On Šostak's fuzzy contra S-pre-continuous mappings

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Abstract

Fuzzy contra S-pre-continuous, fuzzy contra S-preopen, fuzzy contra S-preclosed mappings and fuzzy contra S-pre-homeomorphism with properties are introduced and studied in Šostak's sense. Also a comparison between these new types of functions are established by giving examples.

Keywords

fuzzy contra S-pre-continuous, fuzzy contra S-preopen, fuzzy contra S-preclosed.

AMS Subject Classification

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1. Introduction and Preliminaries

After the introduction of fuzzy sets by Zadeh [29], Chang[5] was the one who initiated the idea of a fuzzy topology in a set X , in which a collection, T , of open sets of fuzzy subsets of X . He also defined new type of fuzzy topology. In [7, 10], the authors defined the idea of gradation of openness of fuzzy subsets of X . They also used the concept of fuzzy topology in the sense of Chang. In 1980 in [11], the basic idea of topology is itself fuzzy. As in 1985 by the customized work of Kubiak [14] and Šostak [21] in Lattice $L = I$ was initiated in 1995 by [12] and further by [15] in 1997, in that topology was used to explain the two membership lattices namely L^X to M , where L and M are appropriate lattices. With all the basic ideas of topologies the basic lattices are fixed in [14, 21] and another significant generalizations of lattices L varies from space to space [13] is also represented. Using the developments of [11–15, 20–24], the notion of a fuzzy topology as a fuzzy subset of powerset is independently consider with the base

work of [11, 14, 21]. At present authors Chattopadhyay [7] et.al., Ramadan [19] and Ying [28] are also worked in this concept.

In this article, fuzzy contra S-pre-continuous, fuzzy contra S-preopen, fuzzy contra S-preclosed mappings and fuzzy contra S-pre-homeomorphism with properties are introduced and studied in Šostak's sense. Also a comparison between these new types of functions are established by giving examples.

X, Y etc., denotes the non empty sets, $I = [0, 1]$ and $I_0 = (0, 1]$. In 1992 Ramadan [19] given the definition of smooth topological spaces (in short, sts) and the operators $C_\tau : I^X \times I_0 \rightarrow I^X$ as $C_\tau(\lambda, r) = \bigwedge \{\mu \in I^X : \lambda \leq \mu, \tau(\bar{1} - \mu) \geq r\}$. and $I_\tau : I^X \times I_0 \rightarrow I^X$ as $I_\tau(\lambda, r) = \bigvee \{\mu \in I^X : \lambda \geq \mu, \tau(\mu) \geq r\}$. Kim et. al.[16] discussed in a sts (X, τ) , $\forall r \in I_0$. λ is called r -fuzzy strongly semiopen (resp. r -fuzzy semiopen & r -fuzzy preopen) (r -fsso (resp. r -fso & r -fpo) for short) iff $\lambda \leq I_\tau(C_\tau(I_\tau(\lambda, r), r), r)$ (resp. $\lambda \leq C_\tau(I_\tau(\lambda, r), r)$ & $\lambda \leq I_\tau(C_\tau(\lambda, r), r)$). The respective compliment sets are closed set.

The operators r -fuzzy strongly semi-interior (resp. r -fuzzy pre-interior, r -fuzzy strongly semi-closure and r -fuzzy pre-closure) of λ , denoted by $SSI_\tau(\lambda, r)$ (resp. $PI_\tau(\lambda, r)$, $SSC_\tau(\lambda, r)$ and $PC_\tau(\lambda, r)$) as $SSI_\tau(\lambda, r)$ (resp. $PI_\tau(\lambda, r)$) = $\bigvee \{\mu \in I^X : \mu \leq \lambda, \mu \text{ is } r\text{-fsso (resp. } r\text{-fpo)}\}$ (resp. $SSC_\tau(\lambda, r)$ (resp. $PC_\tau(\lambda, r)$) = $\bigwedge \{\mu \in I^X : \mu \geq \lambda, \mu \text{ is } r\text{-fssc (resp. } r\text{-fpc)}\}$.

