



On supra ideal topological space via R-I-open sets

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Abstract

In this paper, we introduce a new class of sets and functions in a supra ideal topological space. We give characterizations of supra R-I-open sets and study supra R-I-continuous functions, supra* R-I-continuous functions, supra R-I-irresolute functions, supra R-I-open maps, supra R-I-closed maps, supra R-I-homeomorphism. Separation axioms in terms of supra R-I-open sets are also studied.

Keywords

supra R-I-open sets, supra R-I-continuous functions, supra R-I-open maps, supra R-I-homeomorphism, supra R-I-separation axioms.

AMS Subject Classification

54A05, 54C08, 54C10.

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1. Introduction

The concept of ideal in topological space was first introduced by Kuratowski and Vaidyanathswamy [10]. They have also defined local function in ideal topological space. Further Hamlett and Jankovic in [3] and [9] studied the properties of ideal topological spaces. In 1983, A.S. Mashhour [2] introduced supra topological space and studied s-continuous maps and s*-continuous maps. On 2010, O.R.Sayed and T.Noiri [5] studied supra b-open sets and supra b-continuity. On 2016, Anuradha N. and Baby Chacko [4] studied supra r-open sets and supra r-continuity. In this space, we have introduced via ideal, supra R-I-open sets and supra R-I-continuous functions. Further we have discussed certain properties of the same and new separation axioms are introduced.

2. Preliminaries

By a space (X, τ) , we mean a topological space with a topology τ defined on X on which no separation axioms are assumed unless otherwise explicitly stated. For a given point x in a space (X, τ) , the system of open neighborhoods of x is denoted by $N(x) = \{U \in \tau : x \in U\}$. For a given subset A of a space (X, τ) , $Cl(A)$ and $Int(A)$ are used to denote the closure of A and interior of A , respectively, with respect to the topology.

A nonempty collection of subsets of a set X is said to be an ideal I on X , if it satisfies the following two conditions: (i) If $A \in I$ and $B \subset A$, then $B \in I$; (ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$. An ideal topological space (or ideal space) (X, τ, I) means a topological space (X, τ) with an ideal I defined on X . Let (X, τ) be a topological space with an ideal I defined on X . Then for any subset A of X , $A^*(I, \tau) = \{x \in X / A \cap U \notin I \text{ for every } U \in N(x)\}$ is called the local function of A with respect to I and τ . If there is no ambiguity, we will write $A^*(I)$ or simply A^* for $A^*(I, \tau)$. Also, $Cl^*(A) = A \cup A^*$. It defines a Kuratowski closure operator for the topology $\tau^*(I)$ (or simply τ^*) which is finer than τ . [3]

A subfamily μ of the power set $P(X)$ of a nonempty set X is called a supra topology on X if μ satisfies the following conditions: 1. μ contains ϕ and X , 2. μ is closed under the arbitrary union. The pair (X, μ) is called a supra topological space. If (X, τ) is a topological space and $\tau \subset \mu$, then μ is known as supra topology associated with τ . The member of μ

is called supra open set in (X, μ) . The complement of supra open set is called supra closed set.

Let (X, μ) be a supra topological space and $A \subset X$. Then supra interior and supra closure of A in (X, μ) defined as $\cup\{U : U \subseteq A, U \in \mu\}$ and $\cap\{F : A \subseteq F, X - F \in \mu\}$ respectively. The supra interior and supra closure of A in (X, μ) are denoted as $Int^\mu(A)$ and $Cl^\mu(A)$ respectively. From definition, $Int^\mu(A)$ is a supra open set and $Cl^\mu(A)$ is a supra closed set. [2]

A supra topological space (X, μ) with an ideal I on X is called an **ideal supra topological space** and denoted as (X, μ, I) . [8]

Definition 2.1. [8] Let (X, μ, I) be an ideal supra topological space. A set operator $(\cdot)^{\mu*} : P(X) \rightarrow P(X)$, is called the μ -local function of I on X with respect to μ , is defined as $(A)^{\mu*}(I, \mu) = \{x \in X : U \cap A \notin I, \text{ for every } U \in \mu(x)\}$, where $\mu(x) = \{U \in \mu : x \in U\}$. This is simply called μ -local function and simply denoted as $A^{\mu*}$.

Definition 2.2. [6] A subset A of an ideal topological space (X, τ, I) is said to be **R-I-open** (resp. regular open) if $Int(Cl^*(A)) = A$ (resp. $Int(Cl(A)) = A$). We call a subset A of (X, τ, I) is R-I-closed if its complement is R-I-open.

Lemma 2.3. [6] Let A and B be subsets of an ideal topological space (X, τ, I) . Then the following properties hold:

- (1) $Int(Cl^*(A))$ is R-I-open.
- (2) If A and B are R-I-open, then $A \cap B$ is R-I-open.
- (3) If A is regular open, then it is R-I-open.

Definition 2.4. [1] A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be **R-I-continuous** if for each $x \in X$ and for open set $V \in \sigma$ containing $f(x)$, there exists a R-I open set $U \subset X$ containing x such that $f(U) \subset V$.

