Fuzzy upper and lower generalized e -continuous and irresoluteness in double fuzzy topological spaces

P. Periyasamy¹, V. Chandrasekar², G. Saravanakumar³ and A. Vadivel^{4*}

Abstract

In this article, we induct a fuzzy upper and fuzzy lower generalized e (resp. δ_s , δ_p and β) - continuous and irresoluteness multifunctions. Also many statements and conditions of multifunctions are analyzed in double fuzzy topological spaces.

Keywords

Double (intuitionistic) fuzzy topology, fuzzy multifunction, upper and lower generalized e (resp. δ_s , δ_p and β) continuous, generalized e (resp. δ_s , δ_p and β) irresolute, (r,s) - $FgeT_{\frac{1}{2}}$ (resp. $Fg\delta_sT_{\frac{1}{2}}$, $Fg\delta_pT_{\frac{1}{2}}$ and $Fg\beta T_{\frac{1}{2}}$)-spaces.

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¹Research Scholar, Department of Mathematics, Kandaswamy Kandar's College, P-Velur-638182, Tamil Nadu, India.

²Department of Mathematics, Kandaswamy Kandar's College, P-Velur-638182, Tamil Nadu, India.

³Research Scholar, Department of Mathematics, Annamalai University, Annamalai Nagar-608002, Tamil Nadu, India.

⁴Department of Mathematics, Government Arts College, Karur-639005, Tamil Nadu, India.

*Corresponding author: avmaths@gmail.com;

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Contents

1	Introduction	408
2	Preliminaries	408
3	Fuzzy upper and lower generalized e (resp. δ_s , δ_p and β)-continuous and irresolute multifunctions.	410
	References	413

1. Introduction

Chang [10] in 1968 defined fuzzy topological space as every fuzzy set may be open or not. Goguen [15] developed the concept of Chang and redefines the general lattice L in $[0,1]$. Kubiak [20] represent topology is itself fuzzy in addition to fuzzy set and Sostak [33] also represent the same. The common approach towards the topological system with power sets was unified in [17]-[21] is the open research field for researchers Now.

Atanassov [6]-[8] introduced Intuitionistic fuzzy set a generalization to fuzzy sets. Intuitionistic fuzzy topology was defined by Coker and his team [11]-[13] in Chang sense. Intuitionistic fuzzy gradation of openness was attributed by Kubiak and sense [31, 32], in 2005, Gutierrez Garcia and

Rodabaugh [14] after clearing the doubt's of fuzzy intuitionistic they worked under "double".

Concept of multimapping was coined by Berge [9] in topological space also Popa [29, 30] identified the concept of irresolute multimapping. Many authors [4, 5] [25]-[27]) all worked in continuity of multifunction and in concept of fuzzy topology [10]. At present Kerre et al., [34, 35] identified the continuity of fuzzy multivalued mapping.

All over the paper, nonempty sets will be represent as X , Y etc., $L = [0, 1]$ and $L_0 = (0, 1]$. The group of all fuzzy sets of X is represented by L^X . For $\alpha \in L$, $\underline{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t for $t \in L_0$ is an element of L^X such

that $x_t(y) = \begin{cases} t, & \text{if } y = x \\ 0, & \text{if } y \neq x. \end{cases}$ The group of all fuzzy points in

X is represented by $Pt(X)$. A fuzzy point $x_t \in \lambda$ iff $t < \lambda(x)$. All other symbols are as in standard of fuzzy set theory.

2. Preliminaries

Definition 2.1. [3] Let $F : X \multimap Y$. Then F is called a fuzzy multifunction (FM, for short) iff $F(x) \in L^Y$ for each $x \in X$. The degree of membership of y in $F(x)$ is denoted by $F(x)(y) = G_F(x, y)$ for any $(x, y) \in X \times Y$.

The domain of F , denoted by $\text{dom}(F)$ and the range of F , denoted by $\text{rng}(F)$ for any $x \in X$, $y \in Y$: $\text{dom}(F) = \bigvee_{y \in Y} G_F(x, y)$ and $\text{rng}(F)(y) = \bigvee_{x \in X} G_F(x, y)$.

Definition 2.2. [3] Let $F : X \multimap Y$ be a FM. Then F is called normalized iff for each $x \in X$, there exists $y_0 \in Y$ such that $G_F(x, y_0) = 1$.

Definition 2.3. [3] Let $F : X \multimap Y$ be a FM. Then

- (i) The image of $\lambda \in L^X$ is a fuzzy set $F(\lambda) \in L^Y$ and defined by: $F(\lambda)(y) = \bigvee_{x \in X} [G_F(x, y) \wedge \lambda(x)]$.
- (ii) The lower inverse of $\mu \in L^Y$ is a fuzzy set $F^l(\mu) \in L^X$ and defined by: $F^l(\mu)(x) = \bigvee_{y \in Y} [G_F(x, y) \wedge \mu(y)]$.
- (iii) The upper inverse of $\mu \in L^Y$ is a fuzzy set $F^u(\mu) \in L^X$ and defined by: $F^u(\mu)(x) = \bigwedge_{y \in Y} [G_F^c(x, y) \vee \mu(y)]$.