$f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy continuous (FCts for short) iff $\eta(\mu) \leq \tau(f^{-1}(\mu))$ for each $\mu \in I^Y$. $f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy semi continuous, (resp. fuzzy irresolute and fuzzy S-irresolute continuous) (FSCTs (resp. FI and FS-ICTs) for short) iff $f^{-1}(\mu)$ is r -fso (resp. r -fso and r -fssso) set of X for each $\eta(\mu) \geq r$, (resp. μ is r -fso and r -fssso) set of Y $r \in I_0$. $f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy open (resp. fuzzy closed) (FO (resp. FC) for short) iff $\tau(\lambda) \leq \eta(f(\lambda))$ (resp. $\tau(\bar{1} - \lambda) \leq \eta(\bar{1} - f(\lambda))$) for each $\lambda \in I^X$. $f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy S-irresolute open (resp. fuzzy strongly semi-open and fuzzy strongly semi-closed) (FS-IO (resp. FsSO and FsSCC) for short) iff $f(\mu)$ is r -fssso (resp. r -fssso and r -fssc) set of Y for each r -fssso (resp. $\tau(\mu) \geq r$ & $\tau(\bar{1} - \mu) \geq r$) set $r \in I_0$.

$f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy irresolute preopen (resp. fuzzy irresolute preclosed) [25] iff $f(\mu)$ is r -fpo (resp. r -fpc) set of Y , for each r -fpo (resp. r -fpc) set $\mu \in I^Y$, $r \in I_0$.

$f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy contra continuous [2] (resp. fuzzy contra pre-continuous [8]) (FcCts (resp. FcPCts) for short) if for each $\mu \in I^Y$ (resp. $\eta(\mu) \geq r$) we have $\tau(\bar{1} - f^{-1}(\mu)) \geq \eta(\mu)$ (resp. $f^{-1}(\mu)$ is r -fpc). $f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy contra open (resp. fuzzy contra closed) (FcO (resp. FcC) for short) if for each $\lambda \in I^X$ we have $\eta(\bar{1} - f(\lambda)) \geq \tau(\lambda)$ (resp. $\eta(f(\lambda)) \geq \tau(\bar{1} - \lambda)$). $f : (X, \tau) \rightarrow (Y, \eta)$ is fuzzy contra S-irresolute continuous [26] (resp. fuzzy contra semi-S-irresolute continuous [27]) (FcS-ICTs (resp. FcSS-ICTs) for short) iff $f^{-1}(\mu)$ is r -fssc (resp. r -fsc) set of X for each r -fssso set $\mu \in I^Y$, $r \in I_0$.

The pair (X, τ) is called a smooth topological space (or) Šostak Fuzzy topological space (or) fuzzy topological spaces (in short, sts, sfts, fts).

2. Fuzzy Contra S-pre-continuous Mappings

Definition 2.1. A function $f : (X, \tau) \rightarrow (Y, \eta)$ is called

- (1) fuzzy contra S-pre-continuous (FcS-PCts, for short) iff $f^{-1}(\mu)$ is r -fpc set of X $\forall$ r -fssso set $\mu \in I^Y$, $r \in I_0$.
- (2) fuzzy contra S-preopen (resp. fuzzy contra S-preclosed) (FcS-PO (resp. FcS-PC), for short) iff $f(\mu)$ is r -fpc (resp. r -fpo) set of Y $\forall$ r -fssso (resp. r -fssc) set $\mu \in I^X$, $r \in I_0$.
- (3) fuzzy contra S-pre homeomorphism (FcS-PH, for short) iff f is bijective and both of f & f^{-1} are fuzzy contra S-pre-continuous.

Based on the definitions,

Remark 2.2. The reverse implications not true in general, which is shown in the succeeding example.