Definition 2.5. [7] A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be **completely I-continuous** if $f^{-1}(U)$ is a R-I open set in X for every open set $U \subset Y$.

Definition 2.6. [7] A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be **almost completely I-continuous** if $f^{-1}(U)$ is a R-I open set in X for every R-J open set $U \subset Y$.

Definition 2.7. [7] A function $f : X \rightarrow Y$ is said to be **almost perfectly I-continuous** if $f^{-1}(V)$ is clopen for every R-I open set V in Y .

Definition 2.8. A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be **totally I-continuous** if inverse image of each open set of Y is clopen in X .

3. SUPRA R-I-OPEN SETS

Definition 3.1. Let (X, μ, I) be an ideal supra topological space. A set A is called supra R-I-open if $A = Int^\mu(Cl^*(A))$, the complement of a supra R-I-open set is called a supra R-I-closed set.

Example 3.2. Consider (X, τ^*, I) where $X = \{a, b, c\}$, $\tau^* = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\emptyset, \{a\}\}$. Then $\{a\}$ and $\{b\}$ are supra R-I open sets.

Remark 3.3. Every R-I-open set is supra R-I-open.

Remark 3.4. Every regular open set is supra R-I-open.

Theorem 3.5. Every supra R-I-open set is supra open.

Proof. Since every R-I-open set is open, every supra R-I-open set is supra open. $\square$

Remark 3.6. Converse of the above theorem need not be true.

For example, in example 3.2, $\{a, b\}$ is supra open but not supra R-I-open.

Theorem 3.7. If supra topology equals discrete topology, then every supra open set is supra R-I-open.

Proof. In discrete topology, every open set is R-I-open and hence the result. $\square$

Theorem 3.8. Supra R-I-open sets possess the following properties:

- (i) Finite union of supra R-I-open sets need not be supra R-I-open.
- (ii) Finite intersection of supra R-I-open sets is supra R-I-open.

Proof. (i) In the example 3.2, $\{a\}$ and $\{b\}$ are supra R-I open sets but $\{a, b\}$ is not supra R-I-open.

- (ii) Consider two supra supra R-I-open sets A and B . Then $A = Int^\mu(Cl^*(A))$ and $B = Int^\mu(Cl^*(B))$.
Now $A \cap B = Int^\mu(Cl^*(A)) \cap Int^\mu(Cl^*(B))$
 $= Int^\mu(Cl^*(A) \cap Cl^*(B))$
 $\supset Int^\mu(Cl^*(A \cap B)) \supset Int^\mu(A \cap B)$
 $= Int^\mu(A) \cap Int^\mu(B) = A \cap B$, since A and B are also supra open. $\square$

Theorem 3.9. Supra R-I-closed sets possess the following properties:

- (i) Finite union of supra R-I-closed sets is supra R-I-closed.
- (ii) Arbitrary intersection of supra R-I-closed sets need not be supra R-I-closed.

Proof. (i) Let C and D be supra R-I-closed sets. Then $X - C$ and $X - D$ are supra R-I-open and hence $(X - C) \cap (X - D)$ is supra R-I-open. i.e; $X - (C \cup D)$ is supra R-I-open. Hence $(C \cup D)$ is supra R-I-closed.

- (ii) In the example 3.2, $\{a, c\}$ and $\{b, c\}$ are supra R-I-closed but $\{c\}$ is not supra R-I-closed. $\square$



Remark 3.10. The intersection of a supra R-I-open set and a supra open set need not a supra open set.

Example 3.11. Let $X = \{x, y, z\}$, $I = \{\phi, \{x\}\}$, $\mu = \{\phi, X, \{x\}, \{x, z\}, \{y, z\}\}$. Here $\{x\}$ and $\{x, z\}$ are supra R-I-open sets. Consider $\{x, z\} \cap \{y, z\} = \{z\}$, which is not supra open.

Definition 3.12. Supra R-I-closure of a set A denoted by $\text{supra } R\text{-I-Cl}(A)$ is the intersection of all supra R-I-closed sets containing A .

Definition 3.13. Supra R-I-interior of a set A denoted by $\text{supra } R\text{-I-Int}(A)$ is the union of all supra R-I-open sets contained in A .

Example 3.14. Consider (X, μ, I) where $X = \{a, b, c\}$, $\mu = \{\phi, X, \{a\}, \{a, c\}, \{b, c\}\}$, $I = \{\phi, \{b\}\}$. Then $\text{supra } R\text{-I-Cl}(\{b\}) = \{b, c\}$ and $\text{supra } R\text{-I-Int}(\{b\}) = \phi$.

