij}) \right\rangle\right)$
$= (\sqrt{\max(\max(a_{ij}^2, b_{ij}^2), \max(a_{ij}^2, c_{ij}^2))}, \min(a'_{ij} b'_{ij}, a'_{ij} c'_{ij}))$
$= (A \otimes_{1P} B) \vee (A \otimes_{1P} C)$.
The proof (ii) is similar to that of (i). □

Proposition 5.6. *Let A, B and C be any three PFMs of same size, we have*
$(i) A \otimes_{2P} (B \vee C) = (A \otimes_{2P} B) \vee (A \otimes_{2P} C)$.
$(ii) A \otimes_{2P} (B \wedge C) = (A \otimes_{2P} B) \wedge (A \otimes_{2P} C)$.

Proof. (i)$A \otimes_{2P} (B \vee C)$
$= (a_{ij} \max(b_{ij}, c_{ij}), \sqrt{\max(a_{ij}'^2, \min(b_{ij}'^2, c_{ij}'^2))})$
$= (\max(a_{ij} b_{ij}, a_{ij} c_{ij}),$
$\quad \sqrt{\max(\max(a_{ij}'^2, b_{ij}'^2), \max(a_{ij}'^2, c_{ij}'^2))})$
$= (A \otimes_{2P} B) \vee (A \otimes_{2P} C)$.
The proof (ii) is similar to that of (i). □

Proposition 5.7. *If A and B are any two PFMs of same size, we have*
$(i) A \otimes_{1P} B \geq A \wedge B$.
$(ii) A \otimes_{2P} B \leq A \vee B$.
$(iii) A \otimes_{2P} B \leq A \wedge B$.
$(iv) A \otimes_{1P} B \neq A \vee B$.

Proof. (i)$A \otimes_{1P} B = \left(\left\langle \sqrt{\max(a_{ij}^2, b_{ij}^2)}, a'_{ij} b'_{ij} \right\rangle\right)$ and
$A \wedge B = \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \right\rangle\right)$.
Since $\sqrt{\max(a_{ij}^2, b_{ij}^2)} \geq \min(a_{ij}, b_{ij})$ and
$a'_{ij} b'_{ij} \leq \max(a'_{ij}, b'_{ij})$,
Hence, $A \otimes_{1P} B \geq A \wedge B$.
$(ii) A \otimes_{2P} B = \left(\left\langle a_{ij}, b_{ij}, \sqrt{\max(a_{ij}'^2, b_{ij}'^2)} \right\rangle\right)$ and
$A \vee B = \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \right\rangle\right)$.
Since $a_{ij} b_{ij} \leq \max(a_{ij}, b_{ij})$ and
$\sqrt{\max(a_{ij}'^2, b_{ij}'^2)} \geq \min(a'_{ij}, b'_{ij})$,
Hence, $A \otimes_{2P} B \leq A \vee B$.
(iii) Since $a_{ij} b_{ij} \leq \min(a_{ij}, b_{ij})$ and
$\sqrt{\max(a_{ij}'^2, b_{ij}'^2)} \geq \min(a'_{ij}, b'_{ij})$,
Hence, $A \otimes_{2P} B \leq A \wedge B$.
(iv) Since $a'_{ij} b'_{ij} \leq \min(a'_{ij}, b'_{ij})$,
Hence, $A \otimes_{1P} B \neq A \vee B$. □

Proposition 5.8. *If A and B are any two PFMs of same size, we have*
$(i) (A \vee B) \vee (A \otimes_{1P} B) = A \otimes_{1P} B$.
$(ii) (A \wedge B) \wedge (A \otimes_{2P} B) = A \otimes_{2P} B$.

Proof. (i) $(A \vee B) \vee (A \otimes_{1P} B)$
$= (\max(\max(a_{ij}, b_{ij}), \sqrt{\max(a_{ij}^2, b_{ij}^2)}),$
$\quad \min(\min(a'_{ij}, b'_{ij}), a'_{ij} b'_{ij}))$
$= \left(\left\langle \sqrt{\max(a_{ij}^2, b_{ij}^2)}, a'_{ij} b'_{ij} \right\rangle\right)$
$= A \otimes_{1P} B$.
Hence, $(A \vee B) \vee (A \otimes_{1P} B) = A \otimes_{1P} B$.
The proof (ii) is similar to that of (i). □

Proposition 5.9. *If A and B are any two PFMs of same size, we have*
$(i) (A \wedge B) \otimes_{1P} (A \vee B) = A \otimes_{1P} B$.
$(ii) (A \wedge B) \otimes_{2P} (A \vee B) = A \otimes_{2P} B$.

Proof. (i) $(A \wedge B) \otimes_{1P} (A \vee B)$
$= (\sqrt{\max(\min(a_{ij}^2, b_{ij}^2), \max(a_{ij}^2, b_{ij}^2)}, \max(a'_{ij}, b'_{ij}) \min(a'_{ij}, b'_{ij}))$
$= \left(\left\langle \sqrt{\max(a_{ij}^2, b_{ij}^2)}, a'_{ij} b'_{ij} \right\rangle\right)$
$= A \otimes_{1P} B$.
Hence, $(A \wedge B) \otimes_{1P} (A \vee B) = A \otimes_{1P} B$.
The proof (ii) is similar to that of (i). □

6. Conclusion

This paper, we defined two new multiplicative operations $\otimes_{1P}$ and $\otimes_{2P}$ on PFM and investigated their algebraic properties. These operations are satisfy the De Morgan's laws. Distributive laws of max-min and min-max compositions over $\otimes_{1P}$ and $\otimes_{2P}$ are proved and established their algebraic properties.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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