Theorem 2.4. [3] Let $F : X \multimap Y$ be a FM. Then

- (i) $F(\lambda_1) \leq F(\lambda_2)$ if $\lambda_1 \leq \lambda_2$.
- (ii) $F^l(\mu_1) \leq F^l(\mu_2)$ and $F^u(\mu_1) \leq F^u(\mu_2)$ if $\mu_1 \leq \mu_2$.
- (iii) $F^l(\underline{1} - \mu) = \underline{1} - F^u(\mu)$.
- (iv) $F^u(\underline{1} - \mu) = \underline{1} - F^l(\mu)$.

Definition 2.5. [32, 36] A double fuzzy topology on X is a pair $(\hat{\tau}, \check{\tau})$ of the mappings $\hat{\tau}, \check{\tau} : L^X \rightarrow L$ which satisfy the following conditions:

- (i) $\hat{\tau}(\lambda) + \check{\tau}(\lambda) \leq 1$, $\forall \lambda \in L^X$.
- (ii) $\hat{\tau}(\lambda_1 \wedge \lambda_2) \geq \hat{\tau}(\lambda_1) \wedge \hat{\tau}(\lambda_2)$ and $\check{\tau}(\lambda_1 \wedge \lambda_2) \leq \check{\tau}(\lambda_1) \vee \check{\tau}(\lambda_2)$, $\forall \lambda_1, \lambda_2 \in L^X$.
- (iii) $\hat{\tau}(\bigvee_{i \in \Gamma} \lambda_i) \geq \bigwedge_{i \in \Gamma} \hat{\tau}(\lambda_i)$ and $\check{\tau}(\bigvee_{i \in \Gamma} \lambda_i) \leq \bigvee_{i \in \Gamma} \check{\tau}(\lambda_i)$, $\forall \{\lambda_i\}_{i \in \Gamma} \subset L^X$.

The triplet $(X, \hat{\tau}, \check{\tau})$ is called a double (intuitionistic) fuzzy topological space (dfts, for short). $\hat{\tau}(\lambda)$ and $\check{\tau}(\lambda)$ may be interpreted as a gradation of openness and a gradation of nonopenness for $\lambda \in L^X$, respectively.

Theorem 2.6. [1, 22] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$, we define operators $C_{\hat{\tau}, \check{\tau}}$ and $I_{\hat{\tau}, \check{\tau}} : L^X \times L_0 \times L_1 \rightarrow L^X$ as follows: $C_{\hat{\tau}, \check{\tau}}(\lambda, r, s) = \bigwedge \{\mu \in L^X : \lambda \leq \mu, \hat{\tau}(\underline{1} - \mu) \geq r, \check{\tau}(\underline{1} - \mu) \leq s\}$ and $I_{\hat{\tau}, \check{\tau}}(\lambda, r, s) = \bigvee \{\mu \in L^X : \mu \leq \lambda, \hat{\tau}(\mu) \geq r, \check{\tau}(\mu) \leq s\}$.

For $\lambda, \mu \in L^X$ and $r \in L_0$, $s \in L_1$ the operator $C_{\hat{\tau}, \check{\tau}}$ satisfies the following statements:

- (i) $C_{\hat{\tau}, \check{\tau}}(0, r, s) = 0$.
- (ii) $\lambda \leq C_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$.
- (iii) $C_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \vee C_{\hat{\tau}, \check{\tau}}(\mu, r, s) = C_{\hat{\tau}, \check{\tau}}(\lambda \vee \mu, r, s)$.
- (iv) $C_{\hat{\tau}, \check{\tau}}(\lambda, r, s) = \lambda$ iff $\hat{\tau}(\underline{1} - \lambda) \geq r$ and $\check{\tau}(\underline{1} - \lambda) \leq s$.

$$(v) C_{\hat{\tau}, \check{\tau}}(C_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s) = C_{\hat{\tau}, \check{\tau}}(\lambda, r, s).$$

$$(vi) C_{\hat{\tau}, \check{\tau}}(\underline{1} - \lambda, r, s) = \underline{1} - I_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \text{ and } I_{\hat{\tau}, \check{\tau}}(\underline{1} - \lambda, r, s) = \underline{1} - C_{\hat{\tau}, \check{\tau}}(\lambda, r, s).$$

Definition 2.7. [1] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$: a fuzzy set λ is called an

- (i) (r, s) -generalized fuzzy open (briefly, (r, s) -gfo) iff $\mu \leq I_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$ whenever $\mu \leq \lambda$ and $\hat{\tau}(\underline{1} - \mu) \geq r$, $\check{\tau}(\underline{1} - \mu) \leq s$.
- (ii) (r, s) -generalized fuzzy closed (briefly, (r, s) -gfc) iff $C_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \leq \mu$ whenever $\lambda \leq \mu$ and $\hat{\tau}(\mu) \geq r$, $\check{\tau}(\mu) \leq s$.