Example 2.3. Consider the fts's (X, τ) & (Y, η) with $X = Y = \{x, y, z\}$ and

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1} \\ \frac{1}{2}, & \text{if } \lambda = \alpha_1 \\ 0, & \text{otherwise} \end{cases}$$

$$\eta(\mu) = \begin{cases} 1, & \text{if } \mu = \bar{0} \text{ or } \bar{1} \\ \frac{1}{2}, & \text{if } \mu = \alpha_2 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha_1(x) = 0.2$, $\alpha_1(y) = 0.1$, $\alpha_1(z) = 0.5$; $\alpha_2(x) = 0.7$, $\alpha_2(y) = 0.9$, $\alpha_2(z) = 0.6$; $\alpha_3(x) = 0.8$, $\alpha_3(y) = 1$, $\alpha_3(z) = 0.7$.

Then the identity function $F : X \rightarrow Y$ is

- (i) FcS-PCts but not FcS-ICTs because α_3 is $\frac{1}{2}$ -fssso in (Y, η) but $f^{-1}(\alpha_3) = \alpha_3$ is not $\frac{1}{2}$ -fssc in (X, τ) .
- (ii) FcS-PCts but not FcSS-ICTs continuous because α_3 is $\frac{1}{2}$ -fssso in (Y, η) but $f^{-1}(\alpha_3) = \alpha_3$ is not $\frac{1}{2}$ -fsc in (X, τ) .

Example 2.4. Consider the fts's (X, τ) & (Y, η) with $X = Y = \{x, y, z\}$ and

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1} \\ \frac{1}{2}, & \text{if } \lambda = \alpha_1 \\ 0, & \text{otherwise} \end{cases}$$

$$\eta(\mu) = \begin{cases} 1, & \text{if } \mu = \bar{0} \text{ or } \bar{1} \\ \frac{1}{2}, & \text{if } \mu = \alpha_2 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha_1(x) = 0.2$, $\alpha_1(y) = 0.1$, $\alpha_1(z) = 0$; $\alpha_2(x) = 0.5$, $\alpha_2(y) = 0.7$, $\alpha_2(z) = 0$.

Then the identity function $f : X \rightarrow Y$ is FcSS-ICTs but not FcS-ICTs because α_2 is $\frac{1}{2}$ -fssso in (Y, η) but $f^{-1}(\alpha_2) = \alpha_2$ is not $\frac{1}{2}$ -fssc in (X, τ) .

Theorem 2.5. For a mapping $f : (X, \tau_1) \rightarrow (Y, \tau_2)$, the succeeding statements are equivalent.

- (1) f is FcS-PCts.
- (2) $\forall$ r -fssc $v \in I^Y$, $f^{-1}(v)$ is r -fpo.
- (3) $f(I_{\tau_1}(C_{\tau_1}(\mu, r), r)) \geq SSC_{\tau_2}(f(\mu), r)$, for each $\mu \in I^X$ and $r \in I_0$.
- (4) $I_{\tau_1}(C_{\tau_1}(f^{-1}(v), r), r) \geq f^{-1}(SSC_{\tau_2}(v, r))$, for each $v \in I^Y$ and $r \in I_0$.
- (5) $f^{-1}(v) \geq C_{\tau_1}(I_{\tau_1}(f^{-1}(v), r), r)$, for each r -fssso set $v \in I^Y$ and $r \in I_0$.

Proof. Using the fact $f^{-1}(\bar{I} - v) = \bar{I} - f^{-1}(v)$, (1) $\Rightarrow$ (2), can be obtained directly.

(2) $\Rightarrow$ (3): Suppose $\exists \mu \in I^Y$ & $r \in I_0 \ni$

$$f(I_{\tau_1}(C_{\tau_1}(\mu, r), r)) \not\geq SSC_{\tau_2}(f(\mu), r).$$

There exists $y \in Y$ & $t \in (0, 1) \ni f(I_{\tau_1}(C_{\tau_1}(\mu, r), r))(y) < t < SSC_{\tau_2}(f(\mu), r)(y)$.

If $f^{-1}(\{y\}) = \emptyset$, it contradicts $f(I_{\tau_1}(C_{\tau_1}(\mu, r), r))(y) = 0$.

