A study on normal multi-fuzzy BG-ideals of BG-algebra

R. Muthuraj¹ and S. Devi^{2*}

Abstract

In this paper, the concept of normal multi-fuzzy BG-ideals of BG-algebra is introduced by combining the concepts of multi-fuzzy sets and BG-ideals and its related properties are established. Also the Cartesian product of multi-fuzzy BG-ideals of BG-algebra is defined and its properties are discussed.

Keywords

Multi-fuzzy BG-sub algebra, multi-fuzzy BG-ideals, normal multi-fuzzy BG-ideals, Cartesian product.

AMS Subject Classification

06F35, 03G25, 08A72, 03E72.

¹ PG & Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai - 622 001, Tamil Nadu, India.

² Department of Mathematics, PSNA College of Engineering and Technology, Dindigul-624 622, Tamil Nadu, India.

*Corresponding author: ¹ rmr1973@yahoo.co.in; ² devisaran1982@gmail.com

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1. Introduction

As the extension of fuzzy sets[20], the concept of multi-fuzzy set that is, a set with multi-membership values was introduced by S.Sabu and T.V.Ramakrishnan[17],[18] in 2000. Many researchers applied the concept of fuzzy sets in various abstract algebras and presented some theorems. At first, the two main abstract algebras: BCK-algebras and BCI-algebras were initiated by Y.Imai and K.Iseki[4–6]. It is shown that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. J.Neggers and H.S.Kim[14] introduced a new notion, called a B-algebra. In 2005, C.B.Kim and H.S.Kim[7] introduced the notion of a BG-algebra which is a generalization of B-algebras. S.S.Ahn and H.D.Lee[3] fuzzified this BG-algebra and discussed some properties. The concept of fuzzy ideals of BG-algebra was presented by R.Muthuraj et.al[8] in 2010 and it is normalised by A.Prasanna et.al[16]. By

applying the concept of multi-fuzzy sets in BG-algebra and BG-ideal, R.Muthuraj and S.Devi[9, 13] defined the notion of multi-fuzzy sub algebra and multi-fuzzy BG-ideals of BG-algebra and discussed the properties based on level sets. In this paper, we extend these results to the normal multi-fuzzy BG-ideals and also study some of its related properties. Also the Cartesian product of multi-fuzzy BG-ideals is defined.

2. Preliminaries

In this section, some fundamental definitions are given which will be used in the sequel.

Definition 2.1. [7] A BG-algebra is a non empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:

- (1) $x * x = 0$
- (2) $x * 0 = x$
- (3) $(x * y) * (0 * y) = x, \text{ for all } x, y \in X.$

Example 2.2. [7] Let $X = \{0, 1, 2\}$ be a set with the following table:

Then $(X; *, 0)$ is a BG-algebra.

Definition 2.3. [17] Let X be a non-empty set and let $\{L_i : i \in P\}$ be a family of complete lattices. A multi-fuzzy set A in X is a set of ordered sequences:

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

$A = \{ \langle x, \mu_1(x), \mu_2(x), \dots, \mu_k(x) \rangle : x \in X \}$ where $\mu_i \in L_i^X$, for $i \in P$.

Remark 2.4. [17]

If the sequences of the membership functions have only k -terms (finite number of terms), k is called the dimension of A . If $L_i = [0, 1]$ (for $i = 1, 2, \dots, k$), then the set of all multi-fuzzy sets in X of dimension k is denoted by $M^k FS(X)$. The multi-membership function μ_A is a function from X to I^k such that for all x in X , $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$.

For the sake of simplicity, we denote the multi-fuzzy set $A = \{ \langle x, \mu_1(x), \mu_2(x), \dots, \mu_k(x) \rangle : x \in X \}$ as $A = (\mu_1, \mu_2, \dots, \mu_k)$.