Theorem 2.8. [1] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$, we have

- (i) If $\hat{\tau}(\underline{1} - \lambda) \geq r$, $\check{\tau}(\underline{1} - \lambda) \leq s$, then λ is (r, s) -gfc.
- (ii) If λ is (r, s) -gfc, then λ is (r, s) -gfsc.

Theorem 2.9. [1] A dfts $(X, \hat{\tau}, \check{\tau})$ is said to be: (r, s) -FT $_{\frac{1}{2}}$ iff $\hat{\tau}(\underline{1} - \lambda) \geq r$, $\check{\tau}(\underline{1} - \lambda) \leq s$ for each $\lambda \in L^X$ is (r, s) -gfc.

Definition 2.10. [28] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$: a fuzzy set λ is called an (r, s) -fuzzy regular open (briefly, (r, s) -fro) $\lambda = I_{\hat{\tau}, \check{\tau}}C_{\hat{\tau}, \check{\tau}}(\lambda)$ and (r, s) -fuzzy regular closed (briefly, (r, s) -frc) if $\lambda = C_{\hat{\tau}, \check{\tau}}I_{\hat{\tau}, \check{\tau}}(\lambda)$.

Definition 2.11. [28] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$, we define operators $\delta C_{\hat{\tau}, \check{\tau}}$ and $\delta I_{\hat{\tau}, \check{\tau}} : L^X \times L_0 \times L_1 \rightarrow L^X$ as follows: $\delta C_{\hat{\tau}, \check{\tau}}(\lambda, r, s) = \bigwedge \{\mu \in L^X : \lambda \leq \mu, \mu \text{ is } (r, s)\text{-frc}\}$ and $\delta I_{\hat{\tau}, \check{\tau}}(\lambda, r, s) = \bigvee \{\mu \in L^X : \mu \leq \lambda, \mu \text{ is } (r, s)\text{-fro}\}$.

Definition 2.12. [28] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$: a fuzzy set λ is called an

- (i) (r, s) -fuzzy δ semi open (briefly, (r, s) -f δ so) if $\lambda \leq C_{\hat{\tau}, \check{\tau}}(\delta I_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s)$ and (r, s) -fuzzy δ semi closed (briefly, (r, s) -f δ sc) if $\lambda \geq I_{\hat{\tau}, \check{\tau}}(\delta C_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s)$.
- (ii) (r, s) -fuzzy δ pre open (briefly, (r, s) -f δ po) if $\lambda \leq I_{\hat{\tau}, \check{\tau}}(\delta C_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s)$ and (r, s) -fuzzy δ pre closed (briefly, (r, s) -f δ pc) if $\lambda \geq C_{\hat{\tau}, \check{\tau}}(\delta I_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s)$.
- (iii) (r, s) -fuzzy β open (briefly, (r, s) -f β o) if $\lambda \leq C_{\hat{\tau}, \check{\tau}}(I_{\hat{\tau}, \check{\tau}}(C_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s), r, s)$ and (r, s) -fuzzy β closed (briefly, (r, s) -f β c) if $\lambda \geq I_{\hat{\tau}, \check{\tau}}(C_{\hat{\tau}, \check{\tau}}(I_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s), r, s)$.
- (iv) (r, s) -fuzzy e -open (briefly, (r, s) -f e o) if $\lambda \leq C_{\hat{\tau}, \check{\tau}}(\delta I_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s) \vee I_{\hat{\tau}, \check{\tau}}(\delta C_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s)$ and (r, s) -fuzzy e -closed (briefly, (r, s) -f e c) if $\lambda \geq I_{\hat{\tau}, \check{\tau}}(\delta C_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s) \wedge C_{\hat{\tau}, \check{\tau}}(\delta I_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s)$.

Definition 2.13. [28] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X$, $r \in L_0$ and $s \in L_1$, we define operators $eC_{\hat{\tau}, \check{\tau}}$ (resp. $\delta SC_{\hat{\tau}, \check{\tau}}$, $\delta PC_{\hat{\tau}, \check{\tau}}$ and $\beta C_{\hat{\tau}, \check{\tau}}$) and $eI_{\hat{\tau}, \check{\tau}}$ (resp. $\delta SI_{\hat{\tau}, \check{\tau}}$, $\delta PI_{\hat{\tau}, \check{\tau}}$ and $\beta I_{\hat{\tau}, \check{\tau}}$) : $L^X \times L_0 \times L_1 \rightarrow L^X$ as follows: $eC_{\hat{\tau}, \check{\tau}}$ (resp. $\delta SC_{\hat{\tau}, \check{\tau}}$, $\delta PC_{\hat{\tau}, \check{\tau}}$ and $\beta C_{\hat{\tau}, \check{\tau}}$)

$\delta PC_{\hat{\tau}, \check{\tau}}$ and $\beta C_{\hat{\tau}, \check{\tau}}$ (λ, r, s) = $\bigwedge \{ \mu \in L^X : \lambda \leq \mu, \mu \text{ is } (r, s)\text{-fec (resp. } f\delta sc, f\delta pc \text{ and } f\beta c) \}$.
 $eI_{\hat{\tau}, \check{\tau}}$ (resp. $\delta SI_{\hat{\tau}, \check{\tau}}, \delta PI_{\hat{\tau}, \check{\tau}}$ and $\beta I_{\hat{\tau}, \check{\tau}}$) (λ, r, s) = $\bigvee \{ \mu \in L^X : \mu \leq \lambda, \mu \text{ is } (r, s)\text{-feo (resp. } f\delta so, f\delta po \text{ and } f\beta o) \}$.