If $f^{-1}(\{y\}) \neq \emptyset$, $\exists x \in f^{-1}(\{y\})$ such that

$$\begin{aligned} f(I_{\tau_1}(C_{\tau_1}(\mu, r), r))(y) &\leq I_{\tau_1}(C_{\tau_1}(\mu, r), r)(x) \\ &< t \\ &< SSC_{\tau_2}(f(\mu), r)(f(x)) \quad (2.1) \end{aligned}$$

Since $SSC_{\tau_2}(f(\mu), r)(f(x)) > t$, $\exists r$ -fssc $v \in I^Y$ with $f(\mu) \geq v$ such that

$$SSC_{\tau_2}(f(\mu), r)(f(x)) \geq v(f(x)) > t.$$

And $f(\mu) \geq v$ then $\mu \geq f^{-1}(v)$.

(2) $\Rightarrow f^{-1}(v)$ is r -fpo. Hence

$$I_{\tau_1}(C_{\tau_1}(\mu, r), r)(x) \geq f^{-1}(v)(x) = v(f(x)) > t.$$

It contradicts equation (2.1).

Thus $f(I_{\tau_1}(C_{\tau_1}(\mu, r), r)) \geq SSC_{\tau_2}(f(\mu), r)$.

(3) $\Rightarrow$ (4): $\forall v \in I^Y$, $r \in I_0$, put $\mu = f^{-1}(v)$. From (3),

$$\begin{aligned} f(I_{\tau_1}(C_{\tau_1}(f^{-1}(v), r), r)) &\geq SSC_{\tau_2}(f(f^{-1}(v)), r) \\ &\geq SSC_{\tau_2}(v, r) \end{aligned}$$

It implies

$$\begin{aligned} I_{\tau_1}(C_{\tau_1}(f^{-1}(v), r), r) &\geq f^{-1}(I_{\tau_1}(C_{\tau_1}(f^{-1}(v), r), r)) \\ &\geq f^{-1}(SSC_{\tau_2}(v, r)). \end{aligned}$$

Thus proved (4).

(4) $\Rightarrow$ (3): Let $\mu \in I^X$ & $r \in I_0$. By (4),

$$\begin{aligned} I_{\tau_1}(C_{\tau_1}(\mu, r), r) &\geq I_{\tau_1}(C_{\tau_1}(f^{-1}(f(\mu)), r), r) \\ &\geq f^{-1}(SSC_{\tau_2}(f(\mu), r)) \end{aligned}$$

and hence $f(I_{\tau_1}(C_{\tau_1}(\mu, r), r)) \geq SSC_{\tau_2}(f(\mu), r)$.

Thus (3) is proved.

(3) $\Leftrightarrow$ (1): Take $v \in I^Y$ be a r -fsso set.

Since $f^{-1}(\bar{I} - v) = \bar{I} - f^{-1}(v)$, by (3),

$$\begin{aligned} f(I_{\tau_1}(C_{\tau_1}(f^{-1}(\bar{I} - v), r), r)) &\geq SSC_{\tau_2}(f(f^{-1}(\bar{I} - v), r)) \\ &\geq SSC_{\tau_2}(\bar{I} - v, r) \\ &= \bar{I} - SSI_{\tau_2}(v, r) \\ &\geq \bar{I} - v \end{aligned}$$

and hence $\bar{I} - C_{\tau_1}(I_{\tau_1}(f^{-1}(v), r), r)$

$$\begin{aligned} &= I_{\tau_1}(C_{\tau_1}(\bar{I} - f^{-1}(v), r), r) \\ &= I_{\tau_1}(C_{\tau_1}(f^{-1}(\bar{I} - v), r), r) \\ &\geq f^{-1}f(I_{\tau_1}(C_{\tau_1}(f^{-1}(\bar{I} - v), r), r)) \\ &\geq f^{-1}(\bar{I} - v) \\ &\geq \bar{I} - f^{-1}(v) \end{aligned}$$

Therefore, we obtain

$$f^{-1}(v) \geq C_{\tau_1}(I_{\tau_1}(f^{-1}(v), r), r)$$

$\Rightarrow f^{-1}(v)$ is r -fpc set in X .