Theorem 3.15. The following properties are satisfied by $\text{supra } R\text{-I-Int}(A)$ and $\text{supra } R\text{-I-Cl}(A)$:

- (i) $\text{supra } R\text{-I-Int}(A)$ is a supra R-I-open set.
- (ii) $\text{supra } R\text{-I-Cl}(A)$ is a supra R-I-closed set.
- (iii) $\text{supra } R\text{-I-Int}(A) \subseteq A$ and equality holds if and only if A is a supra R-I-open set.
- (iv) $A \subseteq \text{supra } R\text{-I-Cl}(A)$ and equality holds if and only if A is a supra R-I-closed set.
- (v) $X - \text{supra } R\text{-I-Int}(A) = \text{supra } R\text{-I-Cl}(X - A)$.
- (vi) $X - \text{supra } R\text{-I-Cl}(A) = \text{supra } R\text{-I-Int}(X - A)$.
- (vii) $\text{supra } R\text{-I-Int}(A) \cap \text{supra } R\text{-I-Int}(B) = \text{supra } R\text{-I-Int}(A \cap B)$.
- (viii) $\text{supra } R\text{-I-Cl}(A) \cup \text{supra } R\text{-I-Cl}(B) = \text{supra } R\text{-I-Cl}(A \cup B)$.

4. SUPRA R-I-CONTINUOUS FUNCTIONS

Definition 4.1. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ be an associated supra topology with τ . A function $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ is called a supra R-I-continuous function if the inverse image of each open set of Y is supra R-I-open in X .

Example 4.2. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$, $\mu = \{\phi, X, \{a\}, \{a, c\}, \{b, c\}\}$, $I = \{\phi, \{b\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $J = \{\phi, \{a\}\}$. Define $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ by $f(a) = a, f(b) = b, f(c) = b$. Then f is supra R-I-continuous.

Theorem 4.3. Every completely I-continuous function is supra R-I-continuous.

Remark 4.4. Converse of the above theorem need not be true.

Example 4.5. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$, $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{b\}\}$, $\sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $J = \{\phi, \{a\}\}$. Define $f : (X, \mu, I) \rightarrow (X, \sigma, J)$ by $f(a) = a, f(b) = b, f(c) = b$. Then f is supra R-I-continuous but not completely I-continuous.

Theorem 4.6. Every totally-I continuous function is supra R-I-continuous.

Remark 4.7. Converse of the above theorem holds if X is a discrete space.

Example 4.8. Let $X = \{a, b, c\}$, $\tau = \mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{a\}\}$. Let $f : (X, \mu, I) \rightarrow (X, \tau, I)$ be the identity function. Then f is supra R-I-continuous but not totally-I continuous.

Theorem 4.9. Every perfectly-I continuous function is supra R-I-continuous.

Theorem 4.10. Let Y be a discrete space. If $f : (X, \mu, I) \rightarrow (Y, \sigma)$ is completely I-continuous, then f is supra R-I continuous.

Theorem 4.11. Every almost perfectly I-continuous function into a discrete space is supra R-I-continuous.

Theorem 4.12. Every almost completely I-continuous function into a discrete space is supra R-I-continuous.

Let $(X, \tau, I), (Y, \sigma, J)$ be ideal topological spaces and (Z, ρ) be a topological space. Let μ and ν be the associated supra topologies with τ and σ .

Theorem 4.13. If $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ is supra R-I-continuous and $g : (Y, \sigma, J) \rightarrow (Z, \rho)$ is continuous, then $g \circ f : (X, \mu, I) \rightarrow (Z, \rho)$ is supra R-I-continuous.

Proof. Let $V \in Z$ be open in ρ . Since g is cont $g^{-1}(V)$ is open in σ . Since f is supra R-I-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra R-I-open in X . Thus for any open set $V \in \rho$, $(g \circ f)^{-1}(V)$ is supra R-I-open in μ . $\square$

Theorem 4.14. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ be the associated supra topology with τ . Let $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ be a bijective map. Then the following are equivalent:

- (i) f is supra R-I-continuous.
- (ii) inverse image of a closed set in Y is supra R-I-closed in X .
- (iii) $\text{supra } R\text{-I-Cl}(f^{-1}(V)) \subset f^{-1}(\text{Cl}(V))$ for every $V \subset Y$.
- (iv) $f(\text{supra } R\text{-I-Cl}(U)) \subset \text{Cl}(f(U))$ for every $U \subset X$.
- (v) $f^{-1}(\text{Int}(B)) \subset \text{supra } R\text{-I-Int}(f^{-1}(B))$ for every $B \subset Y$.



Proof. (i) $\Rightarrow$ (ii)

Let V be a closed set in Y . Then $Y - V$ is open. Since f is supra R-I-continuous, $f^{-1}(Y - V)$ is supra R-I-open in X . i.e; $X - f^{-1}(V)$ is supra R-I-open in X . i.e; $f^{-1}(V)$ is supra R-I-closed in X .

(ii) $\Rightarrow$ (iii)

Let $V \subset Y$. Then $Cl(V)$ is closed in Y . So by (ii) $f^{-1}(Cl(V))$ is supra R-I-closed in X . So $\text{supra R-I-Cl}(f^{-1}Cl(V)) = f^{-1}(Cl(V))$. Therefore $f^{-1}(Cl(V)) = \text{supra R-I-Cl}(f^{-1}Cl(V)) \supset \text{supra R-I-Cl}(f^{-1}(V))$.

(iii) $\Rightarrow$ (iv)

Let $U \subset X$ and $f(U) = V \subset Y$.

