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Strong connectivity index of weighted graphs

Jicy N[1]* and Sunil Mathew[2]

Abstract
In a weighted graph model, the reduction of flow value between some pairs of nodes is more relevant and more frequent than the total disruption of the flow or the disconnection of the entire network. So it is necessary to analyze connectivity in terms of connectivity parameters. Strong connectivity index is the sum of minimum flow of each pair of nodes in a network. A characterization of partial trees using strong connectivity index is obtained. Using node strength sequence the strong connectivity index of precisely weighted graphs and path graphs are explained.

Keywords
Weighted graph, strong connectivity index, node strength sequence.

AMS Subject Classification
05C22, 05C38, 05C40.

[1,2] *Department of Mathematics, Indian Institute of Technology Calicut, Calicut - 673601, India.*
***Corresponding author**: [1] jicychanth@gmail.com; [2]sm@nitc.ac.in
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1. Introduction

A *weighted graph* G is a graph in which every arc e is assigned a non-negative number $w(e)$, called the *weight* of e. An unweighted graph can be regarded as a weighted graph in which every arc e is assigned weight $w(e) = 1$. If we have a weighted graph representing some cities and roads connecting them, then the weights may be taken as the populations function and the daily exchange of population between the cities. In such a model, connectivity plays a crucial role. Similar problems arise in all types of networks like communication, computer, biological, etc.

Depending on the strength of a vertex (population, size, capacity), we can have different categories of vertices(A Class city, Big dam, etc). Also depending on the rate of flow (data flow, transport,etc.) between two nodes, we can have different categories of arcs (Broad band, national highway, etc.).

In this article we introduce some new connectivity concepts in weighted graphs. In a weighted graph model, for example, in an information network or electric circuit, the reduction of flow between pairs of nodes is more relevant and may frequently occur than the total disruption of the flow or the disconnection of the entire network. This concept is our motivation. As weighted graphs are generalized structures of graphs, the concepts introduced in this article also generalizes the classic connectivity concepts.

2. Preliminaries

In this section, we recall some definitions and basic results of weighted graphs and connectivity of weighted graphs which will be used throughout the paper. As in graph theory, connectivity plays an important role in weighted graph theory also. For a path P, the weight of a weakest arc is defined as its strength. Some of the basic definitions in [11] are given below

Definition 2.1. *A totally weighted graph is described in [8], is a graph $G : (\delta, \omega)$ where $\delta : V \to R^+$ and $\omega : E \to R^+$ such that $\omega(x,y) \leq \delta(x) \wedge \delta(y)$, for any pair of vertices x, y of G. Let G be a weighted graph, then the strength of a path P of n edges e_i, for $1 \leq i \leq n$, denoted by $s(P)$ is equal to $s(P) = min_{1 \leq i \leq n}\{w(e_i)\}$.*

Definition 2.2. *[11] Let $G(V,E)$ be a weighted graph. The strength of connectedness between two nodes u and v is denoted by $CONN_G(u,v)$, is defined as the maximum of the strengths of all paths between u and v. If u and v are in different components of G, then $CONN_G(u,v) = 0$.*

Example (Figure 1) Consider the following graph $G(V,E)$

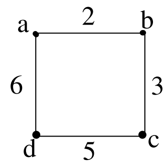


Figure 1: Strength of connectedness

In the above example, $CONN_G(a,b) = 3$, $CONN_G(a,c) = 5$, $CONN_G(a,d) = 6$, $CONN_G(b,c) = 3$, $CONN_G(b,d) = 3$ and $CONN_G(c,d) = 5$.

Proposition [10] Let G be a weighted graph and H be a weighted subgraph of G. Then for any pair of nodes $u,v \in V(G)$, $CONN_H(u,v) \leq CONN_G(u,v)$.

Definition 2.3. *[10] A $u-v$ path in a weighted graph G is called a strongest $u-v$ path if stength of the path $u-v$ is equal to the strength of connectivity between u and v.*

Definition 2.4. *[10] Let G be a weighted graph. A node w is called a partial cutnode (p-cutnode)of G if there exist a pair of nodes u,v in $V(G)$ such that $u \neq v \neq w$ and $CONN_{G-w}(u,v) < CONN_G(u,v)$. A connected weighted graph having no p-cutnodes is called a partial block (p-block).*

Definition 2.5. *[10] Let G be a weighted graph. An arc $e = (u,v)$ is called a partial bridge (p-bridge)of G if $CONN_{G-e}(u,v) < CONN_G(u,v)$.*

Example (Figure 2) Consider the following graph $G(V,E)$,

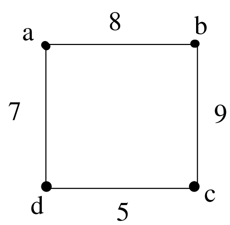


Figure 2: Weighted graph with p-cutnodes and p-bridges

Here node b is a p-cutnode and all the arcs except (c,d) are p-bridges.