For $\lambda, \mu \in L^X$ and $r \in L_0, s \in L_1$ the operator $eC_{\hat{\tau}, \check{\tau}}$ satisfies the following conditions:

- (i) $eC_{\hat{\tau}, \check{\tau}}(\underline{0}, r, s) = \underline{0}$.
- (ii) $\lambda \leq eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$.
- (iii) $eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \vee eC_{\hat{\tau}, \check{\tau}}(\mu, r, s) = eC_{\hat{\tau}, \check{\tau}}(\lambda \vee \mu, r, s)$.
- (iv) $eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s) = \lambda$ iff μ is (r, s) -fec.
- (v) $eC_{\hat{\tau}, \check{\tau}}(eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s), r, s) = eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$.
- (vi) $eC_{\hat{\tau}, \check{\tau}}(\underline{1} - \lambda, r, s) = \underline{1} - eI_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$ and $eI_{\hat{\tau}, \check{\tau}}(\underline{1} - \lambda, r, s) = \underline{1} - eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$.

Similarly the other operators (i.e) $\delta SC_{\hat{\tau}, \check{\tau}}, \delta PC_{\hat{\tau}, \check{\tau}}$ and $\beta C_{\hat{\tau}, \check{\tau}}$ satisfies the above conditions.

Definition 2.14. [28] Let $(X, \hat{\tau}, \check{\tau})$ be a dfts. Then for each $\lambda \in L^X, r \in L_0$ and $s \in L_1$: a fuzzy set λ is called

- (i) (r, s) -generalized fuzzy δ semi open (briefly, (r, s) -gf δso) if $\mu \leq \delta SI_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$ whenever $\mu \leq \lambda$ and $\hat{\tau}(\underline{1} - \mu) \geq r, \check{\tau}(\underline{1} - \mu) \leq s$.
- (ii) (r, s) -generalized fuzzy δ pre open (briefly, (r, s) -gf δpo) if $\mu \leq \delta PI_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$ whenever $\mu \leq \lambda$ and $\hat{\tau}(\underline{1} - \mu) \geq r, \check{\tau}(\underline{1} - \mu) \leq s$.
- (iii) (r, s) -generalized fuzzy β open (briefly, (r, s) -gf βo) if $\mu \leq \beta I_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$ whenever $\mu \leq \lambda$ and $\hat{\tau}(\underline{1} - \mu) \geq r, \check{\tau}(\underline{1} - \mu) \leq s$.
- (iv) (r, s) -generalized fuzzy e open (briefly, (r, s) -gf $e o$) if $\mu \leq eI_{\hat{\tau}, \check{\tau}}(\lambda, r, s)$ whenever $\mu \leq \lambda$ and $\hat{\tau}(\underline{1} - \mu) \geq r, \check{\tau}(\underline{1} - \mu) \leq s$.
- (v) (r, s) -generalized fuzzy δ semi closed (briefly, (r, s) -gf δsc) if $\delta SC_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \leq \mu$ whenever $\lambda \leq \mu$ and $\hat{\tau}(\mu) \geq r, \check{\tau}(\mu) \leq s$.
- (vi) (r, s) -generalized fuzzy δ pre closed (briefly, (r, s) -gf δpc) if $\delta PC_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \leq \mu$ whenever $\lambda \leq \mu$ and $\hat{\tau}(\mu) \geq r, \check{\tau}(\mu) \leq s$.
- (vii) (r, s) -generalized fuzzy β closed (briefly, (r, s) -gf βc) if $\beta C_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \leq \mu$ whenever $\lambda \leq \mu$ and $\hat{\tau}(\mu) \geq r, \check{\tau}(\mu) \leq s$.
- (viii) (r, s) -generalized fuzzy e closed (briefly, (r, s) -gf $e c$) if $eC_{\hat{\tau}, \check{\tau}}(\lambda, r, s) \leq \mu$ whenever $\lambda \leq \mu$ and $\hat{\tau}(\mu) \geq r, \check{\tau}(\mu) \leq s$.

Definition 2.15. [16] Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau}), (Y, \hat{\eta}, \check{\eta})$ and $r \in L_0, s \in L_1$. Then F is called:

- (i) Fuzzy upper generalized continuous (FUG-continuous, for short) at a fuzzy point $x_t \in \text{dom}(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X, \lambda$ is (r, s) -gfo and $x_t \in \lambda$ such that $\lambda \wedge \text{dom}(F) \leq F^u(\mu)$. F is FUG-continuous iff it is FUG-continuous at every $x_t \in \text{dom}(F)$.
- (ii) Fuzzy lower generalized continuous (FLG-continuous, for short) at a fuzzy point $x_t \in \text{dom}(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X, \lambda$ is (r, s) -gfo and $x_t \in \lambda$ such that $\lambda \wedge \text{dom}(F) \leq F^l(\mu)$. F is FLG-continuous iff it is FLG-continuous at every $x_t \in \text{dom}(F)$.