By (iii) $\text{supra R-I-Cl}(f^{-1}(f(U))) \subset f^{-1}(Cl(f(U)))$.

i.e; $\text{supra R-I-Cl}(U) \subset f^{-1}(Cl(f(U)))$.

i.e; $f(\text{supra R-I-Cl}(U)) \subset Cl(f(U))$.

(iv) $\Rightarrow$ (ii)

Let $W \subset Y$ be a closed set and $U = f^{-1}(W) \subset X$. By (iv) $f(\text{supra R-I-Cl}(U)) \subset Cl(f(U)) = Cl(f(f^{-1}(W))) \subset Cl(W) = W$. So $\text{supra R-I-Cl}(U) \subset f^{-1}(f(\text{supra R-I-Cl}(U))) \subset f^{-1}(W) = U$. Thus U is supra R-I-closed in X .

(ii) $\Rightarrow$ (i)

Let $U \subset Y$ be an open set, then $Y - U$ is closed. By (ii) $f^{-1}(Y - U) = X - f^{-1}(U)$ is supra R-I-closed in X . Hence $f^{-1}(U)$ is supra R-I-open in X .

(i) $\Rightarrow$ (v)

Let $B \subset Y$. Then $\text{Int}(B)$ is open in Y and so $f^{-1}(\text{Int}(B))$ is supra R-I-open in X . $f^{-1}(B) \subset f^{-1}(\text{Int}(B)) = \text{supra R-I-Int}(f^{-1}(\text{Int}(B))) \subset \text{supra R-I-Int}(f^{-1}(B))$.

(v) $\Rightarrow$ (i)

Let $U \subset X$ be open. Then by (v) $f^{-1}(\text{Int}(U)) \subset \text{supra R-I-Int}(f^{-1}(U))$ and so $f^{-1}(U) \subset \text{supra R-I-Int}(f^{-1}(U))$. But $\text{supra R-I-Int}(f^{-1}(U)) \subset f^{-1}(U)$. Hence $\text{supra R-I-Int}(f^{-1}(U)) = f^{-1}(U)$. Thus $f^{-1}(U)$ is supra R-I-open in X . $\square$

Theorem 4.15. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ and ν be the associated supra topologies with τ and σ . Let $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ is supra R-I-continuous if one of the following holds.

(i) $f^{-1}(\text{supra R-I-Int}(B)) \subset \text{R-I-Int}(f^{-1}(B))$ for every $B \subset Y$.

(ii) $\text{R-I-Cl}(f^{-1}(B)) \subset f^{-1}(\text{supra R-I-Cl}(B))$ for every $B \subset Y$.

(iii) $f(\text{R-I-Cl}(A)) \subset \text{supra R-I-Cl}(f(A))$ for every $A \subset X$.

Proof. Let V be an open set in Y . Then $f^{-1}(V) \subset X$.

If (i) holds,

$f^{-1}(\text{supra R-I-Int}(V)) \subset \text{R-I-Int}(f^{-1}(V))$. i.e; $f^{-1}(V) \subset \text{R-I-Int}(f^{-1}(V))$. But $f^{-1}(V) \supset \text{R-I-Int}(f^{-1}(V))$. Thus $f^{-1}(V) = \text{R-I-Int}(f^{-1}(V))$. So f^{-1} is R-I-open and so supra R-I-open. Hence f is supra R-I-continuous.

If (ii) holds,

$\text{R-I-Cl}(f^{-1}(V)) \subset f^{-1}(\text{supra R-I-Cl}(V))$. Then $\text{R-I-Int}(f^{-1}(Y - V)) \supset f^{-1}(\text{supra R-I-Int}(Y - V))$. Then by (i) f is supra R-I-continuous.

If (iii) holds,

$f(\text{R-I-Cl}(f^{-1}(V))) \subset \text{supra R-I-Cl}(V)$. So by (ii) f is supra R-I-continuous. $\square$

Definition 4.16. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ and ν be associated supra topologies with τ and σ respectively. A function $f : (X, \mu, I) \rightarrow (Y, \nu, J)$ is called supra* R-I-continuous (resp. supra R-I-irresolute) if the inverse image of each supra open (resp. supra R-I-open) set of Y is supra R-I-open in X .

Theorem 4.17. Every supra* R-I-continuous function is supra R-I-continuous.

Remark 4.18. Converse of the above theorem need not be true.

Example 4.19. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $\mu = \{\emptyset, X, \{a\}, \{a, c\}, \{b, c\}\}$, $I = \{\emptyset, \{b\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\emptyset, X, \{a\}, \{b\}\}$, $\nu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $J = \{\emptyset, \{a\}\}$. Define $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ by $f(a) = b$, $f(b) = a$, $f(c) = b$. Then f is supra R-I-continuous but not supra* R-I-continuous since $f^{-1}(b) = \{a, c\}$ which is not supra R-I-open.

Theorem 4.20. Every supra* R-I-continuous function is supra R-I-irresolute.

Remark 4.21. Converse of the above theorem need not be true.

For example, in example 4.19, f is supra R-I-irresolute also.