Definition 2.6. *[11] A connected weighted graph is called a partial tree if G has a spanning subgraph $F(V,E)$ which is a tree, where for all arcs (u,v) of $E(G)$ which are not in F, we have $CONN_F(u,v) > w(u,v)$. Figure 2, is a partial tree since there exist a spanning subgraph F such that $CONN_F(c,d) > w(c,d)$.*

Definition 2.7. *[11] Let G be a weighted graph. Then an arc $e = (u,v)$ is called α-strong if $CONN_{G-e}(u,v) < w(e)$,β-strong if $CONN_{G-e}(u,v) = w(e)$ and a δ-arc if $CONN_{G-e}(u,v) > w(e)$. A δ-arc e is called a δ^*-arc if e is not a weakest arc of G. In figure 2, all the other arcs except the arc (c,d) are α-strong arcs. The arc (c,d) is a δ arc.*

Definition 2.8. *For any two vertices u and v of G, the θ-evaluation of u and v is defined as $\theta(u,v) = \{\alpha; \alpha \in R\}$ where α is the strength of a strong cycle passing through both u and v [10]. Max $\{\alpha; \alpha \in \theta(u,v); u,v \in \sigma^*\}$ is defined as the cycleconnectivity between u and v in G and is denoted by $C^G_{u,v}$.*

Definition 2.9. *Cycle connectivity of a graph G is defined as $CC(G) = max\{C^G_{u,v}; u,v \in \sigma^*\}$.*

Definition 2.10. *A node w in a weighted graph G is called a cyclic cutvertex if $CC(G-w) < CC(G)$. An edge (u,v) of a weighted graph G is called a cyclic bridge if $CC(G-(u,v)) < CC(G)$.*

Definition 2.11. *[8] weighted graph G is said to be cyclically balanced if G has no cyclic cut vertex and cyclic bridges.*

Definition 2.12. *A precisely weighted graph (PWG)$G : (\delta, \omega)$ is described in [11], is a totally weighted graph $G(V,E)$, where $\delta : V \to R^+$ and $\omega : E \to R^+$ Such that $\omega(x,y) = \delta(x) \wedge \delta(y)$, for any pair of vertices x,y of G.*

Note that if G is a PWG, then it is complete. [11] Let $G(V,E)$ be a weighted graph. The strong degree of a vertex $v \in V$ is defined as the sum of weights of all strong arcs incident at v and is denoted by $d_s(v)$. Also if $N_s(v)$ denote the set of all strong neighbors of v, then $d_s(v) = \sum_{u \in N_s(v)} \omega(u,v)$.

Definition 2.13. *The minimum strong degree of G is $\delta_s(G) = \wedge\{d_s(v; v \in \delta^*\}$ and the maximum strong degree of G is $\Delta_s(G) = \vee\{d_s(v); v \in \delta^*\}$.*

[11] We are denoting $\kappa_w(G)$, $\kappa'_w(G)$ as the weighted vertex connectivity and weighted edge connectivity of the graph G respectively.

The distance $d(u,v)$ between two vertices $u,v \varepsilon V(G)$ is the minimum number of edges in a path between u and v in G. For each $u \in V$, the eccentricity of u, denoted by $e(u)$, is defined by $e(u) = max\{d(u,v); v \in V, v \neq u\}$. The radius of G, is denoted by $r(G)$, is defined by $r(G) = min\{e(u); u \in v\}$. The diameter of G, is denoted by $d(G)$, is defined by $d(G) = max\{e(u); u \in V\}$.

Wiener index of a graph G is described in [7], is the sum of distances between all pairs of vertices of G. Then the Wiener index of graph G is given by $W(G) = \sum_{u,v \in V(G)} d(u,v)$.

3. Strong connectivity in totally weighted graphs

As in the case of Wiener index of totally weighted graphs there is another parameter called the strong connectivity index of a totally weighted graph. For we use the definition of Wiener index in [8].