Hence the mapping f is FcS -PCts. $\square$

Theorem 2.6. Let $f : (X, \tau) \rightarrow (Y, \eta)$ be a bijective mapping. Then (1) & (2) are equivalent.

(1) f is FcS -PCts.

(2) $SSI_{\eta}(f(\mu), r) \geq f(C_{\tau}(I_{\tau}(\mu, r), r))$, $\forall \mu \in I^X$ and $r \in I_0$.

Proof. (1) $\Rightarrow$ (2): Let f be a FcS -PCts mapping and $\mu \in I^X$. Then $f^{-1}(SSI_{\eta}(f(\mu), r))$ is r -fpc set in X .

Since f is 1-1, and by Theorem 2.5, $f^{-1}(SSI_{\eta}(v, r)) \geq C_{\tau}(I_{\tau}(f^{-1}(v), r), r)$. Let $v = f(\mu)$. Then,

$$\begin{aligned} f^{-1}(SSI_{\eta}(f(\mu), r)) &\geq C_{\tau}(I_{\tau}(f^{-1}(f(\mu), r), r)) \\ &= C_{\tau}(I_{\tau}(\mu, r), r). \end{aligned}$$

But, f is onto, so

$$\begin{aligned} SSI_{\eta}(f(\mu), r) &= ff^{-1}(SSI_{\eta}(f(\mu), r)) \\ &\geq f(C_{\tau}(I_{\tau}(\mu, r), r)). \end{aligned}$$

Thus (2) is proved.

(2) $\Rightarrow$ (1): Consider $v \in I^Y$ be a r -fsso set. Using the Theorem in [16], $v = SSI_{\eta}(v, r)$. By (2),

$$\begin{aligned} f(C_{\tau}(I_{\tau}(f^{-1}(v), r), r)) &= SSI_{\eta}(f(f^{-1}(v), r)) \\ &= SSI_{\eta}(v, r) \\ &= v \end{aligned}$$

and

$$\begin{aligned} C_{\tau}(I_{\tau}(f^{-1}(v), r), r) &= f^{-1}f(C_{\tau}(I_{\tau}(f^{-1}(v), r), r)) \\ &\leq f^{-1}(v). \end{aligned}$$

Hence, $C_{\tau}(I_{\tau}(f^{-1}(v), r), r) \leq f^{-1}(v) \Rightarrow f^{-1}(v)$ is r -fpc. Thus (1). $\square$

Theorem 2.7. For a mapping $f : (X, \tau) \rightarrow (Y, \eta)$, the statements

(1) f is FcS -PO.

(2) $f(SS I_{\tau}(\mu, r)) \leq C_{\eta}(I_{\eta}(f(\mu), r), r)$, $\forall \mu \in I^X$ & $r \in I_0$.

(3) $SSI_{\tau}(f^{-1}(v), r) \leq f^{-1}(C_{\eta}(I_{\eta}(v, r), r))$, $\forall v \in I^Y$ & $r \in I_0$.

(4) For any r -fssc $v \in I^Y$ with $f^{-1}(v) \leq \mu$, $\exists r$ -fpo $\rho \in I^Y$ with $v \leq \rho \ni f^{-1}(\rho) \leq \mu$.

are equivalent.

Proof. (1) $\Rightarrow$ (2): Let f be FcS-PO, $\forall \mu \in I^X$, since, $SSI_\tau(\mu, r) \leq \mu$ from Theorem in [16],

$$f(SSI_\tau(\mu, r)) \leq f(\mu).$$

From (1), $f(SSI_\tau(\mu, r))$ is r -fpc. Thus $f(SSI_\tau(\mu, r)) \leq C_\eta(I_\eta(f(\mu), r), r)$. Thus proved (2).

(2) $\Rightarrow$ (3): $\forall v \in I^Y, r \in I_0$ let $\mu = f^{-1}(v)$ from (2). Then,

$$\begin{aligned} f(SSI_\tau(f^{-1}(v), r)) &\leq C_\eta(I_\eta(f(f^{-1}(v)), r), r) \\ &\leq C_\eta(I_\eta(v, r), r). \end{aligned}$$

It implies $SSI_\tau(f^{-1}(v), r) \leq f^{-1}(C_\eta(I_\eta(v, r), r))$.