$\text{supra R-I-continuous} \Leftarrow \text{supra* R-I-continuous} \Rightarrow \text{supra R-I-irresolute}$

Theorem 4.22. If $f : (X, \mu, I) \rightarrow (Y, \nu, J)$ is supra* R-I-continuous and $g : (Y, \nu, J) \rightarrow (Z, \rho)$ is supra R-I-continuous, then $g \circ f : (X, \mu, I) \rightarrow (Z, \rho)$ is supra R-I-continuous.

Theorem 4.23. If $f : (X, \mu, I) \rightarrow (Y, \nu, J)$ is supra R-I-irresolute and $g : (Y, \nu, J) \rightarrow (Z, \rho)$ is supra R-I-continuous, then $g \circ f : (X, \mu, I) \rightarrow (Z, \rho)$ is supra R-I-continuous.

Theorem 4.24. Let $f : (X, \mu, I) \rightarrow (Y, \nu, J)$ be an injective supra* R-I-continuous function. Then $\text{Int}(f(A)) \subset \text{supra R-I-Int}(f(A)) \subset f(\text{supra R-I-Int}(f(A)))$.

Theorem 4.25. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ be the associated supra topology with τ . Let $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ be a bijective map. Then the following are equivalent:



- (i) f is supra* R-I-continuous.
- (ii) inverse image of a closed set in Y is supra R-I-closed in X .
- (iii) $\text{supra R-I-Cl}(f^{-1}(V)) \subset f^{-1}(\text{Cl}(V))$ for every $V \subset Y$.
- (iv) $f(\text{supra R-I-Cl}(U)) \subset \text{Cl}(f(U))$ for every $U \subset X$.
- (v) $f^{-1}(\text{Int}(B)) \subset \text{supra R-I-Int}(f^{-1}(B))$ for every $B \subset Y$.

Theorem 4.26. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ be the associated supra topology with τ . Let $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ be a bijective map. Then the following are equivalent:

- (i) f is supra R-I-irresolute.
- (ii) inverse image of a closed set in Y is supra R-I-closed in X .
- (iii) $\text{supra R-I-Cl}(f^{-1}(V)) \subset f^{-1}(\text{Cl}(V))$ for every $V \subset Y$.
- (iv) $f(\text{supra R-I-Cl}(U)) \subset \text{Cl}(f(U))$ for every $U \subset X$.
- (v) $f^{-1}(\text{Int}(B)) \subset \text{supra R-I-Int}(f^{-1}(B))$ for every $B \subset Y$.

Theorem 4.27. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ be the associated supra topology with τ . Let $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ is supra R-I-irresolute if one of the following holds.

- (i) $f^{-1}(\text{supra R-I-Int}(B)) \subset \text{R-I-Int}(f^{-1}(B))$ for every $B \subset Y$.
- (ii) $\text{R-I-Cl}(f^{-1}(B)) \subset f^{-1}(\text{supra R-I-Cl}(B))$ for every $B \subset Y$.
- (iii) $f(\text{R-I-Cl}(A)) \subset \text{supra R-I-Cl}(f(A))$ for every $A \subset X$.

Theorem 4.28. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and μ be the associated supra topology with τ . Let $f : (X, \mu, I) \rightarrow (Y, \sigma, J)$ is supra* R-I-continuous if one of the following holds.

- (i) $f^{-1}(\text{supra R-I-Int}(B)) \subset \text{R-I-Int}(f^{-1}(B))$ for every $B \subset Y$.
- (ii) $\text{R-I-Cl}(f^{-1}(B)) \subset f^{-1}(\text{supra R-I-Cl}(B))$ for every $B \subset Y$.
- (iii) $f(\text{R-I-Cl}(A)) \subset \text{supra R-I-Cl}(f(A))$ for every $A \subset X$.

5. SUPRA R-I-OPEN MAPS AND SUPRA R-I-CLOSED MAPS

Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and (X, μ, I) and (Y, ν, J) be the associated supra ideal topological spaces.

Definition 5.1. A map $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is supra R-I-open (resp. supra R-I-closed) if image of each open (resp. closed) set in X is supra R-I-open (resp. supra R-I-closed) in Y .

Example 5.2. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\emptyset, \{a\}\}$. Define $f : (X, \mu, I) \rightarrow (X, \mu, I)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Then f is supra R-I-open.

Theorem 5.3. A map $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is supra R-I-open if and only if $f(\text{Int}(A)) \subset \text{supra R-I-Int}(f(A))$ for each $A \subset X$.

Proof. Suppose f is supra R-I-open. Since $f(\text{Int}A)$ is a supra R-I-open set contained in $f(A)$ and $\text{supra R-I-Int}(f(A))$ is the largest supra R-I-open set contained in $f(A)$, $f(\text{Int}A) \subset \text{supra R-I-Int}(f(A))$ for each $A \subset X$. Conversely, suppose that A is open in X . Then $f(A) = f(\text{Int}A) \subset \text{supra R-I-Int}(f(A))$. Therefore $f(A) = \text{supra R-I-Int}(f(A))$. Thus f is a supra R-I-open map. $\square$

Theorem 5.4. A map $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is supra R-I-closed if and only if $\text{supra R-I-Cl}(f(A)) \subset f(\text{Cl}(A))$ for each $A \subset X$.