Definition 3.1. *Let $G : (\delta, \omega)$ be a totally weighted graph. Then the strong connectivity index of G is denoted by $CI(G)$ and is*

$$CI(G) = \sum_{u,v \in V(G)} w(u)w(v)CONN_G(u,v).$$

Also the average connectivity index is,

$$ACI(G) = 1/n_{C_2} \sum_{u,v \in V(G)} w(u)w(v)CONN_G(u,v)$$

Example (Figure 3) Let G : be a weighted graph with $V = \{a,b,c,d\}$ such that $w(a) = 2$, $w(b) = 2$, $w(c) = 3$, $w(d) = 4$, $w(a,b) = 1$, $w(b,c) = 2$, $w(a,c) = 2$ and $w(a,d) = 2$, $w(b,d) = 2$ and $w(c,d) = 3$. Then strong connectivity index of G is $CI(G) = 96$.

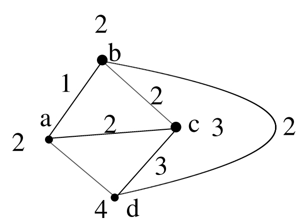


Figure 3: Weighted graph with a connectivity index

Next is a proposition.

Proposition. Let G be a totally weighted graph and H be a totally weighted subgraph of G. Then $CI(H)) \leq CI(G)$.

Proof. Using the result $CONN_H(u,v) \leq CONN_G(u,v)$

(by proposition 2. in [10])

$$\sum_{u,v \in V(G)} CONN_H(u,v) \leq \sum_{u,v \in V(G)} CONN_G(u,v)$$

$$\therefore CI(H)) \leq CI(G).$$ □

The next theorem shows how the deletion of a vertex or an edge will affect the strong connectivity index of the graph.

Theorem 3.2. *Let G be a totally weighted graph with the vertex set $\{u_1, u_2, \cdots u_n\}$. Then*

1) $CI(G-u) < CI(G)$, for all $u \in V(G)$.

2) $CI(G-(u,v)) < CI(G)$, for all $(u,v) \in E(G)$.

Proof. Let G be a totally weighted graph and $\{u_1, u_2, \cdots u_n\}$ be the set of vertices in G. Where $u = u_k$. Then

$$CI(G) = \sum_{1 \leq i,\ j \leq n;\ i \neq j} w(u)w(v)CONN_G(u_i,u_j)$$

$$CI(G-u) = \sum_{1 \leq i,\ j \leq n;\ i \neq j;\ i,j \neq k} w(u)w(v)CONN_G(u_i,u_j)$$

$$\leq \sum_{1 \leq i,\ j \leq n;\ i \neq j} w(u)w(v)CONN_G(u_i,u_j)$$

$$= CI(G).$$

Similarly, consider the edge $e = (u_i, u_j)$. Then

$$CI(G-e) = \sum_{1 \leq i,\ j \leq n;\ i \neq land\ j \neq k} w(u)w(v)CONN_G(u_i,u_j)$$

$\therefore CI(G-e) \leq CI(G)$. □

Corollary Let G be a totally weighted graph with strong connectivity index $CI(G)$. Then

1) $ACI(G-u) < ACI(G)$, for all $u \in V(G)$.

2) $ACI(G-(u,v)) < ACI(G)$, for all $(u,v) \in V(G)$.

Next theorem is a characterization of partial trees using strong connectivity index.

Theorem 3.3. *Let G be a totally weighted graph, then G is a partial tree if and only if $CI(G) = CI(F)$ for every maximal spanning tree F of G.*

Proof. Suppose G be a partial tree,and F is the maximal spanning tree of G. Then edges in F are partial bridges of G from (Theorem 9, in [10]). Also if G is a partial tree and if $e = (u,v)$ is a partial bridge of G, then, $CONN_G(u,v) = w(e)$. Therefore $CONN_G(u,v) = CONN_F(u,v)$ for every $u, v \in V(G)$. Then we have

$$CONN_G(u,v) = CONN_F(u,v)$$

$$\sum_{u,v \in V(G)} w(u)w(v)CONN_G(u,v) = \sum_{u,v \in V(G)} w(u)w(v)CONN_F(u,v)$$

Thus

$$CI(G) = CI(F).$$

Conversely, If $CI(G) = CI(F)$ for every maximal spanning tree F of G. To prove that G is a partial tree. Let $F_1(\tau_1, \upsilon_1)$ and $F_2(\tau_2, \upsilon_1)$ be two maximum spanning trees of G. Then,
$CI(G) = CI(F_1) = CI(F_2)$.