(3) $\Rightarrow$ (4): Take a r -fssc set $\mu \in I^X \ni f^{-1}(v) \leq \mu$. Since $\bar{1} - \mu \leq f^{-1}(\bar{1} - v)$ and $SSI_\tau(\bar{1} - \mu, r) = \bar{1} - \mu$.

$$SSI_\tau(\bar{1} - \mu, r) = \bar{1} - \mu \leq SSI_\tau(f^{-1}(\bar{1} - v), r).$$

(3) implies,

$$\bar{1} - \mu \leq SSI_\tau(f^{-1}(\bar{1} - v), r) \leq f^{-1}(C_\eta(I_\eta(\bar{1} - v), r), r).$$

It implies

$$\begin{aligned} \mu &\geq \bar{1} - f^{-1}(C_\eta(I_\eta(\bar{1} - v), r), r) \\ &= f^{-1}(\bar{1} - C_\eta(I_\eta(\bar{1} - v), r), r) \\ \mu &\geq f^{-1}(I_\eta(C_\eta(v, r), r)) \end{aligned}$$

Thus a r -fpo set $v \in I^Y$ with $v \leq I_\eta(C_\eta(v, r), r) \ni$

$$f^{-1}(I_\eta(C_\eta(v, r), r)) \leq \mu$$

exists. Thus (4) is proved.

(4) $\Rightarrow$ (1): Take a r -fsso set $\omega \in I^X$. Now, we have to prove, $f(\omega) \in I^Y$ is r -fpc. Let $v = \bar{1} - f(\omega)$ & $\mu = \bar{1} - \omega \ni \mu$ is r -fssc. Therefore

$$\begin{aligned} f^{-1}(v) &= f^{-1}(\bar{1} - f(\omega)) = \bar{1} - f^{-1}(f(\omega)) \leq \bar{1} - \omega = \mu \\ \Rightarrow f^{-1}(v) &\leq \mu. \end{aligned}$$

(4) implies, a r -fpo set ρ with $v \leq \rho \ni f^{-1}(\rho) \leq \mu = \bar{1} - \omega$ exists. It implies

$$\omega \leq \bar{1} - f^{-1}(\rho) = f^{-1}(\bar{1} - \rho).$$

Thus,

$$f(\omega) \leq f(f^{-1}(\bar{1} - \rho)) \leq \bar{1} - \rho \quad (2.2)$$

On the other hand, since $v \leq \rho$,

$$f(\omega) = \bar{1} - v \geq \bar{1} - \rho \quad (2.3)$$

Hence from equations (2.2) and (2.3), we get, $f(\omega) = \bar{1} - \rho$.

(i.e) $f(\omega)$ is r -fpc. Thus (1). $\square$

Theorem 2.8. For a mapping $f : (X, \tau) \rightarrow (Y, \eta)$, the statements

(1) f is FcS-PC

(2) $f(SSC_\tau(\mu, r)) \geq I_\eta(C_\eta(f(\mu), r), r) \forall \mu \in I^X$ and $r \in I_0$.

(3) $SSC_\tau(f^{-1}(v), r) \geq f^{-1}(I_\eta(C_\eta(v, r), r)), \forall v \in I^Y$ and $r \in I_0$.

(4) For any $v \in I^Y$ & r -fsso $\mu \in I^X$ with $f^{-1}(v) \leq \mu, \exists r$ -fpc, $\rho \in I^Y$ with $v \geq \rho \ni f^{-1}(\rho) \geq \mu$.

are equivalent.

Proof. (1) $\Rightarrow$ (2): Let f be FcS-PC. For each $\mu \in I^X$ and $r \in I_0$, since $SSC_\tau(\mu, r) \geq \mu$ from Theorem ??(5), Then

$$f(SSC_\tau(\mu, r)) \geq f(\mu).$$

From (1), $f(SSC_\tau(\mu, r))$ is r -fpo. Therefore $f(SSC_\tau(\mu, r)) \geq I_\eta(C_\eta(f(\mu), r), r)$. Thus (2) is proved.