Definition 2.5. [17] Let k be a positive integer and let μ and ν in $M^k FS(X)$, that is

$\mu = (\mu_1, \mu_2, \dots, \mu_k) = \{ \langle x, \mu_1(x), \mu_2(x), \dots, \mu_k(x) \rangle : x \in X \}$ and $\nu = (\nu_1, \nu_2, \dots, \nu_k) = \{ \langle x, \nu_1(x), \nu_2(x), \dots, \nu_k(x) \rangle : x \in X \}$ then we have the following relations and operations:

- (1) $\mu \subseteq \nu$ if and only if $\mu_i \leq \nu_i$ for all $i = 1, 2, \dots, k$;
- (2) $\mu = \nu$ if and only if $\mu_i = \nu_i$ for all $i = 1, 2, \dots, k$;
- (3) $\mu \cup \nu = (\mu_1 \cup \nu_1, \dots, \mu_k \cup \nu_k)$
 $= \{ \langle x, \max(\mu_1(x), \nu_1(x)), \dots, \max(\mu_k(x), \nu_k(x)) \rangle : x \in X \}$
- (4) $\mu \cap \nu = (\mu_1 \cap \nu_1, \dots, \mu_k \cap \nu_k)$
 $= \{ \langle x, \min(\mu_1(x), \nu_1(x)), \dots, \min(\mu_k(x), \nu_k(x)) \rangle : x \in X \}$

Definition 2.6. [9] Let A be a multi-fuzzy set in a BG-algebra X . Then A is called a multi-fuzzy subalgebra of X if $A(x * y) \geq \min\{A(x), A(y)\}$, $\forall x, y \in X$.

Definition 2.7. [13] Let A be a multi-fuzzy set in X . Then A is called a multi-fuzzy BG-ideal in X if it satisfies the following conditions:

- i. $A(0) \geq A(x)$
- ii. $A(x) \geq \min\{A(x * y), A(y)\}$
- iii. $A(x * y) \geq \min\{A(x), A(y)\}$, $\forall x, y \in X$

3. Normal multi-fuzzy BG-ideals of BG-algebra

In this section, normal multi-fuzzy BG-ideals of BG-algebra is defined and discuss some of its related properties.

Definition 3.1. A multi-fuzzy BG-ideal A in X is called normal, if there exists $x \in X$ such that $A(x) = 1_k$ where $1_k = (1, 1, 1, \dots, k \text{ times})$.

Lemma 3.2. A multi-fuzzy BG-ideal A in X is normal if and only if $A(0) = 1_k$.

Example 3.3. Let $X = \{0, 1, 2, 3\}$ be the set with the following table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X, *, 0)$ is a BG-algebra. Define a multi-fuzzy set A in X by $A(0) = A(1) = (1, 1, 1)$ and $A(2) = A(3) = (0.8, 0.6, 0.5)$. Clearly A is a normal multi-fuzzy BG-ideal in X .

Theorem 3.4. Let A be a MFS in a BG-algebra X . Then $A^+(x) = A(x) + 1_k - A(0)$, $\forall x \in X$ is a normal multi-fuzzy BG-ideal of X which contains A .

Proof. Let $x, y \in X$

$$\begin{aligned} \mu_i^+(x) &= \mu_i(x) + 1 - \mu_i(0) \\ &\leq \mu_i(0) + 1 - \mu_i(0) = \mu_i^+(0) \quad \forall i = 1, 2, \dots, k \end{aligned}$$

$$\begin{aligned} \min\{\mu_i^+(x * y), \mu_i^+(y)\} &= \min\{\mu_i(x * y) + 1 - \mu_i(0), \\ &\quad \mu_i(y) + 1 - \mu_i(0)\} \\ &= \min\{\mu_i(x * y), \mu_i(y)\} + 1 - \mu_i(0) \\ &\leq \mu_i(x) + 1 - \mu_i(0) = \mu_i^+(x) \quad \forall i = 1, 2, \dots, k \end{aligned}$$

$$\begin{aligned} \min\{\mu_i^+(x), \mu_i^+(y)\} &= \min\{\mu_i(x) + 1 - \mu_i(0), \\ &\quad \mu_i(y) + 1 - \mu_i(0)\} \\ &= \min\{\mu_i(x), \mu_i(y)\} + 1 - \mu_i(0) \\ &\leq \mu_i(x * y) + 1 - \mu_i(0) \\ &= \mu_i^+(x * y) \quad \forall i = 1, 2, \dots, k \\ \text{and also } A^+(0) &= A(0) + 1_k - A(0) = 1_k. \end{aligned}$$

Therefore A^+ is a normal multi-fuzzy BG-ideal in X . Clearly $A \subseteq A^+$. Thus A^+ is a normal multi-fuzzy BG-ideal which contains A . $\square$

Corollary 3.5. If there is an element $x \in X$ such that $A^+(x) = 0$, then $A(x) = 0$.

Proof. Since $A \subseteq A^+$, $A(x) = 0$.

Corollary 3.6. (i) If A itself is normal then $A = A^+$.
(ii) If A is a multi-fuzzy BG-ideal in X then $(A^+)^+ = A^+$.