Since $CI(F_1) = CI(F_2)$, for every $u, v \in G$. $CONN_{F_1}(u,v) = CONN_{F_2}(u,v)$. For otherwise one of F_1 or F_2 will not be a maximum spanning tree.

Now define a bijection $\phi : F_1 \longrightarrow F_2$ as $\phi = (\phi_1, \phi_2)$ where $\phi_1 : \tau_1 \longrightarrow \tau_2$ as $\phi_1 = I$ and

$\phi_2 : \upsilon_1 \longrightarrow \upsilon_2$ as $\phi_2(e) = e' = (u,v)$, where $CONN_G(x,y) = CONN_F(u,v)$.

Then ϕ_1 and ϕ_2 are well defined since $CI(F_1) = CI(F_2)$.

Clearly $\phi = (\phi_1, \phi_2)$ is an isomorphism. Which implies $F_1 \simeq F_2$. i.e., The two maximal spanning trees are isomorphic. Thus F is the unique maximal spanning tree of G. Therefore G is a partial tree. □

Example

Let G be a totally weighted graph with $V = \{a,b,c,d,e,f\}$ such that $w(a) = 3$, $w(b) = 3$, $w(c) = 4$, $w(d) = 4$, $w(e) = 3$, $w(f) = 3$, $w(a,b) = 3$, $w(b,c) = 3$, $w(c,d) = 4$, $w(d,e) = 3$, $w(a,c) = 2$, $w(c,e) = 2$ and $w(c,f) = 3$. G is a partial tree with maximum spanning tree F. Hence the strong connectivity index of G and strong connectivity index of F are same.

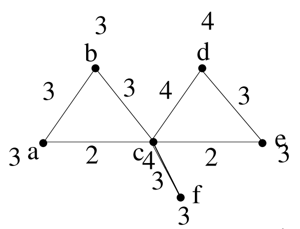
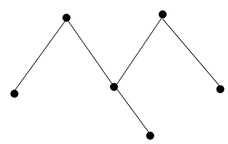


Figure 4: A partial tree



Maximum spanning tree F

Using the definitions of cycle connectivity, strong acyclic level, etc., a new result that the cycle connectivity and strong acyclic level of a precisely weighted graph are the same.

Theorem 3.4. *Let $G : (\delta, \omega)$ be a precisely weighted graph with $|V| = n$. Then the cycle connectivity and the strong acyclic level of G are equal.*

Proof. Let $G : (\delta, \omega)$ be a precisely weighted graph with $|V| = n$ and the cycle connectivity of G, $CC(G) > 0$. The cyclomatic number for each t-cut of G, G^t is defined as $\psi = m^t - n^t + p$, where m^t is the number of strong edges, n^t is the number of vertices and p be the number of connected components in G^t.

G^t has no strong cycles if and only if $t > CC(G)$ (by Theorem 4.1.12 in [9].

Therefore the cyclomatic number of G^t for $t > CC(G)$ is zero.

The acyclic level of a precisely weighted graph is the $\inf\{t\}$ for which $\psi = 0$. Therefore the acyclic level of G is, $CC(G) = s$(say).

Conversely, if $G : (\delta, \omega)$ is a weighted graph with strong acyclic level of G is s, then strong acyclic level of graph G is the $\inf\{t\}$ for which $\psi = 0$. Then there are no strong cycles remaining in the $t - cut$. that is, ψ is non zero if there exist at least one strong cycle in that $t - cut$. It will be the strongest strong cycle in G, giving the cycle connectivity of G. □

4. Strong connectivity index using vertex strength sequence

A totally weighted graph which has no cyclic bridge or cyclic cutnode is called cyclically balanced graph. Next theorem gives an equivalent condition for precisely weighted graphs.

Theorem 4.1. *Let $G(\delta, \omega)$ be a precisely weighted graph with $|V| \geq 3$. Let $\{u_1, u_2, \cdots, u_n\}$ be the n vertices of G, where $\delta(u_i) = p_1$, for $i = 1, 2, ..., n-2$, $\delta(u_{n-1}) = p_2$ and $\delta(u_n) = p_3$ with $p_1 < p_2 < p_3$. Then the following are equivalent.*

1) G is a θ weighted graph.