Proof. Suppose f is supra R-I-closed. Since $f(\text{Cl}(A))$ is a supra R-I-closed set containing $f(A)$ and $\text{supra R-I-Cl}(f(A))$ is the smallest supra R-I-closed containing $f(A)$, $\text{supra R-I-Cl}(f(A)) \subset f(\text{Cl}(A))$ for each $A \subset X$. $\square$

Theorem 5.5. Let (X, τ, I) , (Y, σ, J) and (Z, ρ, K) be ideal topological spaces. Let μ, ν and ξ be the associated supra topologies of τ, σ and ρ respectively. Then

- (i) if $g \circ f : (X, \tau, I) \rightarrow (Z, \xi, K)$ is supra R-I-open and $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is a continuous surjection, then $g : (Y, \sigma, J) \rightarrow (Z, \xi, K)$ is a supra R-I-open map.
- (ii) if $g \circ f : (X, \tau, I) \rightarrow (Z, \rho, K)$ is open and $g : (Y, \nu, J) \rightarrow (Z, \xi, K)$ is a supra R-I-continuous injection, then $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is a supra R-I-open map.
- (iii) if $g \circ f : (X, \tau, I) \rightarrow (Z, \xi, K)$ is supra R-I-open and $g : (Y, \nu, J) \rightarrow (Z, \xi, K)$ is supra R-I-irresolute injection, then $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is a supra R-I-open map.

Proof. (i) Let V be open in Y . Then $f^{-1}(V)$ is open in X . So $g \circ f(f^{-1}(V))$ is supra R-I-open in (Z, ρ, K) . Since f is onto, $g \circ f(f^{-1}(V)) = g(f(f^{-1}(V))) = g(V)$.

(ii) Let U be open in X . Then $g \circ f(U)$ is open in Z . So $g^{-1}(g \circ f(U))$ is supra R-I-open in Y . Since g is one-one, $g^{-1}(g \circ f(U)) = (g^{-1} \circ g)(f(U)) = f(U)$.

(iii) Let U be open in X . Then $g \circ f(U)$ is supra R-I-open in Z . Since g is supra R-I-irresolute, $g^{-1}(g \circ f(U))$ is supra R-I-open in Y . Since g is one-one, $g^{-1}(g \circ f(U)) = (g^{-1} \circ g)(f(U)) = f(U)$. $\square$



Theorem 5.6. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and ν be the associated supra topology with σ . Let $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ be a bijective map. Then the following are equivalent:

- (i) f is a supra R-I-open map.
- (ii) f is a supra R-I-closed map.
- (iii) f^{-1} is a supra R-I-continuous map.

Proof. (i) $\Rightarrow$ (ii)

Suppose that V is a closed subset of X . Then V^c is open in X and $f(V^c)$ is supra R-I-open in Y . Since f is bijective, then $f(V^c) = Y - f(V)$. So $f(V)$ is supra R-I-closed in Y . Thus f is a supra R-I-closed map.

(ii) $\Rightarrow$ (iii)

Suppose f is a supra R-I-closed map and V is a closed subset of X . Then $f(V)$ is supra R-I-closed in Y . Since f is bijective, $(f^{-1})^{-1}(V) = f(V)$. Therefore f^{-1} is a supra R-I-continuous map.

(iii) $\Rightarrow$ (i)

Suppose that U is an open subset of X . Since f^{-1} is a supra R-I-continuous map, $(f^{-1})^{-1}(U)$ is supra R-I-open in Y . Since f is bijective, $f(U) = (f^{-1})^{-1}(U)$. Thus f is a supra R-I-open map. $\square$

In a similar manner, supra* R-I-open and supra* R-I-closed functions can be defined.

6. SUPRA R-I-HOMEOMORPHISM

Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and (X, μ, I) and (Y, ν, J) be the associated supra ideal topological spaces.

Definition 6.1. A map $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be supra R-I-homeomorphism if f is supra R-I-continuous and supra R-I-open.

Remark 6.2. Since every supra R-I-open set is supra open, every supra R-I-homeomorphism is supra homeomorphism.

Theorem 6.3. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and (X, μ, I) and (Y, ν, J) be the associated supra ideal topological spaces. Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a bijective and supra R-I-continuous map. Then the following are equivalent:

- (i) f is supra R-I-homeomorphism.
- (ii) f^{-1} is supra R-I-continuous.
- (iii) f is supra R-I-closed.

Proof. (i) $\Rightarrow$ (ii)

Assume (i). Then f is a supra R-I-open map. So for any open set $U \in \tau$, $f(U)$ is supra R-I-open in ν . Since f is bijective, $(f^{-1})^{-1}(U) = f(U)$. Now $f^{-1} : (Y, \nu, J) \rightarrow (X, \tau, I)$. So for any open $U \in \tau$, $(f^{-1})^{-1}(U)$ is supra R-I-open in ν . Hence f^{-1} is supra R-I-continuous.