Theorem 3.7. Let A and B be two normal multi-fuzzy BG-ideals in X . If $A \subseteq B$ then $X_A \subseteq X_B$.

Proof. Let $X_A = \{x \in X / A(x) = A(0)\}$ and $X_B = \{x \in X / B(x) = B(0)\}$. Let $x \in X_A$.

Then $v_i(x) \geq \mu_i(x) = \mu_i(0) = 1 = v_i(0)$ for every $i = 1, 2, \dots, k$ and so $v_i(x) = v_i(0)$.

This implies $x \in X_B$. Hence $X_A \subseteq X_B$. $\square$

Theorem 3.8. Let A be a multi-fuzzy BG-ideal in X . If there exists a multi-fuzzy BG-ideal B of X such that $B^+ \subseteq A$, then A is normal.

Proof. Since B is a multi-fuzzy BG-ideal in X , B^+ is a normal multi-fuzzy BG-ideal in X .

Then $B^+(0) = 1_k \implies v_i^+(0) = 1$ for every i .

$B^+ \subseteq A \implies v_i^+(x) \leq \mu_i(x), \forall x \in X$ and $i = 1, 2, \dots, k$

$1 = v_i^+(0) \leq \mu_i(0)$ for every i .

That is, $1_k = B^+(0) \leq A(0)$. Hence A is normal. $\square$

Theorem 3.9. Let A be a multi-fuzzy BG-ideal and let $f : [0, A(0)] \rightarrow [0, 1]$ be an increasing function. Then a multi-fuzzy set $A^f : X \rightarrow [0, 1]$ by $A^f(x) = f(A(x)) \forall x \in X$ is a multi-fuzzy BG-ideal in X . Also if $f(A(0)) = 1_k$, then A^f is normal.

Proof. Since f is an increasing function,

$\mu_i^f(0) = f(\mu_i(0)) \geq f(\mu_i(x)) = \mu_i^f(x) \forall i$
 $\implies A^f(0) \geq A^f(x)$

we have

$$\begin{aligned} \mu_i^f(x) &= f(\mu_i(x)) \\ &\geq f(\min\{\mu_i(x * y), \mu_i(y)\}) \\ &= \min\{f(\mu_i(x * y)), f(\mu_i(y))\} \\ &= \min\{\mu_i^f(x * y), \mu_i^f(y)\} \forall i. \end{aligned}$$

and

$$\begin{aligned} \mu_i^f(x * y) &= f(\mu_i(x * y)) \\ &\geq f(\min\{\mu_i(x), \mu_i(y)\}) \\ &= \min\{f(\mu_i(x)), f(\mu_i(y))\} \\ &= \min\{\mu_i^f(x), \mu_i^f(y)\} \forall i. \end{aligned}$$

This implies A^f is a multi-fuzzy BG-ideal in X . If $f(A(0)) = 1_k, A^f(0) = 1_k$. Therefore A^f is normal. $\square$

4. Cartesian product of multi-fuzzy BG-ideals

In this section, the Cartesian product of multi-fuzzy BG-ideals of BG-algebra is defined and its properties are discussed.

Definition 4.1. Let A and B be two MFSs of dimension k . Then Cartesian product of A and B is defined by

$(A \times B)(x_1, x_2) = \{\min\{\mu_1(x_1), v_1(x_2)\}, \min\{\mu_2(x_1), v_2(x_2)\}, \dots, \min\{\mu_k(x_1), v_k(x_2)\}\}$ where $(x_1, x_2) \in X \times X$.

Theorem 4.2. If A and B are two multi-fuzzy BG-ideals of X , then $A \times B$ is also a multi-fuzzy BG-ideal of $X \times X$.

Proof.

$$\begin{aligned} (A \times B)(0, 0) &= \{\min\{\mu_1(0), v_1(0)\}, \\ &\quad \min\{\mu_2(0), v_2(0)\}, \\ &\quad \dots, \min\{\mu_k(0), v_k(0)\}\} \\ &\geq \{\min\{\mu_1(x_1), v_1(x_2)\}, \\ &\quad \min\{\mu_2(x_1), v_2(x_2)\}, \\ &\quad \dots, \min\{\mu_k(x_1), v_k(x_2)\}\} \\ &= (A \times B)(x_1, x_2) \end{aligned}$$