Proposition 2.16. [16] Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau}), (Y, \hat{\eta}, \check{\eta})$ and $r \in L_0, s \in L_1$. If F is normalized, then F is:

- (i) FUG-continuous at $x_t \in \text{dom}(F)$ iff $x_t \in F^u(\mu) \forall \mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X, \lambda$ is (r, s) -gfo and $x_t \in \lambda$ such that $\lambda \leq F^u(\mu)$.
- (ii) FUG-irresolute at $x_t \in \text{dom}(F)$ iff $x_t \in F^u(\mu) \forall \mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X, \lambda$ is (r, s) -gfo and $x_t \in \lambda$ such that $\lambda \leq F^u(\mu)$.

3. Fuzzy upper and lower generalized e (resp. $\delta s, \delta p$ and β)-continuous and irresolute multifunctions

Definition 3.1. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau}), (Y, \hat{\eta}, \check{\eta})$ and $r \in L_0, s \in L_1$. Then F is called:

- (i) Fuzzy upper generalized e (resp. $\delta s, \delta p$ and β) (FUG $e, FUG\delta s, FUG\delta p$ and FUG β , for short) continuous at a fuzzy point $x_t \in \text{dom}(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X, \lambda$ is (r, s) -gfo (resp. gf $\delta so, gf\delta po$ and gf βo) and $x_t \in \lambda$ such that $\lambda \wedge \text{dom}(F) \leq F^u(\mu)$. F is FUG e (resp. FUG $\delta s, FUG\delta p$ and FUG β)-continuous iff it is FUG e (resp. FUG $\delta s, FUG\delta p, FUG\beta$)-continuous at every $x_t \in \text{dom}(F)$.
- (ii) Fuzzy lower generalized e (resp. $\delta s, \delta p$ and β) (FLG $e, FLG\delta s, FLG\delta p$ and FLG β , for short) continuous at a fuzzy point $x_t \in \text{dom}(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X, \lambda$ is (r, s) -gfo (resp. gf $\delta so, gf\delta po$ and gf βo) and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$. F is FLG e (resp. FLG $\delta s, FLG\delta p$ and FLG β)-continuous iff it is FLG e (resp. FLG $\delta s, FLG\delta p$ and FLG β)-continuous at every $x_t \in \text{dom}(F)$.

Definition 3.2. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau}), (Y, \hat{\eta}, \check{\eta})$ and $r \in L_0, s \in L_1$. Then F is called:

- (i) Fuzzy upper generalized e (resp. $\delta s, \delta p$ and β) (FUG $e, FUG\delta s, FUG\delta p$ and FUG β , for short) irresolute at a fuzzy point $x_t \in \text{dom}(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$

and μ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) there exists $\lambda \in L^X$, λ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$. F is $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$)-irresolute iff it is $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$, $FUG\beta$) - irresolute at every $x_t \in dom(F)$.

- (ii) Fuzzy lower generalized ϵ (resp. $\delta S, \delta P, \beta$) ($FUGe$, $FUG\delta S$, $FUG\delta P$ and $FUG\beta$, for short) irresolute at a fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) there exists $\lambda \in L^X$, λ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$. F is $FLGe$ (resp. $FLG\delta S$, $FLG\delta P$ and $FLG\beta$)-irresolute iff it is $FLGe$ (resp. $FLG\delta S$, $FLG\delta P$ and $FLG\beta$)-irresolute at every $x_t \in dom(F)$.

Proposition 3.3. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau})$, $(Y, \hat{\eta}, \check{\eta})$ and $r \in L_0$, $s \in L_1$. Then F is normalized, and F is:

- (i) $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$)-continuous at $x_t \in dom(F)$ iff $x_t \in F^u(\mu) \forall \mu \in L^Y$ and $\hat{\eta}(\mu) \geq r, \check{\eta}(\mu) \leq s$ there exists $\lambda \in L^X$, λ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) and $x_t \in \lambda$ such that $\lambda \leq F^u(\mu)$.
- (ii) $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$)-irresolute at $x_t \in dom(F)$ iff $x_t \in F^u(\mu) \forall \mu \in L^Y$ and μ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) there exists $\lambda \in L^X$, λ is (r,s) -gfeo (resp. $gf\delta so$, $gf\delta po$ and $gf\beta o$) and $x_t \in \lambda$ such that $\lambda \leq F^u(\mu)$.

The following implications hold:

- (i) $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$)-irresolute $\Rightarrow FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$) - continuous.
- (ii) $FLGe$ (resp. $FLG\delta S$, $FLG\delta P$ and $FLG\beta$)-irresolute $\Rightarrow FLGe$ (resp. $FLG\delta S$, $FLG\delta P$ and $FLG\beta$) - continuous.

In general the converses are not true.