(ii) $\Rightarrow$ (iii)

Let $f^{-1} : (Y, \nu, J) \rightarrow (X, \tau, I)$ be supra R-I-continuous. Then for any open $U \in \tau$, $(f^{-1})^{-1}(U)$ is supra R-I-open in ν . Let V be closed in X . Then $X - V$ is open in X . So $(f^{-1})^{-1}(X - V) = f(X - V) = Y - f(V)$ is supra R-I-open in ν . So $f(V)$ is supra R-I-closed in ν . Thus for any closed set V in X , $f(V)$ is supra R-I-closed in ν . Hence $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is a supra R-I-closed map.

(iii) $\Rightarrow$ (i)

Let $f : (X, \tau, I) \rightarrow (Y, \nu, J)$ is a supra R-I-closed map and U be open in τ . Then $f(X - U)$ is supra R-I-closed in ν . By the bijection of f , $Y - f(U)$ is supra R-I-closed in ν and so $f(U)$ is supra R-I-open in ν . Since f is supra R-I-continuous, f is supra R-I-homeomorphism. $\square$

7. NEW SEPARATION AXIOMS

Definition 7.1. Let (X, τ, I) be an ideal topological space and (X, μ, I) be the associated supra ideal topological space. Then (X, μ, I) is called

- (i) supra $R - I - T_0$ if for every two distinct points of X , there exists a supra R-I-open set which contains one, but not the other.
- (ii) supra $R - I - T_1$ if for every two distinct points x and y of X , there exists supra R-I-open sets U and V such that $x \in U, y \notin U$ and $x \notin V, y \in V$.
- (iii) supra $R - I - T_2$ if for every two distinct points x and y of X , there exists supra R-I-open sets U and V such that $x \in U, y \in V$ and $U \cap V = \emptyset$.

Definition 7.2. A subset U of (X, μ, I) is called an supra R-I-nbd of a point $x \in X$ if there exists an supra R-I-open set V such that $x \in V \subset U$.

Theorem 7.3. Let (X, μ, I) be a supra ideal topological space. Then X is supra $R - I - T_0$ if and only if supra $R-I-Cl(\{x\}) \neq$ supra $R-I-Cl(\{y\})$ for every distinct x and y in X .

Proof. Suppose X is supra $R - I - T_0$. Let $x \neq y \in X$. Then there exists a supra R-I-open set U containing x but not y . So $X - U$ is a supra R-I-closed set such that $x \in X - U$ and $y \in X - U$. Thus supra $R-I-Cl(\{x\}) \neq$ supra $R-I-Cl(\{y\})$. Conversely suppose supra $R-I-Cl(\{x\}) \neq$ supra $R-I-Cl(\{y\})$ for any $x \neq y \in X$. Assume X is not supra $R - I - T_0$. So every supra R-I-closed set containing x always contains y . But then supra $R-I-Cl(\{x\}) =$ supra $R-I-Cl(\{y\})$ which is a contradiction. Hence there should exist at least one supra R-I-open set containing x but not y . So X is supra $R - I - T_0$. $\square$



Theorem 7.4. Let (X, μ, I) be a supra ideal topological space. Then X is supra $R-I-T_1$ if and only if every singleton is supra $R-I$ -closed.

Proof. Let X be supra $R-I-T_1$ and $x \in X$. For any $y \in X$, $x \neq y$, there exists a supra $R-I$ -open set V such that $y \in V$ and $x \notin V$. Then $V \cap \{x\} = \emptyset$ or $\{x\} \subseteq V^c$. So for any $y \neq x \in X$, $y \notin \text{supra } R-I-\text{Cl}(\{x\})$. Therefore $\text{supra } R-I-\text{Cl}(\{x\}) = \{x\}$. Thus $\{x\}$ is supra $R-I$ -closed.

Conversely let $x, y \in X$. Then $\{x\}, \{y\}$ are supra $R-I$ -closed sets and so $X - \{x\}$ and $X - \{y\}$ are supra $R-I$ -open sets. Then X is supra $R-I-T_1$. $\square$

Theorem 7.5. Let (X, μ, I) be a supra ideal topological space. Then X is supra $R-I-T_2$ if and only if for each $x \in X$, $\{x\} = \cap \{N_i : N_i \text{ is a supra } R-I\text{-closed nbd of } x\}$.

Proof. Let X be supra $R-I-T_2$. For $x \neq y \in X$, there exists two disjoint supra $R-I$ -open sets U_i and V_i such that $x \in U_i$ and $y \in V_i$ and $U_i \cap V_i = \emptyset$. Then $U_i \subseteq V_i^c$.

So $\text{supra } R-I-\text{Cl}(U_i) \subseteq V_i^c$. Let $N_i = \text{supra } R-I-\text{Cl}(U_i)$. Hence $\{x\} = \cap \{N_i : N_i \text{ is a supra } R-I\text{-closed nbd of } x\}$.