(2) $\Rightarrow$ (3): $\forall v \in I^Y, r \in I_0$ Take $\mu = f^{-1}(v)$ in (2) implies that,

$$\begin{aligned} f(SSC_\tau(f^{-1}(v), r)) &\geq I_\eta(C_\eta(f(f^{-1}(v)), r), r) \\ &\geq I_\eta(C_\eta(v, r), r). \end{aligned}$$

It implies $SSC_\tau(f^{-1}(v), r) \geq f^{-1}(I_\eta(C_\eta(v, r), r))$.

Thus proved (3).

(3) $\Rightarrow$ (4): Let μ be r -fsso set of X such that $f^{-1}(v) \leq \mu$. Since $\bar{1} - \mu \geq f^{-1}(\bar{1} - v)$ and $SSC_\tau(\bar{1} - \mu, r) = \bar{1} - \mu$ $SSC_\tau(\bar{1} - \mu, r) = \bar{1} - \mu \geq SSC_\tau(f^{-1}(\bar{1} - v), r)$. From (3),

$$\begin{aligned} \bar{1} - \mu &\geq SSC_\tau(f^{-1}(\bar{1} - v), r) \\ &\geq f^{-1}(I_\eta(C_\eta(\bar{1} - v), r), r) \end{aligned}$$

It implies

$$\begin{aligned} \mu &\leq \bar{1} - f^{-1}(I_\eta(C_\eta(\bar{1} - v), r), r) \\ &= f^{-1}(\bar{1} - I_\eta(C_\eta(\bar{1} - v), r), r) \\ \mu &\leq f^{-1}(C_\eta(I_\eta(v, r), r)) \end{aligned}$$

Therefore a r -fpc set $v \in I^Y$, with $v \geq C_\eta(I_\eta(v, r), r) \ni f^{-1}(C_\eta(I_\eta(v, r), r)) \geq \mu$ exists.

(4) $\Rightarrow$ (1): Take a r -fssc set $\omega \in I^X$ and $v = \bar{1} - f(\omega)$ and $\mu = \bar{1} - \omega \ni \mu$ is r -fsso. We obtain

$$f^{-1}(v) = f^{-1}(\bar{1} - f(\omega)) = \bar{1} - f^{-1}(f(\omega)) \leq \bar{1} - \omega = \mu.$$

(4) implies, a r -fpc set $\rho \in I^Y$ with $v \geq \rho \ni f^{-1}(\rho) \geq \mu = \bar{1} - \omega$ exists. Thus $\omega \geq \bar{1} - f^{-1}(\rho) = f^{-1}(\bar{1} - \rho)$. Hence

$$f(\omega) \geq f(f^{-1}(\bar{1} - \rho)) = \bar{1} - \rho. \quad (2.4)$$

On the other hand, since $v \leq \rho$,

$$f(\omega) = \bar{1} - v \leq \bar{1} - \rho \quad (2.5)$$

Hence from (2.4) and (2.5), we have $f(\omega) = \bar{1} - \rho$.

(i.e) $f(\omega)$ is r -fpo. Thus (1). $\square$

Theorem 2.9. For a bijective mapping $f : (X, \tau) \rightarrow (Y, \eta)$,

(1) f is a FcS-PC iff $f^{-1}(I_\eta(C_\eta(v, r), r)) \leq SSC_\tau(f^{-1}(v), r), \forall v \in I^Y$ and $r \in I_0$.

(2) f is a FcS-PC iff f is FcS-PO.