Theorem 3.4. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau})$, $(Y, \hat{\eta}, \check{\eta})$ and $\mu \in L^Y$, $r \in L_0$, $s \in L_1$. Suppose that one of the following properties hold:

- (i) $F^u(\mu)$ is (r,s) -gfeo (resp. (r,s) -gf δso , (r,s) -gf δpo and (r,s) -gf βo), for any $\hat{\eta}(\mu) \geq r$, $\check{\eta}(\mu) \leq s$.
- (ii) $F^l(\mu)$ is (r,s) -gfec (resp. (r,s) -gf δsc , (r,s) -gf δpc and (r,s) -gf βc), for any $\hat{\eta}(1-\mu) \geq r$, $\check{\eta}(1-\mu) \leq s$.

Then F is $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$) - continuous.

Proof. (i) $\Rightarrow$ (ii): Let $\mu \in L^Y$ and $\hat{\eta}(1-\mu) \geq r$, $\check{\eta}(1-\mu) \leq s$ hence by (i) $F^u(1-\mu) = 1 - F^l(\mu)$ is (r,s) -gfec. Then $F^l(\mu)$ is (r,s) -gfec.

(ii) $\Rightarrow$ (i): It is similar to that of (i) $\Rightarrow$ (ii).

Suppose that (i) holds. Let $x_t \in dom(F)$, $\hat{\eta}(\mu) \geq r$, $\check{\eta}(\mu) \leq s$ and $x_t \in F^u(\mu)$ we have by (i), $F^u(\mu) = \lambda$ (say) is (r,s) -gfeo then, there exists λ is (r,s) -gfeo and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$. Thus F is $FUGe$ -continuous.

Proof of the other cases are similar. $\square$

Theorem 3.5. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau})$, $(Y, \hat{\eta}, \check{\eta})$ and $\mu \in L^Y$, $r \in L_0$, $s \in L_1$. Suppose that one of the following properties hold:

- (i) $F^u(\mu)$ is (r,s) -gfeo (resp. (r,s) -gf δso , (r,s) -gf δpo and (r,s) -gf βo), for any μ is (r,s) -gfeo (resp. (r,s) -gf δso , (r,s) -gf δpo and (r,s) -gf βo).
- (ii) $F^l(\mu)$ is (r,s) -gfec (resp. (r,s) -gf δsc , (r,s) -gf δpc and (r,s) -gf βc), for any μ is (r,s) -gfec (resp. (r,s) -gf δsc , (r,s) -gf δpc and (r,s) -gf βc).

Then F is $FUGe$ (resp. $FUG\delta S$, $FUG\delta P$ and $FUG\beta$) - irresolute.

Proof. (i) $\Rightarrow$ (ii): Let μ is (r,s) -gfec hence by (i) $F^u(1-\mu) = 1 - F^l(\mu)$ is (r,s) -gfeo. Then $F^l(\mu)$ is (r,s) -gfec.

(ii) $\Rightarrow$ (i): It is similar to that of (i) $\Rightarrow$ (ii).

Suppose that (i) holds. Let $x_t \in dom(F)$, μ is (r,s) -gfeo and $x_t \in F^u(\mu)$ we have by (i), $F^u(\mu) = \lambda$ (say) is (r,s) -gfeo then, there exists λ is (r,s) -gfeo and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$. Thus F is $FUGe$ -irresolute.

Proof of the other cases are similar. $\square$

Theorem 3.6. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau})$, $(Y, \hat{\eta}, \check{\eta})$ and $\mu \in L^Y$, $r \in L_0$, $s \in L_1$. Suppose that one of the following properties hold:

- (i) $F^l(\mu)$ is (r,s) -gfeo (resp. (r,s) -gf δso , (r,s) -gf δpo and (r,s) -gf βo), for any $\hat{\eta}(\mu) \geq r$, $\check{\eta}(\mu) \leq s$.
- (ii) $F^u(\mu)$ is (r,s) -gfec (resp. (r,s) -gf δsc , (r,s) -gf δpc and (r,s) -gf βc), for any $\hat{\eta}(1-\mu) \geq r$, $\check{\eta}(1-\mu) \leq s$.

Then F is $FLGe$ (resp. $FLG\delta S$, $FLG\delta P$ and $FLG\beta$) - continuous.

Proof. This can be proved in similar way as Theorem 3.4 $\square$

Theorem 3.7. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau})$, $(Y, \hat{\eta}, \check{\eta})$ and $\mu \in L^Y$, $r \in L_0$, $s \in L_1$. Suppose that one of the following properties hold:

- (i) $F^l(\mu)$ is (r,s) -gfeo (resp. (r,s) -gf δso , (r,s) -gf δpo and (r,s) -gf βo) for any μ is (r,s) -gfeo (resp. (r,s) -gf δso , (r,s) -gf δpo and (r,s) -gf βo).
- (ii) $F^u(\mu)$ is (r,s) -gfec (resp. (r,s) -gf δsc , (r,s) -gf δpc and (r,s) -gf βc) for any μ is (r,s) -gfec (resp. (r,s) -gf δsc , (r,s) -gf δpc and (r,s) -gf βc).

Then F is $FLGe$ -irresolute.