Let (x_1, x_2) and (y_1, y_2) in $X \times X$. Then

$$\begin{aligned} (A \times B)(x_1, x_2) &= \{\min\{\mu_1(x_1), v_1(x_2)\}, \\ &\quad \min\{\mu_2(x_1), v_2(x_2)\}, \dots \\ &\quad \min\{\mu_k(x_1), v_k(x_2)\}\} \\ &\geq \{\min\{\min\{\mu_1(x_1 * y_1), \mu_1(y_1)\}, \\ &\quad \min\{v_1(x_2 * y_2), v_1(y_2)\}\}, \\ &\quad \min\{\min\{\mu_2(x_1 * y_1), \mu_2(y_1)\}, \\ &\quad \min\{v_2(x_2 * y_2), v_2(y_2)\}\}, \dots \\ &\quad \min\{\min\{\mu_k(x_1 * y_1), \mu_k(y_1)\}, \\ &\quad \min\{v_k(x_2 * y_2), v_k(y_2)\}\}\} \\ &= \min\{\min\{\mu_1(x_1 * y_1), v_1(x_2 * y_2)\}, \\ &\quad \min\{\mu_2(x_1 * y_1), v_2(x_2 * y_2)\}, \dots \\ &\quad \min\{\mu_k(x_1 * y_1), v_k(x_2 * y_2)\}\}, \\ &\quad \{\min\{\mu_1(y_1), v_1(y_2)\}, \\ &\quad \min\{\mu_2(y_1), v_2(y_2)\}, \\ &\quad \dots, \min\{\mu_k(y_1), v_k(y_2)\}\} \\ &= \min\{(A \times B)(x_1 * y_1, x_2 * y_2), \\ &\quad (A \times B)(y_1, y_2)\} \\ &= \min\{(A \times B)((x_1, x_2) * (y_1, y_2)), \\ &\quad (A \times B)(y_1, y_2)\} \end{aligned}$$

$$\begin{aligned} (A \times B)((x_1, x_2) * (y_1, y_2)) &= (A \times B)((x_1 * y_1), \\ &\quad (x_2 * y_2)) \\ &= \{\min\{\mu_1(x_1 * y_1), \\ &\quad v_1(x_2 * y_2)\}, \\ &\quad \min\{\mu_2(x_1 * y_1), \\ &\quad v_2(x_2 * y_2)\}, \dots \\ &\quad \min\{\mu_k(x_1 * y_1), \\ &\quad v_k(x_2 * y_2)\}\} \end{aligned}$$

$$\begin{aligned}
&\geq \min\{\min\{\mu_1(x_1), \mu_1(y_1)\}, \\
&\quad \min\{v_1(x_2), v_1(y_2)\}\}, \\
&\quad \min\{\mu_2(x_1), \mu_2(y_1)\}, \\
&\quad \min\{v_2(x_2), v_2(y_2)\}\}, \dots \\
&\quad \min\{\mu_k(x_1), \mu_k(y_1)\}, \\
&\quad \min\{v_k(x_2), v_k(y_2)\}\} \\
&= \min\{\min\{\mu_1(x_1), v_1(x_2)\}, \\
&\quad \min\{\mu_2(x_1), v_2(x_2)\} \dots \\
&\quad \min\{\mu_k(x_1), v_k(x_2)\}, \\
&\quad \min\{\mu_1(y_1), v_1(y_2)\}, \\
&\quad \min\{\mu_2(y_1), v_2(y_2)\} \dots \\
&\quad \min\{\mu_k(y_1), v_k(y_2)\}\} \\
&= \min\{(A \times B)(x_1, x_2), \\
&\quad (A \times B)(y_1, y_2)\}
\end{aligned}$$

Hence $A \times B$ is a multi-fuzzy BG-ideal in $X \times X$. $\square$

4.1 Corollary

The converse of the above theorem may not be true as given in the following example. Example: Let $X = \{0, 1, 2, 3\}$ be a BG-algebra as in example 3.3. Let $r = (r_1, r_2, \dots, r_k)$ and $s = (s_1, s_2, \dots, s_k)$ where r_i and $s_i \in [0, 1]$ such that $r \leq s$. Define two multi-fuzzy sets A and B by $A(x) = r, B(x) = s$ if $x = 0$ and $B(x) = 1_k$ otherwise.