Proof. (1) $\Rightarrow$: Let f be FcS-PC. Theorem 2.8(2), implies $\forall \mu \in I^X$ & $r \in I_0$,

$$f(SSC_\tau(\mu, r)) \geq I_\eta(C_\eta(f(\mu), r), r).$$

$\forall v \in I^Y, r \in I_0$. Take $\mu = f^{-1}(v)$, and the fact f is surjective implies, $f(f^{-1}(v)) = v$. Thus,

$$\begin{aligned} f(SSC_\tau(f^{-1}(v), r)) &\geq I_\eta(C_\eta(f(f^{-1}(v)), r), r) \\ &= I_\eta(C_\eta(v, r), r). \end{aligned}$$

It implies

$$\begin{aligned} SSC_\tau(f^{-1}(v), r) &= f^{-1}(f(SSC_\tau(f^{-1}(v), r))) \\ &\geq f^{-1}(I_\eta(C_\eta(v, r), r)). \end{aligned}$$

($\Leftarrow$) Take $v = f(\mu)$. Since f is 1-1,

$$\begin{aligned} f^{-1}(I_\eta(C_\eta(f(\mu), r), r)) &\leq SSC_\tau(f^{-1}(f(\mu)), r) \\ &= SSC_\tau(\mu, r). \end{aligned}$$

Since f is surjective,

$$I_\eta(C_\eta(f(\mu), r), r) \leq f(SSC_\tau(\mu, r)).$$

$\Rightarrow f$ is FcS-PC.

(2) It is easily proved from :

$$\begin{aligned} f^{-1}(I_\eta(C_\eta(v, r), r)) &\leq SSC_\tau(f^{-1}(v), r) \\ \Leftrightarrow f^{-1}(\bar{1} - C_\eta(I_\eta(\bar{1} - v, r), r)) &\leq \bar{1} - SSI_\tau(\bar{1} - f^{-1}(v), r) \\ \Leftrightarrow \bar{1} - f^{-1}(C_\eta(I_\eta(\bar{1} - v, r), r)) &\leq \bar{1} - SSI_\tau(f^{-1}(\bar{1} - v), r) \\ \Leftrightarrow f^{-1}(C_\eta(I_\eta(\bar{1} - v, r), r)) &\geq SSI_\tau(f^{-1}(\bar{1} - v), r) \\ \Leftrightarrow \text{Therefore } f \text{ is FcS-PO.} \end{aligned} \quad \square$$

Theorem 2.10. (1) If $f : X \rightarrow Y$ is FS-IO and $g : Y \rightarrow Z$ is FcS-PO, then $g \circ f : X \rightarrow Z$ is FcS-PO.

(2) If $f : X \rightarrow Y$ is FS-IC and $g : Y \rightarrow Z$ is FcS-PC, then $g \circ f : X \rightarrow Z$ is FcS-PC.

(3) If $f : X \rightarrow Y$ is FcS-PO and $g : Y \rightarrow Z$ is FIPC, then $g \circ f : X \rightarrow Z$ is FcS-PO.

(4) If $f : X \rightarrow Y$ is FcS-PC and $g : Y \rightarrow Z$ is FIPO, then $g \circ f : X \rightarrow Z$ is FcS-PC.

(5) If $f : X \rightarrow Y$ is FcS-PCts and $g : Y \rightarrow Z$ is FS-SCts, then $g \circ f : X \rightarrow Z$ is FcPCts.

(6) If $f : X \rightarrow Y$ is FcS-PCts and $g : Y \rightarrow Z$ is FS-ICts, then $g \circ f : X \rightarrow Z$ is FcS-PCts.

Proof. (1) Let λ be r -fsso set in X . As f is FS-IO, $f(\lambda)$ is r -fsso set of Y . Also since $g : Y \rightarrow Z$ is FcS-PO, $g(f(\lambda))$ is r -fpc in Z . (i.e) $(g \circ f)(\lambda) = g(f(\lambda))$ is r -fpc in Z . Thus $g \circ f$ is FcS-PO.

(2) - (6): Similar proof can be given. $\square$

3. Conclusion

In this article, fuzzy contra S-pre-continuous, fuzzy contra S-preopen, fuzzy contra S-preclosed mappings and fuzzy contra S-pre-homeomorphism with properties have been introduced and studied in Šostak's sense. Also a comparison between these new types of functions have been established by giving examples.

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