2) Cycle connectivity of G is p_1.

3) G is a cyclically balanced graph.

4) G has a $(n-2)$-cyclic vertex cut.

Proof. ($1 \Longrightarrow 2$) Let $G : (\delta, \omega)$ be a θ weighted graph with $|V| \geq 3$. let G be a graph with $\{u_1, u_2, \cdots u_n\}$ as the vertex set. Let $\delta(u_i) = p_1$, for every $i = 1, 2, \cdots, n-2$ and vertex strength sequence of G as $({p_1}^{n-2}, p_2, p_3)$.

The strengths of all strong cycles passing through any pair of vertices u and v are equal, since for all edges (u, v) except (p_2, p_3), strengths of all uv- paths are equal to p_1. Also we have $\omega(p_2, p_3) = p_2$. Removal of the edge (p_2, p_3) will not effect the cycle connectivity of the cycle $p_1 p_2 p_3 p_1$, since $C^G(p_1, p_2) = C^G(p_1, p_3) = p_1$. Therefore $CC(G) = p_1$ for precisely weighted graph with $|V| \geq 3$. □

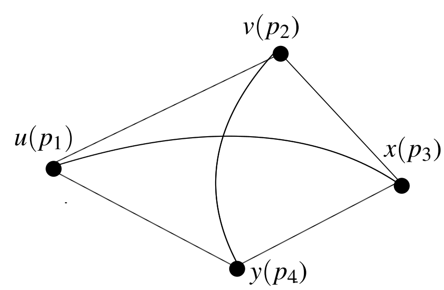


Figure 5: A precisely weighted graph with 4 vertices

Proof. ($2 \Longrightarrow 3$): Let $G : (\delta, \omega)$ be a precisely weighted graph with $CC(G) = p_1$, where p_1 is the weight of u_1,the minimum vertex weight.
All edges except (p_2, p_3) have weight p_1. Then the cycle connectivity of any pair of vertices x, y in G will be p_1 (since $xyuv$ is a cycle). Therefore removal of any of vertex will not effect the cycle connectivity of any other pair of vertices. Hence there does not exist a cyclic cutnode in the weighted graph G. Similarly if an edge of G is removed then there exist another pair of vertices (u,v) with the same cycle connectivity as G. Hence the cycle connectivity of the graph G will not be reduced after removal of any vertex or edge. So the graph has no cyclic cut vertices and cyclic bridges. Hence the graph G is cyclically balanced.

($3 \Longrightarrow 4$) Among $\{u_1, u_2, \cdots, u_n\}$, the set of n vertices of G with $\delta(u_i) = p_1$, for every $i = 1, 2, \cdots, n-2$. Also G has the vertex strength sequence $({p_1}^{n-2}, p_2, p_3)$. Since G has no cyclic cut vertices, removal of any set of r vertices where $r < n-2$ will reduce the cycle connectivity of G. If the first $(n-2)$ vertices are removed from G then a K_2 is obtained, with edge weight is minimum of p_2 and p_3. Therefore G has an $n-2$ cyclic vertex cut.

($4 \Longrightarrow 1$) Let $G : (\delta, \omega)$ be a precisely weighted graph with cyclic vertex cut is $n-2$ vertices (The graph is K_2, which has no strong cycles). That is removal of any single vertex or edge will not reduce the cycle connectivity of the whole graph. Removal of $n-2$ vertices makes the graph a trivial one.
Since all edges except (p_2, p_3) have edge weight p_1. Also for precisely weighted graphs all edges are strong. Then it follows that cycle connectivity of any pair of verices in this graph is the singleton set $\{p_1\}$. Hence G is $\theta - weighted$. □

The next theorem gives a formula for finding the strong connectivity index of a precisely weighted graphs as described as in the above theorem.