If $x \neq 0$, then

$$\begin{aligned}
(A \times B)(x, x) &= \{\min\{\mu_1(x), v_1(x)\}, \min\{\mu_2(x), v_2(x)\} \\
&\quad \dots \min\{\mu_k(x), v_k(x)\}\} \\
&= \{\min\{r_1, 1\}, \min\{r_2, 1\}, \dots, \min\{r_k, 1\}\} \\
&= \{r_1, r_2, \dots, r_k\} = \{r\} \\
&= r
\end{aligned}$$

If $x = 0$, then

$$\begin{aligned}
(A \times B)(x, x) &= \{\min\{\mu_1(x), v_1(x)\}, \min\{\mu_2(x), v_2(x)\} \dots \\
&\quad \min\{\mu_k(x), v_k(x)\}\} \\
&= \{\{\min\{r_1, s_1\}, \min\{r_2, s_2\}, \dots, \min\{r_k, s_k\}\} \\
&= \{r_1, r_2, \dots, r_k\} = \{r\} \\
&= r
\end{aligned}$$

Hence $A \times B$ is a multi-fuzzy BG-ideal of $X \times X$. Clearly A is a multi-fuzzy BG-ideal of X . But B is not a multi-fuzzy BG-ideal of X since for $x = 0, B(0) = s \leq 1_k = B(x)$.

Theorem 4.3. Let A and B be two multi-fuzzy sets in a BG-algebra X such that $A \times B$ is a multi-fuzzy BG-ideal of $X \times X$. Then

- Either $A(0) \geq A(x)$ or $B(0) \geq B(x)$
- If $A(0) \geq A(x)$, then either $B(0) \geq B(x)$ or $B(0) \geq B(x)$.
- If $B(0) \geq B(x)$, then either $A(0) \geq A(x)$ or $A(0) \geq B(x)$.

Proof. Let $A(0) < A(x)$ and $B(0) < B(x)$. Then $\mu_i(0) < \mu_i(x)$ and $v_i(0) < v_i(x), \forall i = 1, 2, \dots, k$

$$\begin{aligned}
(A \times B)(x_1, x_2) &= \{\min\{\mu_1(x_1), v_1(x_2)\}, \\
&\quad \min\{\mu_2(x_1), v_2(x_2)\} \\
&\quad \dots \min\{\mu_k(x_1), v_k(x_2)\}\} \\
&\geq \{\min\{\min\{\mu_1(0), v_1(0)\}, \\
&\quad \min\{\mu_2(0), v_2(0)\} \\
&\quad \dots \min\{\mu_k(0), v_k(0)\}\} \\
&= (A \times B)(0, 0)
\end{aligned}$$

This implies that $(A \times B)(x_1, x_2) > (A \times B)(0, 0)$, which is a contradiction that $A \times B$ is a multi-fuzzy BG-ideal of $X \times X$. Therefore either $A(0) \geq A(x)$ or $B(0) \geq B(x)$.

Let $B(0) < A(x)$ and $B(0) < B(x)$. Then $v_i(0) < \mu_i(x)$ and $v_i(0) < v_i(x), \forall i = 1, 2, \dots, k$

$$\begin{aligned}
(A \times B)(0, 0) &= \{\min\{\mu_1(0), v_1(0)\}, \\
&\quad \min\{\mu_2(0), v_2(0)\} \dots \min\{\mu_k(0), v_k(0)\} \\
&= \{v_1(0), v_2(0), \dots, v_k(0)\} \\
&= B(0) \\
(A \times B)(x_1, x_2) &= \{\min\{\mu_1(x_1), v_1(x_2)\}, \\
&\quad \min\{\mu_2(0), v_2(0)\} \dots \min\{\mu_k(0), v_k(0)\} \\
&\geq \{v_1(0), v_2(0), \dots, v_k(0)\} \\
&= B(0) \\
&= (A \times B)(0, 0)
\end{aligned}$$

This implies that $(A \times B)(x_1, x_2) > (A \times B)(0, 0)$, which is a contradiction that $A \times B$ is a multi-fuzzy BG-ideal of $X \times X$. Therefore either $A(0) \geq A(x)$ then either $B(0) \geq A(x)$ or $B(0) \geq B(x)$.

The third proof is similar to second proof. $\square$

Theorem 4.4. If $A \times B$ is a multi-fuzzy BG-ideal of $X \times X$, then A or B is a multi-fuzzy BG-ideal of X .

Proof. To prove that B is a multi-fuzzy BG-ideal of X .

By theorem 4.3(i), either $A(0) \geq A(x)$ or $B(0) \geq B(x)$.