Conversely for $x \in X$, $\{x\} = \cap \{N_i : N_i \text{ is a supra } R-I\text{-closed nbd of } x\}$. Then for $x \neq y$, there exists supra $R-I$ -closed nbd F such that $x \in F$ and $y \notin F$. So there exist supra $R-I$ -open sets U and V such that $x \in U \subset F$ and $y \in V = F^c$. Also $U \cap V = \emptyset$. Thus X is supra $R-I-T_2$. $\square$

Theorem 7.6. Let (X, μ, I) be a supra ideal topological space. If X is supra $R-I-T_1$, then for each $x \in X$, $\{x\} = \cap \{U_i : U_i \text{ is a supra } R-I\text{-open nbd of } x\}$.

Proof. Suppose X is supra $R-I-T_1$. Fix $x \in X$. Then for each $y_i \in X$, there exists supra $R-I$ -open sets U_i and V_i such that $x \in U_i, y \notin U_i$ and $y \in V_i, x \notin V_i$. Clearly each U_i is a supra $R-I$ -open nbd of x . Hence $\{x\} = \cap \{U_i : U_i \text{ is a supra } R-I\text{-open nbd of } x\}$. $\square$

Theorem 7.7. Let (X, μ, I) be a supra ideal topological space. If singletons are supra $R-I$ -closed, then for each $x \in X$, $\{x\} = \cap \{X - \{y\} : \text{for each } y \neq x \in X\}$.

Proof. Suppose singletons are supra $R-I$ -closed. Fix $x \in X$. Then for any $y \neq x \in X$, $X - \{y\}$ is a supra $R-I$ -open nbd of x . Hence clearly $\{x\} = \cap \{X - \{y\} : \text{for each } y \neq x \in X\}$. $\square$

Theorem 7.8. Every supra $R-I-T_i$ is a supra T_i space.

Proof. Since every supra $R-I$ -open set is supra open, the theorem follows. $\square$

Remark 7.9. Converse of the above theorem need not be true.

For example, let $X = \{a, b, c\}, \mu = \{\emptyset, X, \{a, b\}, \{a, c\}, \{b, c\}\}, I = \{\emptyset, \{a\}\}$. Then (X, μ, I) is supra $I-T_1$ but not supra $R-I-T_1$.

Theorem 7.10. The axioms supra $R-I-T_0$, supra $R-I-T_1$, supra $R-I-T_2$ form a hierarchy of progressively stronger conditions.

Remark 7.11. None of the implications in the theorem above is reversible.

For example, let $X = \{a, b, c\}, \mu = \{\emptyset, X, \{a, b\}, \{a, c\}, \{b, c\}\}, I = \{\emptyset, \{a\}, \{b\}, \{c\}\}$. Then (X, μ, I) is a supra $R-I-T_1$ space but is not a supra $R-I-T_2$ space.

Remark 7.12. Every finite supra $R-I-T_1$ space is not discrete.

For example, let $X = \{a, b, c\}, \mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}, I = \{\emptyset, \{a\}\}$. Then (X, μ, I) is a supra $R-I-T_1$ space but is not discrete.

Theorem 7.13. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and (X, μ, I) and (Y, ν, J) be the associated supra ideal topological spaces. Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a bijective and supra $R-I$ -open map. If (X, μ, I) is a supra $R-I-T_0$ space then (Y, ν, J) is a supra $R-I-T_0$ space.

Proof. Suppose (X, μ, I) is a supra $R-I-T_0$ space. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is bijective there exists $x_1 \neq x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since (X, μ, I) is supra $R-I-T_0$, there exists a supra $R-I$ -open set $U \subseteq X$ such that $x_1 \in U$ and $x_2 \notin U$. Since f is supra $R-I$ -open map, $f(U) \subseteq Y$ is a supra $R-I$ -open set. Also $y_1 \in f(U)$ and $y_2 \notin f(U)$. Thus (Y, ν, J) is a supra $R-I-T_0$ space. $\square$

Theorem 7.14. Let (X, τ, I) and (Y, σ, J) be ideal topological spaces and (X, μ, I) and (Y, ν, J) be the associated supra ideal topological spaces. Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a bijective and supra $R-I$ -continuous map. If (Y, ν, J) is a supra $R-I-T_0$ space then (X, μ, I) is a supra $R-I-T_0$ space.

Proof. Suppose (Y, ν, J) is a supra $R-I-T_0$ space. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijective there exists $y_1 \neq y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since (Y, ν, J) is supra $R-I-T_0$, there exists a supra $R-I$ -open set $V \subseteq Y$ such that $y_1 \in V$ and $y_2 \notin V$. Since f is supra $R-I$ -continuous, f^{-1} is supra $R-I$ -open and so $f^{-1}(V) \subseteq X$ is a supra $R-I$ -open set. Also $x_1 \in f^{-1}(V)$ and $x_2 \notin f^{-1}(V)$. Thus (X, μ, I) is a supra $R-I-T_0$ space. $\square$

Remark 7.15. The above two theorems are true for supra $R-I-T_i$ spaces also for $i = 1, 2$.

Remark 7.16. The theorem 7.14 is true, if the bijective function f is a supra $*$ $R-I$ -continuous map and also if the bijective function f is a supra $R-I$ -irresolute map.

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