Theorem 4.2. *Let $G : (\delta, \omega)$ be a precisely weighted graph with $|V| \geq 3$. Let $\{u_1, u_2, \cdots u_n\}$ be the n vertices of G, where $\delta(u_i) = p_1$, for $i = 1, 2, ..., n-2$, $\delta(u_{n-1}) = p_2$ and $\delta(u_n) = p_3$ where $p_1 < p_2 < p_3$. Then the strong connectivity index of G is, $CI(G) = p_1^3(1 + 2 + \cdots + (n-3)) + (n-2)p_1^2(p_2 + p_3) + p_2^2 p_3$.*

Proof. Suppose $G : (\delta, \omega)$ is a precisely weighted graph with $|V = 3$. Let $\{u_1, u_2, u_3\}$ be the 3 vertices of G, where $\delta(u_1) = p_1$, $\delta(u_2) = p_2$ and $\delta(u_3) = p_3$ with $p_1 < p_2 < p_3$. Then the strong connectivity index, $CI(G) = p_1^2(p_2 + p_3) + p_2^2 p_3$. So the result is true for $|V| = 3$.

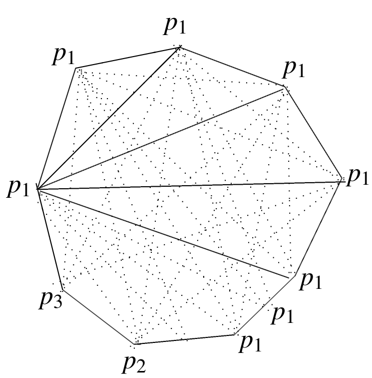


Figure 6: Precisely weighted graph with n vertices

Assume the result is true for $|V| = k$. Let the strong connectivity index of G is, $CI(G_k) = p_1^3(1 + 2 + \cdots + (n-3)) + (n-2)p_1^2(p_2 + p_3) + p_2^2 p_3$.

For $|V| = k+1$, suppose that the $k+1^{th}$ vertex be u_{k+1} with $\delta(u_{k+1}) = p_1$. Since the graph is precisely weighted the new vertex u_{k+1} share adjacent edges with all the other k vertices including $p_2 and p_3$. Then the strong connectivity index of the new graph is,

$CI(G_{k+1}) = p_1^3(1 + 2 + \cdots + (n-3)) + (n-2)p_1^2(p_2 + p_3) + p_2^2 p_3 + p_1^3 + p_1^2 p_2 + p_1^2 p_3$

$CI(G_{k+1}) = p_1^3(1 + 2 + \cdots + (n-3+1)) + (n-2+1)p_1^2(p_2 + p_3) + p_2^2 p_3.$

$= p_1^3(1 + 2 + \cdots + (n-2)) + (n-1)p_1^2(p_2 + p_3) + p_2^2 p_3$

□

Theorem 4.3. *Let $G : (\delta, \omega)$ be a precisely weighted path graph, P_n with vertex set $\{u_1, u_2, \cdots, u_n\}$. $\delta(u_i) = e_i$ for $i = 1, 2, ..., n$ where $e_1 < e_2 \cdots < e_n$. Then the strong connectivity index of path graph P_n is,*

$$CI(P_n) = e_1^2(e_2 + e_3 + \cdots + e_n) + e_2^2(e_3 + e_4 + \cdots + e_n) + e_{n-1}^2 e_n$$

Figure 7: A precisely weighted path graph with n vertices

Proof. Let $G : (\delta, \omega)$ be a totally weighted graph with $|V| = n$. We can prove the result using induction method. If $k = 2$. Let P_2 be a path graph from u to v of length 1 with weight e_1. Then
$CI(P_2) = e_1^2 e_2$

Assume that the result is true for $k = n$. That is
$CI(P_n) = e_1^2(e_2 + e_3 + \cdots + e_n) + e_2^2(e_3 + e_4 + \cdots + e_n) + e_{n-1}^2 e_n$

Then for $k = n+1$, only one edge is added to the graph G_4. Then
$CI(P_{n+1}) = e_1^2(e_2 + e_3 + \cdots + e_n) + e_2^2(e_3 + e_4 + \cdots + e_n) + e_{n-1}^2 e_n + e_1^2 e_{n+1} + e_2^2 e_{n+1} + \cdots + e_n^2 e_{n+1}$

That is,
$CI(P_{n+1}) = e_1^2(e_2 + e_3 + \cdots + e_n + e_{n+1}) + e_2^2(e_3 + e_4 + \cdots + e_n + e_{n+1}) + \cdots + e_n^2 e_{n+1}$

Hence the proof. □

5. Conclusion

New connectivity parameters for totally weighted graphs are introduced. A characterization for partial trees using strong connectivity index is provided. Using vertex strength sequences, strong connectivity indices of precisely weighted graphs and path graphs are obtained. It is proved that acyclic level and cycle connectivity of precisely weighted graphs are equal.

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