Proof. This can be proved in similar way as Theorem 3.5 $\square$

Definition 3.8. A dfts $(X, \hat{\tau}, \check{\tau})$ is said to be (r, s) -Fge $T_{\frac{1}{2}}$ (resp. (r, s) -Fg $\delta sT_{\frac{1}{2}}$, (r, s) -Fg $\delta pT_{\frac{1}{2}}$ and (r, s) -Fg $\beta T_{\frac{1}{2}}$) iff $\hat{\tau}(\underline{1} - \lambda) \geq r$, $\check{\tau}(\underline{1} - \lambda) \leq s$ for each $\lambda \in L^X$ is (r, s) -gfec (resp. (r, s) -gf δsc , (r, s) -gf δpc and (r, s) -gf βc).

Example 3.9. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F : X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = 0.8$, $G_F(x_1, y_3) = 0.3$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = 0.3$, $G_F(x_2, y_3) = 0.7$. Define double fuzzy topologies $\hat{\tau}, \check{\tau} : L^X \rightarrow L$ and $\hat{\eta}, \check{\eta} : L^Y \rightarrow L$ as follows:

$$\hat{\tau}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.4} \\ 0, & \text{otherwise} \end{cases} \quad \check{\tau}(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.4} \\ 1, & \text{otherwise} \end{cases}$$

$$\hat{\eta}(\mu) = \begin{cases} 1, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.65} \\ 0, & \text{otherwise} \end{cases} \quad \check{\eta}(\mu) = \begin{cases} 0, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.65} \\ 1, & \text{otherwise} \end{cases}$$

(i) F is FUG δs (resp. FLG δs)-continuous, FUG δp (resp. FLG δp)-continuous but not FUG (resp. FLG)-continuous because $\hat{\eta}(\underline{0.65}) = \frac{1}{2} = \check{\eta}(\underline{0.65})$ in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.65}) = \underline{0.65}$ (resp. $F^l(\underline{0.65}) = \underline{0.65}$) is not $(\frac{1}{2}, \frac{1}{2})$ -gfo.

(ii) F is FUG δs (resp. FLG δs)-irresolute, FUG δp (resp. FLG δp)-irresolute but not FUG (resp. FLG)-irresolute because $\underline{0.65}$ is $(\frac{1}{2}, \frac{1}{2})$ -gfo in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.65}) = F^l(\underline{0.65}) = \underline{0.65}$ is not $(\frac{1}{2}, \frac{1}{2})$ -gfo.

Example 3.10. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F : X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = 0.8$, $G_F(x_1, y_3) = 0.3$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = 0.3$, $G_F(x_2, y_3) = 0.7$. Define double fuzzy topologies $\hat{\tau}, \check{\tau} : L^X \rightarrow L$ and $\hat{\eta}, \check{\eta} : L^Y \rightarrow L$ as follows:

$$\hat{\tau}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.65} \\ 0, & \text{otherwise} \end{cases} \quad \check{\tau}(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.65} \\ 1, & \text{otherwise} \end{cases}$$

$$\hat{\eta}(\mu) = \begin{cases} 1, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.4} \\ 0, & \text{otherwise} \end{cases} \quad \check{\eta}(\mu) = \begin{cases} 0, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.4} \\ 1, & \text{otherwise} \end{cases}$$

(i) F is FUGe (resp. FLGe)-continuous, but not FUG δs (resp. FLG δs)-continuous because $\hat{\eta}(\underline{0.4}) = \frac{1}{2} = \check{\eta}(\underline{0.4})$ in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.4}) = \underline{0.4}$ (resp. $F^l(\underline{0.4}) = \underline{0.4}$) is not $(\frac{1}{2}, \frac{1}{2})$ -gf δso .

(ii) F is FUGe (resp. FLGe)-irresolute, but not FUG δs (resp. FUG δs)-irresolute because $\underline{0.4}$ is $(\frac{1}{2}, \frac{1}{2})$ -gf δso in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.4}) = F^l(\underline{0.4}) = \underline{0.4}$ is not $(\frac{1}{2}, \frac{1}{2})$ -gf δso .

Example 3.11. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F : X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.1$, $G_F(x_1, y_2) = 0.9$, $G_F(x_1, y_3) = 0.3$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = 0.3$, $G_F(x_2, y_3) = 0.9$. Define double fuzzy topologies $\hat{\tau}, \check{\tau} : L^X \rightarrow L$ and $\hat{\eta}, \check{\eta} : L^Y \rightarrow L$ as follows:

$$\hat{\tau}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \{\underline{0.1}, \underline{0.6}\} \\ 0, & \text{otherwise} \end{cases} \quad \check{\tau}(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \{\underline{0.1}, \underline{0.6}\} \\ 1, & \text{otherwise} \end{cases}$$

$$\hat{\eta}(\mu) = \begin{cases} 1, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.4} \\ 0, & \text{otherwise} \end{cases} \quad \check{\eta}(\mu) = \begin{cases} 0, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.4} \\ 1, & \text{otherwise} \end{cases}$$

(i) F is FUGe (resp. FLGe)-continuous, but not FUG δp (resp. FLG δp)-continuous because, $\hat{\eta}(\underline{0.4}) = \frac{1}{2} = \check{\eta}(\underline{0.4})$ in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.4}) = \underline{0.4}$ (resp. $F^l(\underline{0.4}) = \underline{0.4}$) is not $(\frac{1}{2}, \frac{1}{2})$ -gf δpo .