Assume that $B(0) \geq B(x)$. Also from theorem 4.3(iii), either $A(0) \geq A(x)$ or $A(0) \geq B(x)$. If $A(0) \geq B(x)$, then $\mu_i(0) \geq v_i(x), \forall i = 1, 2, \dots, k$.

$$\begin{aligned}
(A \times B)(0, x) &= \{\min\{\mu_1(0), v_1(x)\}, \\
&\quad \min\{\mu_2(0), v_2(x)\} \dots \\
&\quad \min\{\mu_k(0), v_k(x)\}\} \\
&= \{v_1(x), v_2(x), \dots, v_k(x)\} \\
&= B(x)
\end{aligned}$$

$$\begin{aligned}
B(x) &= (A \times B)(0, x) \\
&\geq \min\{(A \times B)((0, x) * (0, y)), \\
&\quad (A \times B)(0, y)\} \\
&= \min\{(A \times B)((0 * 0), (x * y)), \\
&\quad (A \times B)(0, y)\} \\
&= \min\{B(x * y), B(y)\}
\end{aligned}$$

$$\begin{aligned}
\text{Also } B(x * y) &= (A \times B)(0, x * y) \\
&= (A \times B)(0 * 0, x * y) \\
&= (A \times B)((0, x) * (0, y)) \\
&\geq \min\{(A \times B)(0, x), (A \times B)(0, y)\} \\
&= \min\{B(x), B(y)\}
\end{aligned}$$

Hence B is a multi-fuzzy BG-ideal of X .

To prove that A is a multi-fuzzy BG-ideal of X .

By theorem 4.3(i), either $A(0) \geq A(x)$ or $B(0) \geq B(x)$.

Assume that $A(0) \geq A(x)$. Also from theorem 4.3(ii), either $B(0) \geq A(x)$ or $B(0) \geq B(x)$.

If $B(0) \geq A(x)$, then $v_i(0) \geq \mu_i(x) \forall i = 1, 2, \dots, k$.

$$\begin{aligned}
(A \times B)(x, 0) &= \{\min\{\mu_1(x), v_1(0)\}, \\
&\quad \min\{\mu_2(x), v_2(0)\}, \dots \\
&\quad \min\{\mu_k(x), v_k(0)\}\} \\
&= \{\mu_1(x), \mu_2(x), \dots, \mu_k(x)\} = A(x)
\end{aligned}$$

$$\begin{aligned}
A(x) &= (A \times B)(x, 0) \\
&\geq \min\{(A \times B)((x, 0) * (y, 0)), (A \times B)(y, 0)\} \\
&= \min\{(A \times B)((x * y), (0 * 0)), \\
&\quad (A \times B)(y, 0)\} \\
&= \min\{A(x * y), A(y)\}
\end{aligned}$$

$$\begin{aligned}
\text{Also } A(x * y) &= (A \times B)(x * y, 0) \\
&= (A \times B)(x * y, 0 * 0) \\
&= (A \times B)((x, 0) * (y, 0)) \\
&\geq \min\{(A \times B)(x, 0), (A \times B)(y, 0)\} \\
&= \min\{A(x), A(y)\}
\end{aligned}$$

Hence A is a multi-fuzzy BG-ideal of X . $\square$

Theorem 4.5. If A and B are two normal multi-fuzzy BG-ideals of X , then $A \times B$ is a normal multi-fuzzy BG-ideal of $X \times X$.

Proof. Let A and B be two normal multi-fuzzy BG-ideals of X . Then $A(0) = 1_k$ and $B(0) = 1_k$. This implies that $\mu_i(0) = 1, v_i(0) = 1$ for $i = 1, 2, \dots, k$.

$$\begin{aligned}
(A \times B)(0, 0) &= \{\min\{\mu_1(0), v_1(0)\}, \min\{\mu_2(0), v_2(0)\}, \dots \\
&\quad \min\{\mu_k(0), v_k(0)\}\} = \{\min\{1, 1\}, \min\{1, 1\}, \dots, \min\{1, 1\}\} \\
&= (1, 1, 1, \dots, k \text{ times}) = 1_k
\end{aligned}$$

Hence $A \times B$ is a normal multi-fuzzy BG-ideal of $X \times X$. $\square$

5. Conclusion

The notion of normal multi-fuzzy BG-ideals of BG-algebra is introduced by applying the concept of multi-fuzzy sets in ideals of BG-algebra and its properties are studied. Also the cartesian product of multi-fuzzy BG-ideals of BG-algebra is defined and its properties are discussed.

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