(ii) F is FUGe (resp. FLGe)-irresolute, but not FUG δp (resp. FUG δp)-irresolute because $\underline{0.4}$ is $(\frac{1}{2}, \frac{1}{2})$ -gf δpo in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.4}) = F^l(\underline{0.4}) = \underline{0.4}$ is not $(\frac{1}{2}, \frac{1}{2})$ -gf δpo .

Example 3.12. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $F : X \multimap Y$ be a FM defined by $G_F(x_1, y_1) = 0.2$, $G_F(x_1, y_2) = 0.8$, $G_F(x_1, y_3) = 0.3$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = 0.3$, $G_F(x_2, y_3) = 0.7$. Define double fuzzy topologies $\hat{\tau}, \check{\tau} : L^X \rightarrow L$ and $\hat{\eta}, \check{\eta} : L^Y \rightarrow L$ as follows:

$$\hat{\tau}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.8} \\ 0, & \text{otherwise} \end{cases} \quad \check{\tau}(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.8} \\ 1, & \text{otherwise} \end{cases}$$

$$\hat{\eta}(\mu) = \begin{cases} 1, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.2} \\ 0, & \text{otherwise} \end{cases} \quad \check{\eta}(\mu) = \begin{cases} 0, & \text{if } \mu \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \mu = \underline{0.2} \\ 1, & \text{otherwise} \end{cases}$$

(i) F is FUG β (resp. FLG β)-continuous but not FUGe (resp. FLGe)-continuous because $\hat{\eta}(\underline{0.2}) = \frac{1}{2} = \check{\eta}(\underline{0.2})$ in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.2}) = \underline{0.2}$ (resp. $F^l(\underline{0.2}) = \underline{0.2}$) is not $(\frac{1}{2}, \frac{1}{2})$ -gf eo .

(ii) F is FUG β (resp. FLG β)-irresolute but not FUGe (resp. FLGe)-irresolute because $\underline{0.2}$ is $(\frac{1}{2}, \frac{1}{2})$ -gf eo in $(Y, \hat{\eta}, \check{\eta})$ and $F^u(\underline{0.2}) = F^l(\underline{0.2}) = \underline{0.2}$ is not $(\frac{1}{2}, \frac{1}{2})$ -gf eo .

Corollary 3.13. Let $F : X \multimap Y$ be a FM between two dfts's $(X, \hat{\tau}, \check{\tau})$, $(Y, \hat{\eta}, \check{\eta})$. The following statements hold.

(i) If $(X, \hat{\tau}, \check{\tau})$ is (r, s) -Fge $T_{\frac{1}{2}}$ (resp. Fg $\delta sT_{\frac{1}{2}}$, Fg $\delta pT_{\frac{1}{2}}$ and Fg $\beta T_{\frac{1}{2}}$) then the concepts of FUGe (resp. FUG δs , FUG δp and FUG β)-continuous are equivalent. Also the concepts of FLGe (resp. FLG δs , FLG δp and FLG β)-continuous are equivalent.

- (ii) If $(Y, \hat{\eta}, \check{\eta})$ is (r, s) - $FgeT_{\frac{1}{2}}$ (resp. $Fg\delta sT_{\frac{1}{2}}$, $Fg\delta pT_{\frac{1}{2}}$ and $Fg\beta T_{\frac{1}{2}}$) then the concepts of $FUGe$ (resp. $FUG\delta s$, $FUG\delta p$ and $FUG\beta$)-continuous and $FUGe$ (resp. $FUG\delta s$, $FUG\delta p$ and $FUG\beta$)-irresolute are equivalent. Also the concepts of $FLGe$ (resp. $FLG\delta s$, $FLG\delta p$ and $FLG\beta$)-continuous and $FLGe$ (resp. $FLG\delta s$, $FLG\delta p$ and $FLG\beta$)-irresolute are equivalent.
- (iii) If $(X, \hat{\tau}, \check{\tau})$ and $(Y, \hat{\eta}, \check{\eta})$ are (r, s) - $FgeT_{\frac{1}{2}}$ (resp. $Fg\delta sT_{\frac{1}{2}}$, $Fg\delta pT_{\frac{1}{2}}$ and $Fg\beta T_{\frac{1}{2}}$) then the concepts of $FUGe$ (resp. $FUG\delta s$, $FUG\delta p$ and $FUG\beta$)-continuous, $FUGe$ (resp. $FUG\delta s$, $FUG\delta p$ and $FUG\beta$)-irresolute are equivalent. Also the concepts of $FLGe$ (resp. $FLG\delta s$, $FLG\delta p$ and $FLG\beta$)-continuous, $FLGe$ (resp. $FLG\delta s$, $FLG\delta p$ and $FLG\beta$)-irresolute are equivalent.

Remark 3.14. From the above examples it is clear that the following implications are true:

In general the converse are not true as shown in above examples.

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